M A Mackenzie
University of Toronto

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JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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THE INSTITUTE OF ACTUARIES.

The New Mortality Experience.

AT the First Ordinary General Meeting for the Session 1893–4, held on 27 November 1893, Mr. AUGUSTUS HENDRIKS (the President) said he had to announce the appointment of Mr. S. L. Loney as Examiner for Part I. Mr. Loney was Third Wrangler at Cambridge in 1882. He need hardly refer to his other qualifications, or say that he was in every way competent to undertake the duties that they trusted he would successfully perform. In past years it had been customary for the President to deliver an address at the first of each of the sessional meetings during the two years of his term of office. That custom had, he thought, been very wisely abandoned by his predecessor in office, and in opening the present meeting he desired only to refer very briefly to one subject. He would speak of a matter which had been occupying the attention of all the members of this Institute, as well as the body of actuaries in Scotland—the compilation of a table of mortality based upon the experience which had occurred during the 30 years which had elapsed since the last tables were formed by the Institute of Actuaries. This Institute and the Faculty of Actuaries in Scotland had respectively appointed committees, and those two committees had

been incessantly labouring in the cause during the eight or nine months since the subject was first started. Communications of a very friendly character had been exchanged, and they had unanimously come to conclusions on some points, and also upon the chief point which at present interested them all—the issue of a circular emanating from the Institute of Actuaries and the Faculty of Actuaries in Scotland to the insurance companies, calling upon them each and severally to contribute the results of their mortality. He had every reason to anticipate that that circular would be replied to and met by the directors of the several companies in the spirit in which it was proposed. He trusted the directors, officials, and last, but not least, the insurance press, would do their utmost to bring the matter to the attention of all those who were interested in life assurance; so that at no very distant date they might bring the labours which they had undertaken for the compilation of tables of mortality made up to the present time to a successful conclusion. He then called upon Mr. Hardy to read the paper which follows.

An enquiry into the methods of representing and giving effect to the Experience of a Friendly Society: with some account of the Hearts of Oak Benefit Society, and its Experience for the years 1884-91. *By* RALPH PRICE HARDY, *one of the Vice-Presidents of the Institute of Actuaries; Actuary to the United Kingdom Temperance and General Provident Institution.*

[Read before the Institute, 27 November 1893.]

1. CONSIDERING the important part played in the social system by the various Friendly Societies of the Empire, both as fosterers of the inestimable principle of self-reliance and as trainers in the art of practical government, it appears to me strange that hardly anything respecting their aims, or their working or experience, has been brought before this Institute.

2. It cannot be contended that the nature and sufficiency of the provision made by so large a number of our fellow Citizens (mostly of the class to whom small sums are of importance) for the day of Sickness, for their decent burial, and for the other inevitable accidents of life—a provision intended, and passionately clung to, to secure them and their families from the degradation of Parish relief—is not a subject worthy of the passing attention of our members.

Nor, ought we, who are daily preaching the doctrine of Thrift and urging broadcast the necessity of the abnegation of the day for the imperative needs of the morrow, to look patronizingly, or even unconcerned, upon these humble endeavours to realize those manifold responsibilities that ever attend the life of Labour—those precursors of larger efforts that will follow a social rise and increased opportunities. May we not consider, that it is from the ranks of the sons of those that now make such sacrifices, often so disproportionate to their incomes, in recognition of their duties and in reliance upon that principle of combination that lies at the root of all Life Assurance, many of our future customers will come, fully educated to listen to our solicitations?

3. Further, such of us as lay even a moderate claim to be serious political students cannot ignore the abundant evidence of the advance in the art of self-government, and that healthy growing distrust of the bureaucratic system of meddling and regulation so rife upon the Continent, which is mainly due to the experience had in the administration of the local Friendly Societies, and especially of the great Centralized Societies and the Affiliated Orders.

It is, fortunately, becoming increasingly difficult for ignorant agitators (to use the words of Hooker) "to persuade a multitude that they are not so well governed as they ought to be": because, so many, daily growing in numbers and influence and enlightened by handling the minute, but varied, affairs of the Friendly Societies, now understand "the secret lets and difficulties, which in public proceedings are innumerable and inevitable." If this be thought to claim a perfection in political advancement that circumstances do not justify, I still say that the light has broken in upon the dark places and that, though it may not yet be noon, the sun is rising high in the heavens, to the improved security of all sustainable rights in Property and to the steadying of all those portions of our institutions that are fit to survive through the growing evolution.

4. Lastly, I would protest (though, in this Meeting, I have no occasion) against the uncalled for ridicule that æsthetic persons, and many that are otherwise, are apt to cast upon the social surroundings of some of the Friendly Societies. To many of us here present, such nice distinctions as that Odd Fellows should congregate in "Lodges", that Foresters should enter "Courts", or Rechabites dwell in "Tents", appear to be unnecessary periphrases. Or, again, that the business head of a

serious organization should sometimes be formally styled "Worthy Grand", or be gravely addressed as "Primo"; or, that those deserving and often hard-worked functionaries should seek to add to a natural dignity that is involved in election to office by their fellows, by a Regalia that violates the Laws of Heraldry and confuses colours in an impossible blend. These appeals to the imagination, too often amidst sordid surroundings, and the air of mystery and romance imported into the proceedings probably form an oasis in the lives of many and help to lift them towards an ideal not possible in their ordinary life. A man clothed in the outward insignia of rank, and treated with the deference that befits his temporary royalty, may truly expand, and even retain on resuming his daily garb and round of monotonous labour some of the more ennobling sentiments that fell naturally to him in his short-lived reign. It is within my own observation that amongst members of Friendly Societies (as in other communities) exalted office, involving both respect and responsibility, does enlarge the man in the highest sense, and does add to the best class of our Citizens.

5. But, I say that, so long as Nobles scramble for Ribands (just as they did at the Court of Lilliput for silken cords), or Aldermen intrigue for titles, or restless women thirst for the pinchbeck honours of a Primrose League "Habitation", or persons continue to believe in Masonry and practice travesties of solemn Rites, just so long should these innocent, but very human, excrescences be tolerated and not be made the subject of an illogical and cheap fun. Leaving to others to defend the general case of Titles and Decorations, they, so far as the Friendly Societies are concerned, have a better claim to respect. That such have arisen (and the user has been long), and continue despite the criticism of the matter-of-fact day, suggests that they are probably based upon ineradicable instincts and point to some deep seated, but unknown and as yet unabsorbed, principle in our Human Nature, evidencing the longing for the outward and visible sign as satisfying some inward impelling emotion—to the universal desire for a common symbol of Collectivity to which Individualism can pay tribute, to that grand and inspiring sentiment—the Solidarity of the Human Race.

Let me add that, should any one think that, because the members of some Friendly Societies appear to set their hearts upon these glittering externals, such, and such alone, are the subjects of their real intensities, he would make as great a

mistake as those did that misjudged the Puritans upon the ground of their distinctive peculiarities. As Macaulay puts it, the emotions being satisfied, the intellectual forces were released for serious efforts—and, it will be found that the most decorated Official, when at the Council Board, remembers his Robe only for the duties it typifies, and deals with the business before him with the coolness and deliberation of a man in a common frock coat.

6. Apart from these considerations, the whole subject possesses special attractions.

The Rules constitute a Literature of their own, and register ideas and standards of conduct current at various stages of our development. Some, in their endeavours to define all rights and provide for all possible cases, are models of a lengthy, but futile, precision; in others, the Puritan cast, the reflection of the strong minds and stern beliefs of their framers, is very apparent. Such mundane matters, as the sufficiency of the rates of Contribution and provision for periodical adjustments of the several pecuniary interests, stand aside for peremptory regulations prescribing the moral demeanour. "Whoever", declared the Rules of a West Country Society, "profanes the most holy name of God, shall be fined—sixpence." Casuists must decide whether the crime and the penalty were in due relation; but, the intention was as obvious as, probably, the necessity.

7. A whole chapter could be written upon the names and circumscribed areas adopted by the old-fashioned Societies, and the romantic and political associations they connoted. Caste and exclusiveness were not confined to Court circles, but appeared naturally in the Tradesmen separating themselves from the Artizans, and these again from the tillers of the soil—while scarcely any would recognize either the needs or the claims of Women. A village would shroud itself in its inaccessibility, and even a town would stand strong against the foreigner; and it was only in later times that the broadening spirit of the age led to the expansion of the political boundary to the greater sweep of the County.

There were, doubtless, some administrative advantages in this system of isolation; but, while it fostered the spirit of brotherhood, it left the composing units miserably weak and open to a disintegration that Time and changed circumstances have started and are rapidly completing. It was only some months ago, that I was consulted by a despairing Committee to know what could be done with the remanet of a body, composed

chiefly of old men, upon whose visible resources each year made serious demands. There was nothing, that I could see, left for these unhappy men but the Workhouse—that scarcely blessed haven of retreat, so different from what their long cherished expectations had pictured as the protector of their Old Age—and that was evidently distressingly near at hand.

No doubt the Managers of this and other like Societies talked much in the same way as the paid advocates of the outrageous fraud of Assessmentism (or, of its cunningly devised equivalent, the so-called "Natural-Premium" system) now do, urging the same ignorant and dishonest arguments against the sound Actuarial view; and, the fate of bitter disappointment and bankruptcy that has deservedly befallen many of these Societies will, in like manner, repeat itself in the case of those Assurance Companies that are now enveigling the Public into their rotten schemes. The Press of the Country should warn the Public against these cleverly concocted, but utterly false schemes, now being sedulously propagated by certain recently established American and British Companies.

8. A fair sized general village Society, of the old-fashioned type, was a microcosm, and reflected proportionately the order in which it was centered. A sprinkling of contributors from the yeoman and farming ranks joined as a matter of good feeling, but never claimed the Benefits. These classes, at a later stage, became elevated into Honorary Members, or were dignified as Patrons; but, the real link was broken by these changes. There would be a section that never claimed upon the Fund, that representative group of the backbone of all social aggregation: the shiftless, coming on and off the Fund, would write their histories there as clearly as their mismanagement of life had upon their affairs: the recurring rheumatic would reappear every autumn: and, that dead-weight, the permanently crippled or maimed member would, while "on the box", be allowed a very free interpretation of the Rules against work.

9. I have placed many of these Societies under the microscope, and very curious are the revelations they afford. In one case, I was informed, with a severity born of confidence, that I had made a mistake in the estimate of liability, that the Society could not be insolvent as it had been going for 150 years! Without admitting the "reason", the case was one that fairly called for enquiry, when it turned out that the history of the Society was practically as before referred to, namely, for nearly up to recent

years, a partly charitable association, largely supported by the contributions of non-claiming members.

In another, I noticed that the finally reduced pay was no aliquot part of the full allowance, and for some time my enquiries into the cause of this were met by a diplomatic evasion. But, I insisted upon knowing what was behind, and at last learned that the rate had been specially fixed at sixpence below "weeding-price"—so as to force the claimant to go to work, as they put it, "for the sake of his tobacco." I will not discuss the morals of this piece of state-craft.

Again: once, I tracked an excess of Sickness to the single case of a man that invariably took a month upon the sick-list, in order to recruit his energies, dissipated, not by work, but by the yearly indulgence he gave himself with his harvest-money. It was, in fact, his long vacation, and was acquiesced in as an institution.

10. But, while Societies of the types mainly above referred to are passing away, there still remain a sufficient number of them with others of a like, but somewhat improved, class that have sprung up within modern periods, to make the question of their Valuation treatment a subject of both anxiety and difficulty to the Actuary that approaches the problem with a proper sense of responsibility.

Speaking from my own experience, I cannot recall a case (save that of the Hearts of Oak), where my recommendations for adjustment were accepted—but, I do not consider that any such possible rejection would justify us in not endeavouring to find and show grounds for the conclusions come to, although such conclusions, to an expert, may be obvious. It is a serious matter to pass sentence of death upon a Society, that may be a centre of other unimpeachable movements, and I hold that sufficient reasons for the opinion arrived at should be put on record. To the ordinary rural mind, a Valuation Balance Sheet is quite incomprehensible, and a "present value" conveys no meaning that can be used as a practical standard of measurement. However, I have found that the expectation of sickness during the period under examination, when contrasted with the outcome at the same group of ages, is a relation that can be grasped, as one that has both prevailed in the past and may rule in the future. This comparison can be easily produced by an approximate method. Hence, if this expectation be taken out, and if it be explained that the Valuation is made on similar principles, and provides for the future like

payments with the interest taken off, the true position of the Society may be realized by a few of the best minds. But, even when this has been done, the difficulty of carrying any adjustment is almost insurmountable: the management is chiefly composed of the elder members, whose interests would the more largely suffer in a reconstruction, and a majority of such are sure to oppose any reduction that would touch their vested rights. Very rigid management, backed by an unsparing enforcement of the letter of the Rules, may prolong the life of many of these Societies, but, the real financial interests of the young and middle aged members can be met only by a Dissolution.

I do not think it necessary to make any further exhibition of the experience.

11. I have seen Valuations of these, and even larger Societies, where no attempt has been made to test the experience, or otherwise to ascertain whether the Tables employed are a sufficient measure of the probable outcome, and where the liability was estimated by the Manchester Unity Life Sickness Tables, full effect being given to the reduction of pay allowed by the Rules, with interest at 4 per-cent, a rate that could not be made upon the funds.

12. In the case of larger Societies, and especially where they are expanding, I submit that the experience should be fully investigated, and upon lines similar to those hereinafter described. Generally, the mischief (if any) has not gone too far for remedy, and an improvement may be judiciously commenced by revising the scale of Contributions, followed by endeavouring to have the Sickness-for-life Benefit converted into a Sickness allowance ceasing at some reasonable age, with a fixed Life-pension then to follow. A Benefit in this latter shape can be valued more closely, as there are no existing trustworthy Tables of Life-sickness.

13. I make no special reference to the Affiliated Orders, as their Managing bodies (at least, of the more important) are fully alive to these points, and—so far as the Odd Fellows are concerned—possess their own Actuarial Staff: consequently, the cases of this class of Society are not likely to come before us.

But, with respect to that important Centralized Society, the Hearts of Oak, to which I shall refer at some length, a proper and frequently repeated investigation into the working, and an exhibition of its real position, are of vital importance, regard being had not only to the numerous interests involved but also to the

fact that it is a single Society upon a vast scale, and not a federation of a mass of small groups each of financial independence.

I will, therefore, give some account of this great Society, and will detail the anxious care taken by the Management to watch and test its movement, to educate the general body in the first principles of financial soundness, and to place and maintain the Society in a position of Security commensurate with its claim as a representative Provident Association of the expanding Working Classes.

14. The Hearts of Oak Benefit Society was established on 20 June 1842, and was, it is believed, an offshoot from the "Royal Standard", a still existing body. As the careers of these two Societies have been widely different, and, in the case of the younger, with important achieved social results, pointing to an equally important growing influence in the future, it may not be out of place to advert briefly to some of the circumstances by which, it would appear, the Hearts of Oak has been fortunately affected.

15. By general consent, the first marked departure from the ordinary administration of a Friendly Society was taken by the appointment of the late Mr. Thos. Marshall, as Secretary, on 29 July 1865, the number of members then being 10,171 and the realized assets £35,414. 14*s*. 5*d*. This gentleman, who was a nephew of the well-known Mr. Matthew Marshall, of the Bank of England, was an administrator of no common order, single-eyed in his devotion to the laborious duties of his office, generally at his post from 10 o'clock in the morning to 10 o'clock at night. No detail was too insignificant for his personal attention: no principle affecting the basis upon which the finances rested was slothfully slurred over as too difficult for his attempting, and usually succeeding in, its mastery. As a consequence, the management became distinctly superior to that of any similar body; and, slowly but surely, the Society emerged from the obscurity of a mere Club to its well deserved position as a leading Working-class organization.

A rigorous and upright administration of this class cleared the air of all "jobbery", and attracted the support of honest and capable men in sympathy with such principles, who, in their turn, aided in the general development.

Mr. Marshall died on 1 March 1891, and was succeeded by Mr. T. W. Galloway, C.A. (Edin.), the present Secretary.

16. The second contributing cause may fairly be attributed to the "Friendly Societies Act 1875."

About the time of the passing of this Act, the realized assets of the Society stood at about £200,000: the largeness of this amount in the eyes of some of the members had induced a demand for an increase in the Benefits, it being erroneously supposed, by a section, that the accumulated fund represented, not Reserve, but monies put aside by way of extra caution. Since the provisions of the above Act prescribed a Valuation of the assets and liabilities at some time within the coming quinquennium, those members of the Management that understood the finance availed themselves of the opportunity and succeeded in getting the question postponed until the results of an immediate Valuation for 31 December 1876 were reported. This Valuation was entrusted to me, and I stipulated that, in order to secure a sound basis, the experience of the Society should be investigated for the period 1871–6. This was accordingly done, when it was found that instead of the resources permitting of the further proposed burthen, an increase upon the Contributions of 2*d.* per month was required in order to make the Society solvent. Amongst other important conclusions, it was ascertained (as confirmed by subsequent investigation) that, owing to the liberty of doing a limited amount of work whilst upon Superannuation, the Reserves for that Benefit had to be doubled upon the Manchester Annuity expectation.

It was at that time that I first fully appreciated the sterling qualities of the late Mr. Marshall, who insisted (not upon following the arithmetical details) upon understanding the meaning of the processes employed, their object, and how the results would be employed as modifications, or confirmations, of the standards proposed to be employed in the Valuation. I believe that the thorough grasp Mr. Marshall secured over the principles, (coupled, I venture to trust, by the various explanatory memoranda prepared by me) enabled him to bring the real issues, and their vital importance to the true interests of the members, before the Management, who succeeded in inducing the general body to accept my recommendation of an increased Contribution and an abandonment of the desired enhancement of the Benefits.

It is no exaggeration to say that this step saved the Society, as will now be fully admitted.

I have always considered this culmination to have been the first fruits of the Act of 1875, and probably the most important result directly achieved by that piece of legislation.

17. In Table XV, will be found an abstract of the income and outgo of the Society for each of the eight years 1884–91, and for the consolidated period.

The following Summary shows the amount of Benefits paid to the members, both from the commencement and for the period under observation.

Summary of Benefits paid to Members.

	1842–83	1884–91	1842–91
Sickness and Superannuation .	933,443	1,008,762	1,942,205
Wives' Lyings-in	362,625	253,622	616,247
Funerals (Members and Wives) .	230,638	216,737	447,375
Fire Losses	14,596	10,742	25,338
Imprisonment for Debt . . .	80	61	141
Interests purchased up . . .	4,627	8,622	13,249
£	1,546,009	1,498,546	3,044,555

18. The statement below shows the movement in the membership, with the average age of the whole body at the end of each of the eight years, and the total accumulated funds at each point, irrespective of the sundry credits brought to account at each Valuation.

The assets, which are set out in detail in the printed yearly accounts, are chiefly loans to Counties and Boroughs, under Parliamentary authority, and Freehold Ground Rents: and their own Freehold House, in Charlotte Street, Fitzroy Square.

Summary of movement 1884–91.

Year	No. of Members at beginning of the year	New Entrants in the year	Left in the Year		Adjustment upon recount	Average age at end of year	Assurance Fund at end of the year
			By Death	By other Causes			
1884	102,239	7,809	859	3,567	...	34·58	696,206
1885	105,622	7,296	874	2,970	...	34·98	766,350
1886	109,074	6,899	849	3,187	...	35·44	832,232
1887	111,937	7,541	891	2,784	...	35·80	907,489
1888	115,803	11,388	934	3,034	...	35·80	989,371
1889	123,223	12,104	987	3,283	...	35·83	1,081,245
1890	131,057	14,755	1,124	3,228	191	35·70	1,157,518
1891	141,269	17,210	1,335	3,549	...	35·48	1,236,111
1892	153,595	...	...	...	...	...	...
	...	85,002	7,853	25,602	...	...	...

The number of members at the commencement of the observations (1 January 1884) and the number at the close (31 December 1891) namely 102,239 and 153,595 respectively, distributed at the several quinquennial groups of ages as under:

Ages	1 Jan. 1884	31 Dec. 1891	Ages	1 Jan. 1884	31 Dec. 1891
-19	156	665	Brought over	97,771	140,827
20-24	7,552	15,913	50-54	2,248	7,665
25-29	22,098	30,734	55-59	1,286	2,691
30-34	29,434	32,014	60-64	644	1,372
35-39	20,655	25,457	65-69	239	727
40-44	13,135	20,863	70-74	46	250
45-49	4,741	15,181	75-79	5	61
			80-	...	2
Carried over	97,771	140,827	All Ages	102,239	153,595

19. *Constitution.*—The Society is a registered Friendly Society, and is governed by 200 Delegates, who are annually elected by universal suffrage, and who hold annual or more frequent Meetings as business may necessitate. Their sessions are held in the Society's own Freehold House and are conducted with due formality and with order and decorum, the business being taken from a detailed printed Agenda, which is strictly adhered to. There are also Committees, some of which meet nightly to admit applicants, for dealing with the ordinary questions arising out of the current administration, and there is a medical officer in attendance every evening (except Saturday) for the purpose of examining candidates for admission. Moreover, there is a permanent Court of Arbitration, composed of public men not belonging to the Society, for the hearing of appeals of members against decisions of the Management and for generally safeguarding the rights of all. This latter provision, admirable in my view, renders justice speedily accessible to the humblest member, and practically without cost.

The practice of the Society in the matter of publicity goes beyond the requirements of the Friendly Societies Acts, since, in addition to a long-standing and continuous audit of the accounts by a Public Accountant, to the publication of all accounts in far greater detail than the said Acts prescribe, and to their extensive distribution, for many years past there has been a yearly examination into, and a full Report upon, the Experience of the Society in respect of the Sickness suffered, the Deaths, and the Secessions, accompanied

by an Actuarial Valuation of the assets and liabilities—and these Reports have been printed and circulated throughout the whole body. One effect of this thorough-going enlightened publicity and the discussion consequent thereon, has been the creation of a standard of intelligence amongst the governing body far beyond that elsewhere prevalent, to the protection of the less advanced members, and the furtherance of the cause of soundly based Provident Association—while the outside public are rapidly responding to the appeals made to their sober reason and not directed to those elementary instincts that fade before an advancing Civilization. Speaking from a long and intimate contact with the leading members of the Society and from a somewhat extensive practice in questions affecting the finance of Provident Associations, I am satisfied that this generous policy of fearless Publicity, of constantly testing prognostications against the issue in facts, of insisting upon knowing the Truth, whether palatable or not, is the grand foundation of all Safety. And, I go further and say that it is the imperative duty of all (whatever functions they may exercise) that dare to meddle with the savings of the People, to spare no efforts to place and keep this Safety beyond reasonable dispute, to deal unresentingly with internal or even outside criticism, however ignorantly based, and to await patiently that general enlightenment that is sure to emerge and finally recognise the permanent advance so secured.

20. *Membership.*—Membership is restricted to males, above the age of 18 and under that of 30, of medically certified health, and not being afflicted with lameness, defect of sight, or other bodily infirmity. A declaration is required that the Wife is in good health; but, if she be otherwise, the applicant may be admitted, subject to a waiver of the Funeral money on her account. The entrance fee is two shillings and sixpence. The limits of age and the amounts of the entrance fees have varied from time to time.

No disqualification now exists by reason of a limit of Wages, or on account of the trade, or other occupation, followed by the proposed member, either at the time of his application and admission, or thence after; but, up to the year 1887 there was a Wages limit of 24*s.*, and persons engaged as under were ineligible for election, entrance upon any of the scheduled employments bringing about exclusion, namely:

"Drug or colour grinder; water gilder; worker in white or red lead or quicksilver; gunpowder, firework, or lucifer match maker; typefounder; miner; puddler; sugar baker; cigar maker; brass founder; game-keeper; police constable; soldier; sailor; brewer's drayman; grinder in dry cutlery; baker; miller; cement maker; or any other occupation which the Committee may conceive dangerous or injurious to health."

As a fact, exclusion was not practically enforced for the assumption of any of the prohibited trades, but was confined to non-payment of the Contributions and Levies, fraudulent claims upon the Society, and Felony generally.

It may be of interest to state that, on the average, about 500 changes of address are received every day, occupying the entire time of 4 clerks to carry the necessary alterations through the Books. The general correspondence shows an average daily receipt of about 4,000 letters: and the letters despatched amount to over 700,000 per annum, in addition to the Money Orders transmitted.

21. *Contributions.*—The Contributions are uniform for all ages at entry and attained, namely:

Sickness and Superannuation: fixed at	7*s.*	per quarter.
All Other Benefits: levy per member, now averaging	3*s.*	do.
Total present Contribution ...	10*s.*	do.

All Contributions must be made direct to the Society, and are not subject to any commission, allowance for postage, or other charge for remittance—neither can they be paid in part, or set off against accruing Benefits.

Superannuated members, of whatever age, are exempt from Contributions.

There are Fines for non-payment of the Contributions and Levies—and, as will be seen from the appended abstract of accounts, these yield a considerable sum in the course of the year; and, while they used to defray nearly all the Expenses of management, they still afford a substantial aid on that account.

About 12,000 Contributions are received at the counter every week, and nearly half-a-million Postal Notes and Orders in the course of the year.

22. *Benefits.*—The Benefits allowed are as under—to members not in full benefit, and in some special cases hereinafter stated, slightly reduced allowances are paid:

1. Sickness	First 6 months	18/- weekly	Provided by the fixed Contributions and the existing Assurance Fund
	Second do.	9/- „	
2. Superannuation	Remainder of disability *Earnings up to 12/- weekly allowed to be made during receipt of Superannuation — Contributions also remitted*	4/- „	
3. Funeral	Member	£20	Provided by general Levy per Member, to meet the emerged claims and no more
	Wife	£10	
4. Wife's Lying-in	Each Confinement	£1. 10 -	
5. Fire	Loss of Tools and Implements of Trade	£15	
6. Imprisonment for Debt	For 3 months	5/- weekly	
7. Drawn for Militia	As price of substitute	£5	
8. Temporary Assistance	Remission of Contributions for a limited period during distress	...	Provided by special Levy
9. Convalescent Homes	Assistance to promote recovery	...	

23. A member is not entitled to the full Sickness Benefit for the first 12 months, unless he purchases immediate freedom at entry, which, since the year 1888, he can do for 5*s.* In the absence of such purchase, the following scale applies for the first 12 months.

Member first ill when under 3 months standing	2*s.* 6*d.* weekly	with proportionate reduction when on half-pay.
3 and under 6 „ „	6*s.* 0*d.* „	
6 „ 9 „ „	9*s.* 0*d.* „	
9 „ 12 „ „	12*s.* 0*d.* „	

In the appended Tables of the Society's experience, those cases where freedom was bought are not distinguished; but, a brief account will be subsequently found of a recent enquiry into the financial effect of these purchases.

A medical certificate showing the nature of the Sickness (certain causes of illness disqualifying for benefit) has to be furnished in support of a declaration upon the Fund, and the same has to be renewed every 14 days; and the sick man (unless in Hospital) has to submit to visitation by a rota of members living in his neighbourhood. There are also regulations intended to secure that the claimant is having proper medical treatment, and that the disability is genuine—and, further, there are provisions for checking declarations off the Fund before complete recovery, which otherwise might be attempted so as to avoid coming under the reduced scale of allowances.

24. Table No. XIV, appended hereto, shows the number of declarations on and off the Fund for every month in each of the years 1884-91, from which may be gathered some idea of the heavy money work involved in disposing of Sickness claims, and especially since it is in only a very few cases that a single payment represents the settlement. When it is remembered that in addition, there are the reports of the Medical attendants and those of the Sick-visitors to receive and consider, the payments to post to each separate account, and the duration of the illness to be watched so that only the proper rate of allowance is paid, it will be seen that the administration of a Society with a large membership involves very important inside work in merely the Claim Department, none of which will brook delay.

Sickness and Superannuation allowances are paid, at the Society's House, at the close of each week, to the authorised representatives of the member, or are sent by post at the cost of the claimant. About 4,500 Post Office orders are obtained every week to discharge these and the Superannuation claims, involving an additional Post Office Staff of 15, from 6—8.30 p.m. on Thursday and all day Friday to 6 o'clock p.m. In addition, from 1,200 to 1,500 are settled with over the counter. During the Influenza epidemic of 1890, the claims rose to so high a number, that relays of clerks, working night and day, were required to keep down the work.

25. Superannuation allowance, which in all cases carries a remission of all Contributions, is at the following rates, namely:

Under 6 years' standing	2*s*. weekly.
6 years and under 8 ,, ,,	3*s*. ,,
8 years and upwards ,,	4*s*. ,,

Superannuated members are not entitled to any other Benefit than the weekly allowance as above, and the Funeral monies as allowed in that case.

The special provision in the Rules, permitting a member to do a limited amount of work whilst in receipt of this allowance (provided the earnings do not exceed 12*s*. weekly) should be borne in mind, when contrasting the outcome of the Society's experience of this particular Benefit with that of other bodies, where the right to the allowance is guarded by enforced abstinence from all remunerative labour.

26. A member is not free to Funeral benefit until of 6 months standing, and then for the next 3 months the allowance is £6, during the following 3 months £9, rising to the fixed sum of £20

after 12 months membership. The Wife's Funeral benefit is one-half that of her husband—except where the member has been superannuated for 2 years, when her allowance is restricted to £5.

Claims in respect of Second-taken Wives are not recognized, except that such marriage has endured for 3 years, and has been registered (the charge for which is one guinea) for 12 months. The Rules do not appear to provide for cases of divorce.

27. The "Lying-in" Benefit attaches to the Wives of only free members, and does not cover the case of miscarriage, or of still-birth, unless of a child of at least seven months. The Wife must, if second-taken, be qualified and registered as provided for in the case of the Wives' Funeral allowance. These claims average about 600 per week, and involve the despatch of about 400 Post Office Orders in settlement.

28. The "Fire Insurance" Benefit does not call for any special observation; but, amongst the curiosities on the subject, may be mentioned that a magisterial decision decided that the dress-coat of a Waiter was held to be a Tool of Trade.

29. The "Imprisonment for Debt" Benefit is not now of any financial weight; and that of "Drawn for the Militia" can hardly be made the subject of claim in these days.

30. The "Temporary Assistance" Benefit, which has only recently been passed, is derived under Rule 59, which is as follows:

"Any member of one year's standing and upwards, who is unable, from want of employment or other distressing circumstances, to pay his current contributions to the Society, may lay his case before the Secretary, accompanied by suitable proofs of its genuineness; and all such cases shall be laid before any sectional Sub-Committee, who shall be empowered, at their discretion, to remit the payments due from such distressed member to any amount not exceeding one quarter's Contribution. No member to be thus assisted oftener than once in twelve months, nor more than six times during the entire period of his membership."

The commendable altruistic feeling that has prompted the members to take upon themselves collectively these burthens that weigh down so heavily their weaker brethren in some of their desperate struggles for life, should be noted as constituting a distinct advance over the mere business element in this and like Societies.

31. The "Convalescent Homes" Benefit, in like manner, is

another example of the altruistic sentiment, perhaps even extended beyond the object of the last referred to Benefit.

I do not consider that the possibilities of like timely help to the unfortunate few, to be rendered by the more fortunate many, are exhausted by these two last named Benefits.

32. The movement in respect of the entries, deaths, secessions and numbers sick are shown in Table I for the same quinquennial groups of ages for each of the quadriennial periods 1884-87, 1888-91, and for the combined period 1884-91. Thus, in the combined period, 940,224 lives attained the ages in question, 85,002 entered within the period, 7,853 died, and 25,602 passed away by secession: 230,181 sick members (not necessarily different lives) drew Full pay, 8,969 Half pay, and 8,560 Superannuation allowance—the total claimants within the period being 238,787, some of which latter received each class of pay.

In Table II the same details are given, but for each year of the period separately.

33. In Table III will be found the numbers that became exposed to the risk of Sickness, and number of weeks of expected claim according to the Manchester Unity standard (1866-70, R. T. and C. combined), and the number of weeks actually suffered—shown for each of the quadriennial periods and for the eight years combined. The mean numbers at risk were derived by allowing half a year's exposure for each new entrant, and a like period for those dying or seceding.

Thus, for the combined period, the mean number exposed to risk were 965,987, and their expectation of Sickness during the first 6 months of membership, aggregating to 848,066 weeks, was met by an actual demand of 1,014,164 weeks; and so on.

In Table IV, the same facts are set out for each separate year of the combined period.

From the raw materials shown in Tables I to IV, Tables V, VI, VII, and VIII were prepared.

34. Table V shows, in a summary form, the main facts of the Sickness experience, and for each quadriennial and for the combined period.

Actual Sickness per member exposed to risk.—The ratios here shown were obtained by dividing the quantity of actual Sickness (Table III) of the group of ages by the corresponding numbers exposed to risk. The rates deduced may be compared with those according to the "expectation" basis, shown in another division of the same Table.

Actual Sickness per member Sick.—In like manner, these ratios were found by dividing the last mentioned quantity of actual Sickness by the number of claimants under their respective rates of pay. There is no general standard by which these results can be tested.

Expected Sickness per member exposed to risk.—These are the Manchester Unity rates, obtained by operating with the numbers actually at risk. They serve as the standard for testing the actual outcome of the working.

Numbers Sick out of each 100 *exposed to risk.*—These results were obtained by dividing the number of each class of Claimant by the mean numbers exposed to risk. Again, no general standard exists for testing these ratios.

The fluctuations in each separate year will be seen by referring to Tables VI, VII, and VIII.

By help of the foregoing, any marked change in the character of the Sickness-claims is brought to light and can probably be traced to its source. Thus, should the proportion of the numbers exposed that become sick remain steady, in the face of a general rise in the observed rate, the change will probably disclose itself in the greater average duration of the illness: or, again, it may be found that the excess of sickness was due to a larger proportion of claimants for short periods. When this latter occurs, speaking generally, it suggests a want of sufficient supervision, and perhaps the growth of a habit of resorting to the Fund upon scarcely warrantable grounds.

In considering these results, it should be remembered that the financial year 1886 comtained 53 weeks; and that the years 1890–1 were those of the Influenza epidemic.

I knew the case of an old Society that had to be broken up, owing to the irreclaimable practice of certain members declaring upon the Fund for a few days prior to the falling due of the quarterly Contributions, in order to provide for their periodical payments.

35. Tables IX and X exhibit the money-effect upon the finances due to the deflections of the Sickness experience from the provision made by the periodical Valuations for the several payments, the former for the grouped periods and the latter for each separate year—this provision being, in respect of the Sickness claims, the Manchester Unity rates for the full and half payments respectively, and double the same tabular rate for the Superannuation allowances (with adjustment for loss of

Contributions), which latter, under the special Rule of the Society, may be claimed during only partial disability.

The general outcome of the working against the Valuation-provision made in that behalf was as undershown:

Summary of financial outcome compared with the Valuation-provision.

Year	Money loss per-cent upon the Valuation-provision	Total actual Sickness and Super-annuation payments	Total Sickness and Super-annuation payments provided by the Valuation	Loss upon the estimates	Remarks
		£	£	£	
1884	14·13	102,083	89,444	12,639	
1885	15·12	107,988	93,809	14,179	
1886	20·32	118,155	98,209	19,946	
1887	13·41	116,715	102,924	13,791	
1888	13·48	123,903	109,182	14,721	
1889	7·91	126,198	116,950	9,248	
1890	25·40	158,017	126,003	32,014	Influenza epidemic
1891	25·65	171,887	136,799	35,088	Do.

36. Table XI gives the numbers that became exposed to the risk of Secession and of Death, with the numbers thereout so passing away, actual and expected, with the rates of such exits. The expectation was based upon the Society's own experience as investigated for the period 1871-76; that for the Deaths being adjusted merely for working purposes, but that for the Secessions abated one-third for margin.

The combined flux, or rate of leaving from all causes, is also shown, and should be compared with the like expected rate, which has been the measure of the relief afforded by the periodical Valuation-estimates. These results are given for the usual quinquennial groups of ages, and for each of the quadriennial periods, and for the combined period 1884-91.

Tables XII and XIII show the Secessions and Deaths for each of the separate years 1884–91.

37. The results of the several yearly Valuations are set out in Table XVI.

The basis of Valuation was as under:

1. Interest at 4 per-cent.
2. Manchester Unity Sickness rate for the Full and Half Payments, at the allowances shown.

3. (1) Double the Manchester Unity rate for the Superannuation Pay, with 1*s.* weekly added for loss of Contributions: treated as an effective 9*s.* weekly up to age 80.
   (2) A Pension of 4*s.* weekly, from and after age 80.
4. Society's own death-rate (1871–76).
5. Two-thirds of Society's own Secession rate (1871–76).
6. Negative values thrown out.
7. All Contributions to cease at age 80.
8. No direct provision for Expenses, the fines and sundry profits having been taken as a sufficient source.

38. It may be enquired why, in face of the losses upon the Sickness expectation as shown in paragraph 34, the annual Surplus still showed a moderate growth. The reason is mainly to be found in the collection of the "negative values" excluded from the Valuation-assets, and perhaps in some additional relief by the Secessions beyond the estimate.

The difficulties in exactly tracing the several items of profit and loss are very great, where the facts to be sifted exist in such enormous numbers. I never yet succeeded in quite satisfying myself as to the complete incidence of each variation from the expectation. But, since the machinery for record is being improved, I expect shortly to be in a position to ear-mark the profit and loss with a precision equal to that attained by the best Assurance Offices.

39. The results of some special investigations into specific points in the working are not without interest.

In 1890, the question was raised as to the effectiveness of the medical examinations made in the Country, and the single "policy-year" risk and outcome was taken for a representative section, and investigated with the following results:

	Number	Expected Sickness	Actual Sickness
Examined at the Society's House	469	358 weeks	366 weeks
„ in the Country . . .	1,007	770 „	869 „

I do not consider that these figures, drawn from a limited area and representing only a short period, necessarily proved that

the Country examinations were generally defective; but, when we remember how serious a recurring liability is entailed by a Sickness Benefit (where the member can be repeatedly a claimant, whereas for an ordinary assurance the liability can emerge but once, namely, by death), it would appear that great care should be exercised in the admissions, and special regard had to the fitness of a man to bear the strain of his particular trade. The Society's Medical Officer, Dr. J. Beresford Ryley, is specially experienced in these questions, and it is not impossible that the favourable outcome of the London examinations may be due to his trained skill. But, the risk of Country examinations has to be run, and it is for the Mangement to devise means for making them effective.

40. The policy of the removal of the disqualification for membership implied in the Interdicted Trades (*vide,* 19) has occasioned great discussion, and a brief examination was, in 1890, made into the results so far as they affected Miners and Policemen, also for a single "Policy-year." The results for 169 members showed an actual claim of 230 weeks against 127 expected. Again, deeming the surface of observation too limited, I advised that, at present, no sufficient ground was disclosed for retracing the liberal step of unrestricted membership. I take leave to quote the exact words used by me, as indicating the path that, in my judgment, a Society claiming to be National, instead of local, should strive to pursue. I entirely adhere to the views then expressed:

"I see no reason for requiring the exclusion of the Trades " heretofore interdicted, nor, looking to the relief heavy labour " will receive when the pending eight-hours law is passed, and " the improvement in personal health and self-respect that (it " is sincerely trusted) will thereby ensue, for at present excluding " Miners and others from the Benefits of the Society—the present " numbers following this occupation forming but an insignificant " proportion of the whole.

"The legitimate extension of such a Society as the " Hearts of Oak is in a distinctly National direction: it should " aim to embrace all classes of the workers in our varied industries, " and to rest ultimately upon the basis of the entire Population. " To achieve these results, it must act upon broad and com- " prehensive principles, retracing its steps as seldom as possible, " and remembering that it will deal on a large scale, where the " incidence of a small section of the body becomes neutralized by

"that of another group. With a membership so wide-spread, "and with the great advance in Education and in the improving "knowledge of the conditions upon which the solvency of these "Institutions depend, the Management may rest assured that an "appeal to the whole body will meet with adequate support for "improving the practice on any point that may be found to need "amending."

41. But, the question has not (most properly, as I think), been allowed to rest, and a full investigation into the results attending the entries for the years 1888–91 was ordered, to include an enquiry into both the effect of allowing the purchase of immediate freedom (*vide*, 22) and the Sickness attributable to the Interdicted Trades.

The following is a general summary of the results, affecting 55,457 entrants and ranging from ages 17–33, namely:

	Freedom Paid		Freedom not Paid		Total
	Number	Per-cent	Number	Per-cent	
Interdicted Trades (excluding Miners)	787	27·5	2,079	72·5	2,866
Miners	1,511	51·5	1,424	48·5	2,935
	2,298	...	3,503	...	5,801
All other Trades combined	...	...	...	...	49,656
Total New Entrants					55,457

	Interdicted Trades		Miners		Society's other experience
	Freedom Paid	Freedom not Paid	Freedom Paid	Freedom not Paid	
Numbers exposed to risk	1,003	2,624	1,587	1,565	244,913
Actual Sickness (weeks)	1,551	3,136	3,474	2,673	224,911
Expected Sickness (weeks)	755	2,131	1,163	1,275	205,351
Excess of actual Sickness per-cent	105·7	47·2	199·1	109·8	9·5

42. The general conclusions these figures would appear to support are:

1. That there was a tendency for the proportion of the original Interdicted Trades and Miners combined to increase.

2. That the proportion of Miners purchasing immediate freedom was larger than that found in the other combined groups.

It may be further stated that, while for the groups of ages observed, the total excess-strain upon the Society was 11·9 per-cent, which, by excluding the Interdicted Trades and Miners, would have been reduced to 9·5 per-cent only. The difference represented an estimated money-loss of about £1,000 per annum in the four years under observation.

43. I consider that, whether these results were welcome or not, it was very important to know this piece of the inner working, and to learn the necessary lesson. I hold so firmly to the principle of broad trading that (with the exception of the Miners, whose case I feel compelled to abandon), I have advised that, before recalling the Rules as to immediate freedom, and for the admission of all Trades (except Miners), endeavour should be made to improve the medical examinations, by requiring a direct Report as to the bodily fitness of the applicant for the strain of his Trade, and that the results be retested by the light of the new experience.

44. I desire to place these facts on record, so that it may be understood that, while the Management decline to join in the vulgar struggle for a factious increase of mere numbers, they desire that the advantages and matured organization of the Society should be open to all well-conducted workers in our varied industries—but, they will not permit the common interests to be degraded by the membership of persons whose inferior civilization leads them to mismanage their lives and inflict unnecessary burthens upon their fellows.

45. Some interesting features have been elicited by means of an examination that was made into the composing items for which the Levy was raised, *e.g.* for Funerals (members and wives), Lying-in allowances, and the few sundry Benefits.

That, so important a Benefit as that of the Death-monies should be raised by Levy, instead of according to the sounder principle of a graduated level Contribution, is, to me, a matter of great regret; and, I consider that the evils of this system would have become apparent to the members by this time had not the

Lyings-in shown a marked tendency to fall and consequently have counter-met the natural rise in the Funeral demands.

The following Table shows the figures for each of the years 1878–1890:

Year	Levy per Member							
	Total		Funeral Money				Other Benefits	
			Members		Wives			
	s.	d.	s.	d.	s.	d.	s.	d.
1878	11	7	3	2	1	5	7	0
79	12	4	3	3	1	3	7	10
1880	12	2	3	4	1	3	7	7
81	12	2	3	5	1	5	7	4
82	11	10	3	2	1	5	7	3
83	11	11	3	5	1	4	7	2
84	11	10	3	6	1	4	7	0
1885	11	11	3	6	1	6	6	11
86	11	6	3	5	1	6	6	7
87	11	5	3	6	1	5	6	6
88	11	4	3	7	1	5	6	4
89	10	9	3	7	1	4	5	10
1890	10	10	3	9	1	6	5	7

This fall in the Lying-in rate, reckoned over the whole body (young and old, married and single) suggests a check upon the fertility of the members, probably due to artificial economy in reproduction.

46. To show how closely the Management keeps in touch with the movements of the day, the question of utilizing the resources and machinery of the Society, with the view of providing an Old-age Pension for the members, was taken into serious consideration, and the cost of the change and of the other necessary arrangements to that end made the subject of an elaborate investigation.

In brief, taking the financial position as at 31 December 1890, it was found that, in order to convert the present Superannuation allowance of 4*s.* (now treated as a pension from age 80) into a Life-pension, commencing at some earlier age, all Sickness allowances then ceasing, the cost would have been as follows:

Accelerated Pension Age	Additional Cost, viz., required addition to Reserve, or equivalent increase of Contributions
	£
75	45,907
70	280,948
65	902,071
60	2,068,531

This cost made the proposed conversion financially impossible. Alternative proposals were then considered for amending the Rule allowing partial work to be done whilst on Superannuation, so as to bring that benefit to accord with the Manchester Unity experience: but, notwithstanding the eager desire of the members for a settled Old-age allowance, commencing at some practicable point, the real, or supposed, advantages of the Rule were deemed to outweigh those of the Pension, and the attempt in this direction had to be abandoned; and even the formation of a separate Pension-fund was found too costly for general support. When we remember that the members of the Hearts of Oak Society are accustomed to pay a Contribution at the rate of 40*s.* per annum, and therefore do not belong to the very poor classes, and when it is found that even such a body shrinks from imposing upon itself a further fixed annual charge, we can understand with what disfavour such Pension Schemes as those put forward by Mr. Chamberlain have been received by the Working Classes generally, and how little chance there is of the solution of the problem in the direction advocated by that gentleman.

47. The foregoing and the appended Tables exemplify the methods that I have found most useful in exhibiting the experience of a Friendly Society, and which I have used for many years past. To these, there may be added, as occasionally helpful, a proportionate distribution of the total Sickness suffered, showing the percentage claimed under the various rates of pay.

It sometimes happens that the periods for which the different rates of pay are allowed do not correspond with those shown in any of the tabulated experiences generally used for Valuation; in these cases, I have found that the measure of risk may be conveniently found by taking the total observed money cost at each age, and comparing it with the total cost of an unreduced allowance of £1 by the standard, and adjusting the Reserve brought out according to this last in proportion to the former.

I lay no claim to any novelty of treatment, having gathered indications of method from various quarters; but, I have given most suggestions a trial. I have derived most help from the form in which the facts were shown in the Tables prepared by our late respected member, Mr. A. G. Finlaison.

I would refer the Appendix for a method, whereby a Valuation upon any standard basis can be made to correspond exactly with any substituted experience, or with an adjustment upon such standard.

48. It may be asked, what advantage accrues to the Society from these several investigations, and from the yearly Reports upon the working and as to the financial position? I reply, emphatically, that such have been, still are, and will be of the highest possible educational benefit.

They have conclusively shown to the members, by an appeal to their own working and not as a matter of mere abstract theory, that the Sickness rate increases with age; and that, although it may apparently remain fairly constant over the whole body for a period, the unsound assessment principle, viz., that such can be perpetually maintained at a fixed level, by the continual importation of new lives, is opposed to all safe finance; and many of the leading members are now trained to defend the outworks and oppose the disastrous system of increased Benefits without a corresponding addition to the Contributions.

These several Reports form the subject of earnest discussion at the meetings of the Local Associations promoted by the members; and, consequently, the number of centres of light is daily being multiplied.

While their future usefulness must largely depend upon the spirit in which the investigations are made, it cannot be doubted that their influence will be progressively favourable—since, when a body of men have once grasped the proper aspect of viewing these important questions, they will not be put off with counterfeits.

49. I have endeavoured to trace the rise and growth of the Hearts of Oak Society, to show the results of repeated examinations into its working, and unreservedly to lay bare the present estimated financial position. Neither, have I hesitated to disclose the more inner matters, such as the hopes cherished by the members that their resources would supply a sufficient Old-age Pension; but, I have had equally the legitimate pride of recording that, upon

technical enquiry, the disappointment entailed was acquiesced in, and no attempt made to peril the Society. Again, I have not shrunk from admitting that doubts have arisen as to the operation of some of the recently enlarged Rules, nor from showing how far the conclusions arrived at by the investigations specially set on foot to probe those very points have, up to the present, sustained, or disproved, the original views of the framers of such liberal measures.

I can only further say that, such being the enlightened views taken by the Management of their responsibilities to the Public and of the position of trust in which they stand to the members, and such being the evidence of the serious spirit of investigation and test that has manifestly actuated, and still penetrates, the whole body, coupled with the united firm resolve to maintain the integrity of the finances, the Society may be safely left to administer its own affairs without outside interference.

50. Two great schemes for the administration of Provident Organizations that depend upon the operation of the law of average are being worked out before our eyes. One, representing the principle of Federation, is being developed by the Affiliated Orders; the other, involving the larger idea of a Republic, one and indivisible, is being evolved by such Societies of which the Hearts of Oak is the grand type. The former, no doubt, affords considerable play to local feeling, and supplies a vent for local energies; the latter appeals to the broader instincts, rests upon a diffused and National support, and relies upon drawing from its wide electoral areas that talent that finds in large questions and profound interests its proper scope.

Either experiment is worthy of the practical genius of the People; and, both have their historical roots and are but restatements of the old problem, whether Government is better administered from the centre or from points in the circumference.

51. Time alone will decide which of these principles shall prevail. But, when the history of the development of Provident Association—the only sound form of Communism—is written, and some philosophic mind, with a broad survey, traces the dawn of the ideas that lie at the root of the principle, pourtrays their gradual emergement into practical application, depicts the salutary struggles endured, recounts the stimulating difficulties met and overcome, and enumerates the diseases (such as Assessmentism) repelled, we shall then recognize, what at present only a few

dimly discern, how a high Civilization, based upon a scientific Order, became possible, that under the blighting influence of unchecked Individualism would have perished.

In that grand record, the Hearts of Oak Benefit Society will claim, and will receive, a full chapter.

APPENDIX.

On a method of exhibiting the results of a Valuation, showing, for each age to be subsequently attained, the present values of the total sums at risk then emerging, and those of the Premiums to be then received.

1. The ordinary form of the Valuation-units and of the values of Benefits therefrom deduced show the results according to the present ages of the lives, and represent the sum of the values of the risks, or of the receipts, for such present ages and for all older ages to be thereafter attained. It is obvious that the risk is the same for all lives (of whatever present age) when passing through the same older ages, though such risk emerges at different times; likewise, the probability of the receipt of the Premiums is the same at each common age the groups may, in various future years, attain.

2. Thus, if S_1, S_2, &c., represent the gross sums assured at the present ages x, $x+1$, &c., the values of the same may be written:

$$A_x S_1 = S_1 \frac{C_x}{D_x} + S_1 \frac{C_{x+1}}{D_x} + S_1 \frac{C_{x+2}}{D_x} + \&c.$$

$$A_{x+1} S_2 = \qquad S_2 \frac{C_{x+1}}{D_{x+1}} + S_2 \frac{C_{x+2}}{D_{x+1}} + \&c.$$

$$A_{x+2} S_3 = \qquad\qquad S_3 \frac{C_{x+2}}{D_{x+2}} + \&c.$$

Therefore,

$$A_x S_1 + A_{x+1} S_2 + A_{x+2} S_3 + \ldots = \frac{S_1}{D_x} C_x + \left(\frac{S_1}{D_x} + \frac{S_2}{D_{x+1}}\right) C_{x+1}$$

$$+ \left(\frac{S_1}{D_x} + \frac{S_2}{D_{x+1}} + \frac{S_3}{D_{x+2}}\right) C_{x+2} + \&c.$$

Here, for instance, $\left(\frac{S_1}{D_x} + \frac{S_2}{D_{x+1}} + \frac{S_3}{D_{x+2}}\right) C_{x+2}$ represents the

present value of all death-claims in respect of the total assurances that will emerge upon any of the lives passing from age $x+2$ to $x+3$; and so on.

In like manner, the value of such of the entire Premium-income as will be received at age $x+3$ is $\left(\frac{P_1}{D_x}+\frac{P_2}{D_{x+1}}+\frac{P_3}{D_{x+2}}\right)D_{x+3}$, taking the annuity as curtate; and so on.

Suitable modifications should be made in the above formulæ, to correspond with the period of the payment of the death-claims and to the Valuation-adjustment made to the ordinary value of a_x.

3. The value of a Sickness Benefit is, in form, the value of an annuity, where the respective payments (taken as payable in the middle of the year) are s_x, s_{x+1}, s_{x+2}, &c., where these symbols represent the average quantity of Sickness experienced per member, well or ill, whilst paying from one age to the next older.

Hence, if in (2), for the sums assured and Valuation-unit C, there be respectively substituted the amount of weekly allowances at risk and the columnar value $s_x l_x v^{x+\frac{1}{2}}$ (designated L by Ratcliff), there will be shown the values of the Benefits according to, as may be styled, "ages to be passed through." The columns $\frac{S}{D}$ and $\frac{P}{D}$ should be first formed, and summed, before multiplying by the factor of Valuation.

4. These results at any given age (here called x) are based upon the numerical values of s_x: consequently, if the actual experience of the Society makes it necessary to provide for a higher or lesser rate of Sickness-claim, the values recorded should be proportionately adjusted. In practice, it will be sufficient to group the ages in fives for the adjustment.

5. It will be observed that the method indicated is of general application, and may be employed in the Valuation of Deferred Annuities and other Benefits. Also, a Surplus could be appropriated towards finding the common age at which all the Premiums of the mass would be extinguished, or the ordinary Assurance be converted into Endowment-Assurances at some other common age.

The summation of various probabilities, and of the composing units of many Benefits, in the form sketched, is a very instructive exercise, and presents some curious (but hitherto barren) forms of series.

TABLE I.

Numbers entering and leaving, and numbers Sick: shown for the two periods 1884–87, 1888–91, *and for the combined period* 1884–91.

Ages	Numbers attaining Ages	New Entrants	Left		Numbers Sick				Ages
			By Death	By other Causes	Full Pay	Half Pay	Super-annuation	All Claimants	
				1884–1887					
-19	560	1,638	1	129	140	1	...	140	-19
20–24	29,736	13,417	131	2,209	6,402	99	21	6,426	20–24
25–29	84,786	14,490	456	4,073	17,677	422	151	17,841	25–29
30–34	117,794	...	728	3,261	24,701	802	389	25,127	30–34
35–39	88,218	...	688	1,609	19,602	849	609	20,227	35–39
40–44	60,545	...	634	810	14,610	767	640	15,266	40–44
45–49	26,152	...	346	261	6,718	428	403	7,125	45–49
50–54	10,457	...	151	85	3,069	276	254	3,320	50–54
55–59	5,884	...	140	56	1,896	210	302	2,202	55–59
60–64	3,183	...	113	21	1,119	194	325	1,423	60–64
65–69	1,237	...	58	12	438	95	266	682	65–69
70–74	293	...	23	2	105	24	82	182	70–74
75–79	27	...	4	...	4	1	13	16	75–79
80–	...	...	...	...	...	...	...	...	80–
All Ages	428,872	29,545	3,473	12,508	96,481	4,168	3,455	99,977	All Ages
				1888–1891					
-19	1,447	4,700	7	231	828	...	1	828	-19
20–24	38,901	26.273	164	2,718	12,590	136	35	12,628	20–24
25–29	93,489	24,484	386	4,079	23,832	392	142	23,987	25–29
30–34	117,474	...	631	3,144	28,138	644	338	28,510	30–34
35–39	101,284	...	779	1,478	24,281	782	753	25,015	35–39
40–44	74,619	...	777	753	19,204	866	846	20,088	40–44
45–49	49,289	...	679	395	13,675	746	807	14,480	45–49
50–54	19,110	...	354	126	5,732	437	519	6,247	50–54
55–59	8,170	...	229	60	2,654	296	418	3,066	55–59
60–64	4,529	...	151	61	1,637	222	450	2,072	60–64
65–69	2,181	...	130	38	824	185	471	1,282	65–69
70–74	708	...	69	9	266	79	260	506	70–74
75–79	142	...	20	2	38	15	61	96	75–79
80–	9	...	4	...	1	1	4	5	80–
All Ages	511,352	55,457	4,380	13,094	133,700	4,801	5,105	138,810	All Ages
				1884–1891					
-19	2,007	6,338	8	360	968	1	1	968	-19
20–24	68,637	39,690	295	4,927	18,992	235	56	19,054	20–24
25–29	178,275	38,974	842	8,152	41,509	814	293	41,828	25–29
30–34	235,268	...	1,395	6,405	52,839	1,446	727	53,637	30–34
35–39	189,502	...	1,467	3,087	43,883	1,631	1,362	45,242	35–39
40–44	135,164	...	1,411	1,563	33,814	1,633	1,486	35,354	40–44
45–49	75,441	...	1,025	656	20,393	1,174	1,210	21,605	45–49
50–54	29,567	...	505	211	8,801	713	773	9,567	50–54
55–59	14,054	...	369	96	4,550	506	720	5,268	55–59
60–64	7,712	...	264	82	2,756	416	775	3,495	60–64
65–69	3,418	...	188	50	1,262	280	737	1,964	65–69
70–74	1,001	...	92	11	371	103	342	688	70–74
75–79	169	...	24	2	42	16	74	112	75–79
80–	9	...	4	...	1	1	4	5	80–
All Ages	940,224	85,002	7,853	25,602	230,181	8,969	8,560	238,787	All Ages

TABLE II.

Numbers entering and leaving, and numbers Sick: shown for each of the Eight Years 1884–1891.

Year	Numbers attaining Ages	New Entrants	Left: By Death	Left: By other Causes	Numbers Sick: Full Pay	Numbers Sick: Half Pay	Numbers Sick: Super-annuation	Numbers Sick: All Claimants	Year
				–19					
1884	156	458	1	47	36	...	...	36	1884
1885	143	417	...	27	25	...	...	25	1885
1886	152	349	...	34	44	...	...	44	1886
1887	109	414	...	21	35	1	...	35	1887
1888	159	812	...	31	95	...	...	95	1888
1889	319	958	1	58	167	...	...	167	1889
1890	395	1,258	3	63	217	...	...	217	1890
1891	574	1,672	3	79	349	...	1	349	1891
				20–24					
1884	7,552	3,686	36	665	1,631	26	3	1,636	1884
1885	7,612	3,295	44	535	1,606	17	5	1,610	1885
1886	7,479	3,084	28	574	1,676	28	6	1,681	1886
1887	7,093	3,352	23	435	1,489	28	7	1,499	1887
1888	7,099	5,352	29	543	1,829	27	7	1,835	1888
1889	8,756	5,642	33	640	2,467	28	9	2,476	1889
1890	10,278	6,977	48	722	3,642	36	11	3,656	1890
1891	12,768	8,302	54	813	4,652	45	8	4,661	1891
				25–29					
1884	22,098	3,665	146	1,172	4,521	131	44	4,570	1884
1885	21,343	3,584	112	941	4,395	97	41	4,442	1885
1886	20,969	3,466	85	1,052	4,408	103	32	4,443	1886
1887	20,376	3,775	113	908	4,353	91	34	4,386	1887
1888	20,572	5,224	79	893	4,271	89	42	4,313	1888
1889	22,080	5,504	97	1,002	4,751	95	33	4,784	1889
1890	23,953	6,520	91	1,019	7,127	101	30	7,159	1890
1891	26,884	7,236	119	1,165	7,683	107	37	7,731	1891
				30–34					
1884	29,434	...	202	924	6,163	208	90	6,260	1884
1885	29,700	...	182	804	6,168	214	96	6,285	1885
1886	29,389	...	174	815	6,365	198	104	6,471	1886
1887	29,271	...	170	718	6,005	182	99	6,111	1887
1888	28,822	...	150	769	5,966	159	84	6,054	1888
1889	28,947	...	145	833	5,852	144	76	5,937	1889
1890	29,330	...	150	726	8,180	193	85	8,277	1890
1891	30,375	...	186	816	8,140	148	93	8,242	1891

TABLE II—(*continued*).

Numbers entering and leaving, and numbers Sick: shown for each of the Eight Years 1884–1891.

Year	Numbers attaining Ages	New Entrants	Left: By Death	Left: By other Causes	Numbers Sick: Full Pay	Numbers Sick: Half Pay	Numbers Sick: Super-annuation	Numbers Sick: All Claimants	Year
				35–39					
1884	20,655	...	173	470	4,656	183	130	4,787	1884
1885	21,446	...	170	391	4,754	201	147	4,907	1885
1886	22,578	...	186	388	5,050	237	164	5,216	1886
1887	23,539	...	159	360	5,142	228	168	5,317	1887
1888	24,694	...	183	367	5,217	192	203	5,409	1888
1889	25,396	...	188	397	5,297	194	189	5,480	1889
1890	25,742	...	197	367	6,952	199	182	7,128	1890
1891	25,452	...	211	347	6,815	197	179	6,998	1891
				40–44					
1884	13,135	...	134	202	3,114	163	128	3,242	1884
1885	14,668	...	160	186	3,444	170	148	3,598	1885
1886	15,918	...	160	210	3,887	219	173	4,059	1886
1887	16,824	...	180	212	4,165	215	191	4,367	1887
1888	17,436	...	159	185	4,093	209	190	4,293	1888
1889	18,191	...	192	207	4,047	208	209	4,270	1889
1890	19,045	...	180	187	5,535	228	215	5,756	1890
1891	19,947	...	246	174	5,529	221	232	5,769	1891
				45–49					
1884	4,741	...	63	53	1,231	83	72	1,302	1884
1885	5,802	...	85	53	1,429	106	87	1,517	1885
1886	7,065	...	91	77	1,918	110	103	2,025	1886
1887	8,544	...	107	78	2,140	129	141	2,281	1887
1888	10,111	...	145	108	2,510	171	178	2,672	1888
1889	11,676	...	126	100	2,895	174	188	3,092	1889
1890	13,138	...	198	94	4,018	186	208	4,226	1890
1891	14,364	...	210	93	4,252	215	233	4,490	1891
				50–54					
1884	2,248	...	33	22	646	67	56	695	1884
1885	2,406	...	41	17	716	58	60	778	1885
1886	2,721	...	31	19	839	82	67	905	1886
1887	3,082	...	46	27	868	69	71	942	1887
1888	3,487	...	69	43	1,005	95	89	1,098	1888
1889	4,181	...	76	22	1,136	117	130	1,266	1889
1890	5,144	...	97	28	1,626	96	146	1,765	1890
1891	6,298	...	112	33	1,965	129	154	2,118	1891

Table II—(*continued*).

Numbers entering and leaving, and numbers Sick: shown for each of the Eight Years 1884–1891.

Year	Numbers attaining Ages	New Entrants	Left: By Death	Left: By other Causes	Numbers Sick: Full Pay	Numbers Sick: Half Pay	Numbers Sick: Super-annuation	Numbers Sick: All Claimants	Year
				55–59					
1884	1,286	...	29	4	415	48	65	481	1884
1885	1,420	...	37	8	478	56	73	548	1885
1886	1,524	...	37	11	518	57	82	595	1886
1887	1,654	...	37	13	485	49	82	578	1887
1888	1,791	...	48	33	561	67	95	649	1888
1889	1,930	...	59	10	585	80	93	674	1889
1890	2,068	...	63	9	694	80	105	814	1890
1891	2,381	...	59	8	814	69	125	929	1891
				60–64					
1884	644	...	18	3	228	34	56	284	1884
1885	739	...	31	6	248	50	79	320	1885
1886	857	...	37	5	319	58	95	400	1886
1887	943	...	27	7	324	52	95	419	1887
1888	1,032	...	36	34	344	58	92	437	1888
1889	1,066	...	28	9	369	46	113	477	1889
1890	1,170	...	46	7	450	49	123	567	1890
1891	1,261	...	41	11	474	69	122	591	1891
				65–69					
1884	239	...	14	5	75	16	50	122	1884
1885	276	...	6	1	99	22	56	148	1885
1886	344	...	17	2	116	21	72	180	1886
1887	378	...	21	4	148	36	88	232	1887
1888	442	...	22	21	141	41	102	244	1888
1889	490	...	23	4	184	40	110	288	1889
1890	569	...	28	5	220	44	124	336	1890
1891	680	...	57	8	279	60	135	414	1891
				70–74					
1884	46	...	8	...	14	3	14	26	1884
1885	63	...	5	1	23	4	19	41	1885
1886	71	...	3	...	25	7	18	43	1886
1887	113	...	7	1	43	10	31	72	1887
1888	135	...	11	6	41	11	43	83	1888
1889	163	...	17	1	59	19	58	113	1889
1890	182	...	17	1	81	24	68	143	1890
1891	228	...	24	1	85	25	91	167	1891

TABLE II—(*continued*).

Numbers entering and leaving, and numbers Sick: shown for each of the Eight Years 1884–1891.

Year	Numbers attaining Ages	New Entrants	Left		Numbers Sick				Year
			By Death	By other Causes	Full Pay	Half Pay	Super-annuation	All Claimants	
				75–79					
1884	5	...	2	...	...	...	3	3	1884
1885	4	...	1	...	...	...	2	2	1885
1886	7	...	...	...	1	...	3	4	1886
1887	11	...	1	...	3	1	5	7	1887
1888	23	...	3	1	7	4	8	15	1888
1889	26	...	2	...	8	4	11	19	1889
1890	40	...	5	...	8	3	21	27	1890
1891	53	...	10	1	15	4	21	35	1891
				80–					
1884	...	...	...	...	...	...	...	...	1884
1885	...	...	...	...	...	...	...	...	1885
1886	...	...	...	...	...	...	...	...	1886
1887	...	...	...	...	...	...	...	...	1887
1888	...	...	...	...	...	...	...	...	1888
1889	2	...	...	...	...	...	1	1	1889
1890	3	...	1	...	...	...	2	2	1890
1891	4	...	3	...	1	1	1	2	1891

Table III.

Numbers exposed to risk of Sickness, and weeks of expected and actual Sickness: shown for the two periods 1884–87, 1888–91, *and for the combined period* 1884–91.

Ages	Numbers exposed to risk	Expected Sickness (Weeks)				Actual Sickness (Weeks)				Ages
		Full Pay	Half Pay	Super-annuation	Total	Full Pay	Half Pay	Super-annuation	Total	
					1884–87					
–19	1,313	850	31	8	889	546	1	...	547	–19
20–24	35,274	24,238	1,387	831	26,456	23,824	841	603	25,268	20–24
25–29	89,762	64,176	4,575	4,242	72,993	66,857	4,228	3,891	74,976	25–29
30–34	115,800	89,309	7,799	9,736	106,844	104,165	8,482	12,197	124,844	30–34
35–39	87,068	73.743	6,989	11,769	92,501	89,275	9,084	22,807	121,166	35–39
40–44	59,823	57,166	6,735	12,164	76,065	72,977	8,232	24,336	105,545	40–44
45–49	25,848	29,086	4,234	8,901	42,221	37,441	4,763	16,006	58,210	45–49
50–54	10,339	14,530	2,517	5,879	22,926	19,485	3,280	10,034	32,799	50–54
55–59	5,795	10,460	2,167	5,512	18,139	12,999	2,221	13,595	28,815	55–59
60–64	3,114	7,377	1,910	5,364	14,651	9,138	2,532	14,180	25,850	60–64
65–69	1,202	3,792	1,240	3,790	8,822	4,228	1,296	12,248	17,772	65–69
70–74	280	1,070	447	1,678	3,195	970	300	3,684	4,954	70–74
75–79	25	100	50	246	396	14	26	592	632	75–79
80–	...	...	...	...	...	...	...	...	...	80–
All Ages	435,643	375,897	40,081	70,120	486,098	441,919	45,286	134,173	621,378	All Ages
					1888–91					
–19	3,680	2,387	81	19	2,487	2,672	...	22	2,694	–19
20–24	50,598	34,705	1,965	1,158	37,828	41,874	1,254	974	44,102	20–24
25–29	103,499	73,944	5,255	4,847	84,046	81,835	3,832	4,098	89,765	25–29
30–34	115,586	89,108	7,774	9,697	106,579	106,523	6,466	11,867	124,856	30–34
35–39	100,155	84,974	8,061	13,620	106,655	102,820	8,834	28,608	140,262	35–39
40–44	73,854	70,775	8,379	15,171	94,325	90,989	9,011	34,137	134,137	40–44
45–49	48,752	55,240	8,091	17,114	80,445	69,797	8,381	31,854	110,032	45–49
50–54	18,870	26,313	4,529	10,557	41,399	33,955	4,963	21,656	60,574	50–54
55–59	8,024	14,459	2,991	7,610	25,060	18,029	3,566	17,269	38,864	55–59
60–64	4,422	10,548	2,748	7,738	21,034	12,747	2,969	19,360	35,076	60–64
65–69	2,098	6,614	2,171	6,619	15,404	7,735	2,422	21,349	31,506	65–69
70–74	668	2,567	1,067	4,066	7,700	2,783	1,177	11,534	15,494	70–74
75–79	131	535	281	1,313	2,129	460	262	2,463	3,185	75–79
80–	7	...	...	364	364	26	14	141	181	80–
All Ages	530,344	472,169	53,393	99,893	625,455	572,245	53,151	205,332	830,728	All Ages
					1884–91					
–19	4,993	3,237	112	27	3,376	3,218	1	22	3,241	–19
20–24	85,872	58,943	3,352	1,989	64,284	65,698	2,095	1,577	69,370	20–24
25–29	193,261	138,120	9,830	9,089	157,039	148,692	8,060	7,989	164,741	25–29
30–34	231,386	178,417	15,573	19,433	213,423	210,688	14,948	24,064	249,700	30–34
35–39	187,223	158,717	15,050	25,389	199,156	192,095	17,918	51,415	261,428	35–39
40–44	133,677	127,941	15,114	27,335	170,390	163,966	17,243	58,473	239,682	40–44
45–49	74,600	84,326	12,325	26,015	122,666	107,238	13,144	47,860	168,242	45–49
50–54	29,209	40,843	7,046	16,436	64,325	53,440	8,243	31,690	93,373	50–54
55–59	13,819	24,919	5,158	13,122	43,199	31,028	5,787	30,864	67,679	55–59
60–64	7,536	17,925	4,658	13,102	35,685	21,885	5,501	33,540	60,926	60–64
65–69	3,300	10,406	3,411	10,409	24,226	11,963	3,718	33,597	49,278	65–69
70–74	948	3,637	1,514	5,744	10,895	3,753	1,477	15,218	20,448	70–74
75–79	156	635	331	1,559	2,525	474	288	3,055	3,817	75–79
80–	7	...	...	364	364	26	14	141	181	80–
All Ages	965,987	848,066	93,474	170,013	1,111,553	1,014,164	98,437	339,505	1,452,106	All Ages

Table IV.

Numbers exposed to risk of Sickness, and expected and actual Sickness: shown for each of the Eight Years 1884–1891.

Year	Numbers exposed to risk	Expected Time (Weeks)				Actual Time (Weeks)				Year
		Full Pay	Half Pay	Super-annuation	All Payments	Full Pay	Half Pay	Super-annuation	All Payments	
					–19					
1884	360	234	9	2	245	102	...	...	102	1884
1885	338	219	8	2	229	120	...	...	120	1885
1886	310	200	7	2	209	151	...	...	151	1886
1887	305	197	7	2	206	173	1	...	174	1887
1888	550	356	13	3	372	299	...	...	299	1888
1889	769	500	17	4	521	561	...	...	561	1889
1890	992	643	20	5	668	640	...	...	640	1890
1891	1,369	888	31	7	926	1,172	...	22	1,194	1891
					20–24					
1884	9,043	6,213	355	213	6,781	5,777	212	121	6,110	1884
1885	8,970	6,162	352	212	6,726	6,364	159	163	6,686	1885
1886	8,720	5,994	344	204	6,542	6,212	246	169	6,627	1886
1887	8,541	5,869	336	202	6,407	5,471	224	150	5,845	1887
1888	9,489	6,518	373	221	7,112	6,567	299	133	6,999	1888
1889	11,241	7,714	439	259	8,412	8,384	276	302	8,962	1889
1890	13,382	9,175	517	305	9,997	11,704	329	358	12,391	1890
1891	16,486	11,298	636	373	12,307	15,219	350	181	15,750	1891
					25–29					
1884	23,270	16,642	1,189	1,105	18,936	17,192	1,251	1,119	19,562	1884
1885	22,608	16,163	1,153	1,068	18,384	16,788	997	1,048	18,833	1885
1886	22,133	15,823	1,127	1,044	17,994	16,583	1,086	889	18,558	1886
1887	21,751	15,548	1,106	1,025	17,679	16,294	894	835	18,023	1887
1888	22,698	16,226	1,155	1,071	18,452	15,478	878	1,169	17,525	1888
1889	24,283	17,353	1,234	1,141	19,728	17,230	896	919	19,045	1889
1890	26,658	19,040	1,353	1,244	21,637	23,214	1,049	796	25,059	1890
1891	29,860	21,325	1,513	1,391	24,229	25,913	1,009	1,214	28,136	1891
					30–34					
1884	28,870	22,257	1,943	2,421	26,621	26,243	2,155	2,858	31,256	1884
1885	29,207	22,523	1,967	2,456	26,946	26,208	2,182	2,967	31,357	1885
1886	28,895	22,287	1,946	2,430	26,663	26,828	2,138	3,080	32,046	1886
1887	28,828	22,242	1,943	2,429	26,614	24,886	2,007	3,292	30,185	1887
1888	28,362	21,897	1,915	2,398	26,210	24,559	1,727	2,931	29,217	1888
1889	28,458	21,952	1,917	2,397	26,266	23,329	1,548	2,729	27,606	1889
1890	28,892	22,263	1,940	2,417	26,620	29,101	1,885	2,842	33,828	1890
1891	29,874	22,996	2,002	2,485	27,483	29,534	1,306	3,365	34,205	1891

Table IV—(*continued*).

Numbers exposed to risk of Sickness, and expected and actual Sickness: shown for each of the Eight Years 1884–1891.

Year	Numbers exposed to risk	Expected Time (Weeks)				Actual Time (Weeks)				Year
		Full Pay	Half Pay	Super-annuation	All Payments	Full Pay	Half Pay	Super-annuation	All Payments	
35–39										
1884	20,333	17,240	1,636	2,760	21,636	21,027	2,053	4,644	27,724	1884
1885	21,165	17,930	1,701	2,862	22,493	21,840	2,071	5,283	29,194	1885
1886	22,291	18,865	1,786	3,006	23,657	23,907	2,508	6,252	32,667	1886
1887	23,279	19,708	1,866	3,141	24,715	22,501	2,452	6,628	31,581	1887
1888	24,419	20,703	1,961	3,310	25,974	23,456	2,068	7,972	33,496	1888
1889	25,103	21,297	2,022	3,415	26,734	22,972	2,026	7,021	32,019	1889
1890	25,460	21,609	2,051	3,467	27,127	28,054	2,396	6,947	37,397	1890
1891	25,173	21,365	2,027	3,428	26,820	28,338	2,344	6,668	37,350	1891
40–44										
1884	12,966	12,359	1,450	2,615	16,424	15,405	1,841	4,638	21,884	1884
1885	14,496	13,842	1,627	2,940	18,409	17,514	1,892	5,497	24,903	1885
1886	15,733	15,042	1,774	3,204	20,020	19,533	2,482	6,652	28,667	1886
1887	16,628	15,923	1,884	3,405	21,212	20,525	2,017	7,549	30,091	1887
1888	17,264	16,568	1,966	3,562	22,096	20,902	2,144	7,699	30,745	1888
1889	17,992	17,258	2,047	3,708	23,013	19,988	2,043	8,772	30,803	1889
1890	18,861	18,066	2,137	3,868	24,071	24,621	2,426	8,751	35,798	1890
1891	19,737	18,883	2,229	4,033	25,145	25,478	2,398	8,915	36,791	1891
45–49										
1884	4,682	5,279	769	1,621	7,669	6,769	901	2,777	10,447	1884
1885	5,733	6,457	941	1,979	9,377	7,971	1,118	3,392	12,481	1885
1886	6,981	7,843	1,139	2,392	11,374	10,701	1,222	4,210	16,133	1886
1887	8,452	9,507	1,385	2,909	13,801	12,000	1,522	5,627	19,149	1887
1888	9,985	11,277	1,647	3,473	16,397	14,164	2,035	6,594	22,793	1888
1889	11,563	13,081	1,914	4,042	19,037	14,188	1,912	7,816	23,916	1889
1890	12,992	14,730	2,158	4,568	21,456	19,768	2,174	8,288	30,230	1890
1891	14,212	16,152	2,372	5,031	23,555	21,677	2,260	9,156	33,093	1891
50–54										
1884	2,220	3,133	545	1,272	4,950	4,349	755	1,932	7,036	1884
1885	2,377	3,352	581	1,361	5,294	4,511	801	2,243	7,555	1885
1886	2,696	3,781	653	1,526	5,960	5,419	873	2,916	9,208	1886
1887	3,046	4,264	738	1,720	6,722	5,206	851	2,943	9,000	1887
1888	3,431	4,806	830	1,934	7,570	6,776	1,034	3,702	11,512	1888
1889	4,132	5,765	993	2,317	9,075	6,920	1,230	5,444	13,594	1889
1890	5,082	7,083	1,219	2,843	11,145	9,119	1,203	6,045	16,367	1890
1891	6,225	8,659	1,487	3,463	13,609	11,140	1,496	6,465	19,101	1891

TABLE IV—(*continued*).

Numbers exposed to risk of Sickness, and expected and actual Sickness: shown for each of the Eight Years 1884–1891.

Year	Numbers exposed to risk	Expected Time (Weeks)				Actual Time (Weeks)				Year
		Full Pay	Half Pay	Super-annuation	All Payments	Full Pay	Half Pay	Super-annuation	All Payments	
					55–59					
1884	1,269	2,300	477	1,216	3,993	2,762	505	2,856	6,123	1884
1885	1,397	2,527	524	1,335	4,386	3,248	637	3,178	7,063	1885
1886	1,500	2,700	558	1,419	4,677	3,659	650	3,645	7,954	1886
1887	1,629	2,933	608	1,542	5,083	3,330	429	3,916	7,675	1887
1888	1,750	3,155	653	1,660	5,468	4,251	979	3,818	9,048	1888
1889	1,895	3,424	709	1,808	5,941	4,252	844	3,821	8,917	1889
1890	2,032	3,669	760	1,936	6,365	4,221	905	4,294	9,420	1890
1891	2,347	4,211	869	2,206	7,286	5,305	838	5,336	11,479	1891
					60–64					
1884	632	1,496	387	1,084	2,967	1,835	407	2,637	4,879	1884
1885	720	1,706	440	1,239	3,385	2,127	675	3,352	6,154	1885
1886	836	1,977	511	1,434	3,922	2,681	794	3,903	7,378	1886
1887	926	2,198	572	1,607	4,377	2,495	656	4,288	7,439	1887
1888	997	2,376	618	1,741	4,735	2,816	802	4,003	7,621	1888
1889	1,047	2,504	653	1,841	4,998	2,851	609	5,080	8,540	1889
1890	1,143	2,730	711	2,006	5,447	3,430	661	5,161	9,252	1890
1891	1,235	2,938	766	2,150	5,854	3,650	897	5,116	9,663	1891
					65–69					
1884	228	719	234	713	1,666	793	249	2,170	3,212	1884
1885	273	856	277	845	1,978	866	314	2,524	3,704	1885
1886	335	1,061	349	1,070	2,480	1,107	228	3,477	4,812	1886
1887	366	1,156	380	1,162	2,698	1,462	505	4,077	6,044	1887
1888	421	1,328	435	1,330	3,093	1,439	449	4,901	6,789	1888
1889	476	1,499	492	1,496	3,487	1,603	630	4,884	7,117	1889
1890	553	1,744	572	1,747	4,063	1,994	633	5,566	8,193	1890
1891	648	2,043	672	2,046	4,761	2,699	710	5,998	9,407	1891
					70–74					
1884	42	160	65	244	469	96	51	555	702	1884
1885	60	227	97	355	679	187	51	894	1,132	1885
1886	69	266	112	427	805	303	56	899	1,258	1886
1887	109	417	173	652	1,242	384	142	1,336	1,862	1887
1888	126	484	199	758	1,441	464	121	1,987	2,572	1888
1889	154	590	246	932	1,768	595	268	2,638	3,501	1889
1890	173	664	276	1,051	1,991	780	387	2,960	4,127	1890
1891	215	829	346	1,325	2,500	944	401	3,949	5,294	1891

TABLE IV—(*continued*).

Numbers exposed to risk of Sickness, and expected and actual Sickness: shown for each of the Eight Years 1884–1891.

Year	Numbers exposed to risk	Expected Time (Weeks)				Actual Time (Weeks)				Year
		Full Pay	Half Pay	Super-annuation	All Payments	Full Pay	Half Pay	Super-annuation	All Payments	
					75–79					
1884	4	16	8	38	62	...	...	152	152	1884
1885	4	14	7	34	55	...	...	53	53	1885
1886	7	28	14	68	110	4	...	159	163	1886
1887	10	42	21	106	169	10	26	228	264	1887
1888	21	85	44	207	336	137	73	355	565	1888
1889	25	102	53	248	403	74	30	543	647	1889
1890	37	153	80	374	607	92	77	910	1,079	1890
1891	48	195	104	484	783	157	82	655	894	1891
					80–					
1884	...	...	...	...	...	...	...	...	...	1884
1885	...	...	...	...	...	...	...	...	...	1885
1886	...	...	...	...	...	...	...	...	...	1886
1887	...	...	...	...	...	...	...	...	...	1887
1888	...	...	...	...	...	...	...	...	...	1888
1889	2	...	...	104	104	...	...	52	52	1889
1890	2	...	...	130	130	...	...	66	66	1890
1891	3	...	...	130	130	26	14	23	63	1891

TABLE V.

Sickness Experience for the four years 1884–87.

1884–87

Ages	Actual Sickness (Weeks) per Member exposed to risk: Full Pay	Half Pay	Super-annuation	All Claimants	Ages	Actual Sickness (Weeks) per Member sick: Full Pay	Half Pay	Super-annuation	All Claimants
–19	·42	...	...	·42	–19	3·90	1·00	...	3·91
20–24	·68	·02	·02	·72	20–24	3·72	8·50	28·71	3·93
25–29	·75	·05	·04	·84	25–29	3·78	10·02	25·77	4·20
30–34	·90	·07	·11	1·08	30–34	4·22	10·58	31·37	4·97
35–39	1·03	·10	·26	1·39	35–39	4·56	10·70	37·45	5·99
40–44	1·22	·14	·40	1·76	40–44	5·00	10·73	38·03	6·91
45–49	1·45	·18	·62	2·25	45–49	5·57	11·13	39·73	8·17
50–54	1·89	·31	·97	3·17	50–54	6·35	11·89	39·49	9·88
55–59	2·24	·38	2·35	4·97	55–59	6·86	10·58	45·03	13·09
60–64	2·94	·81	4·55	8·30	60–64	8·17	13·05	43·63	18·17
65–69	3·52	1·07	10·19	14·78	65–69	9·65	13·64	46·05	26·06
70–74	3·46	1·07	13·16	17·69	70–74	9·24	12·50	44·93	27·22
75–79	·56	1·04	23·68	25·28	75–79	3·50	26·00	45·54	39·50

Ages	Expected Sickness (Weeks) per Member exposed to risk (M.U.): Full Pay	Half Pay	Super-annuation	All Claimants	Ages	Numbers Sick out of each 100 exposed to risk: Full Pay	Half Pay	Super-annuation	All Claimants
–19	·65	·02	·01	·68	–19	10·66	·08	...	10·66
20–24	·69	·04	·02	·75	20–24	18·15	·28	·06	18·22
25–29	·71	·05	·05	·81	25–29	19·70	·47	·17	19·87
30–34	·77	·07	·08	·92	30–34	21·33	·69	·34	21·70
35–39	·85	·08	·13	1·06	35–39	22·52	·98	·70	23·24
40–44	·96	·11	·20	1·27	40–44	24·43	1·28	1·07	25·53
45–49	1·12	·16	·35	1·63	45–49	25·98	1·66	1·56	27·56
50–54	1·41	·24	·57	2·22	50–54	29·68	2·67	2·46	32·11
55–59	1·81	·37	·95	3·13	55–59	32·71	3·62	5·21	37·99
60–64	2·37	·61	1·72	4·70	60–64	35·93	6·23	10·44	45·70
65–69	3·16	1·03	3·15	7·34	65–69	36·44	7·90	22·13	56·74
70–74	3·82	1·60	5·99	11·41	70–74	37·50	8·57	29·28	65·00
75–79	4·00	2·00	9·84	15·84	75–79	16·00	4·00	52·00	64·00

Table V—(*continued*).

Sickness Experience for the four years 1888–91.

1888–91

Ages	Actual Sickness (Weeks) per Member exposed to risk				Ages	Actual Sickness (Weeks) per Member sick			
	Full Pay	Half Pay	Super-annuation	All Claimants		Full Pay	Half Pay	Super-annuation	All Claimants
–19	·73	...	...	·73	–19	3·23	...	22·00	3·25
20–24	·83	·02	·02	·87	20–24	3·33	9·22	27·83	3·49
25–29	·79	·04	·04	·87	25–29	3·44	9·78	28·86	3·74
30–34	·92	·06	·10	1·08	30–34	3·78	10·04	35·12	4·38
35–39	1·03	·09	·28	1·40	35–39	4·23	11·30	37·99	5·61
40–44	1·23	·12	·47	1·82	40–44	4·74	10·41	40·36	6·67
45–49	1·43	·17	·66	2·26	45–49	5·10	11·24	39·46	7·60
50–54	1·80	·26	1·15	3·21	50–54	5·93	11·36	41·73	9·70
55–59	2·25	·44	2·15	4·84	55–59	6·79	12·05	41·31	12·67
60–64	2·88	·67	4·38	7·93	60–64	7·79	13·37	43·02	16·93
65–69	3·69	1·15	10·18	15·02	65–69	9·39	13·09	45·33	24·58
70–74	4·17	1·76	17·26	23·19	70–74	10·46	14·90	44·34	30·61
75–79	3·51	2·00	18·80	24·31	75–79	12·11	17·47	40·38	33·17
80–	3·72	2·00	20·14	25·86	80–	26·00	14·00	35·25	36·20

Ages	Expected Sickness (Weeks) per Member exposed to risk (M.U.)				Ages	Numbers Sick out of each 100 exposed to risk			
	Full Pay	Half Pay	Super-annuation	All Claimants		Full Pay	Half Pay	Super-annuation	All Claimants
–19	·65	·02	·01	·68	–19	22·50	...	·03	22·50
20–24	·69	·04	·02	·75	20–24	24·88	·27	·07	24·96
25–29	·71	·05	·05	·81	25–29	23·03	·38	·14	23·18
30–34	·77	·07	·08	·92	30–34	24·34	·56	·29	24·66
35–39	·85	·08	·14	1·07	35–39	24·23	·78	·75	24·96
40–44	·96	·11	·21	1·28	40–44	26·00	1·17	1·15	27·20
45–49	1·13	·17	·35	1·65	45–49	28·06	1·53	1·66	29·70
50–54	1·39	·24	·56	2·19	50–54	30·37	2·32	2·75	33·11
55–59	1·80	·37	·95	3·12	55–59	33·08	3·69	5·21	38·21
60–64	2·39	·62	1·75	4·76	60–64	37·02	5·02	10·18	46·86
65–69	3·15	1·04	3·15	7·34	65–69	39·27	8·82	22·45	61·11
70–74	3·84	1·60	6·09	11·53	70–74	39·82	11·82	38·92	75·75
75–79	4·08	2·15	10·00	16·23	75–79	29·01	11·45	46·56	73·28
80–	...	...	52·00	52·00	80–	14·29	14·29	57·15	71·43

TABLE V—(*continued*).

Sickness Experience for the eight years 1884–91.

1884–91									
Ages	Actual Sickness (Weeks) per Member exposed to Risk				Ages	Actual Sickness (Weeks) per Member sick			
	Full Pay	Half Pay	Super-annuation	All Claimants		Full Pay	Half Pay	Super-annuation	All Claimants
–19	·65	...	...	·65	–19	3·32	1·00	22·00	3·35
20–24	·77	·02	·02	·81	20–24	3·46	8·92	28·16	3·64
25–29	·77	·04	·04	·85	25–29	3·58	9·90	27·26	3·94
30–34	·91	·07	·10	1·08	30–34	3·99	10·34	33·10	4·66
35–39	1·03	·10	·27	1·40	35–39	4·38	10·98	37·75	5·78
40–44	1·23	·13	·43	1·79	40–44	4·85	10·56	39·35	6·78
45–49	1·44	·18	·64	2·26	45–49	5·26	11·19	39·55	7·78
50–54	1·83	·28	1·09	3·20	50–54	6·07	11·56	40·99	9·76
55–59	2·25	·42	2·23	4·90	55–59	6·82	11·44	42·86	12·85
60–64	2·91	·73	4·45	8·09	60–64	7·94	13·23	43·28	17·43
65–69	3·62	1·13	10·18	14·93	65–69	9·48	13·28	45·58	25·10
70–74	3·96	1·56	16·05	21·57	70–74	10·12	14·34	44·50	29·72
75–79	3·04	1·85	19·58	24·47	75–79	11·29	18·00	41·29	34·08
80–	3·72	2·00	20·14	25·86	80–	26·00	14·00	35·25	36·20
Ages	Expected Sickness (Weeks) per Member exposed to risk (M.U.)				Ages	Numbers Sick out of each 100 exposed to risk			
	Full Pay	Half Pay	Super-annuation	All Claimants		Full Pay	Half Pay	Super-annuation	All Claimants
–19	·65	·02	·01	·68	–19	19·39	·02	·02	19·39
20–24	·69	·04	·02	·75	20–24	22·12	·27	·07	22·19
25–29	·71	·05	·05	·81	25–29	21·48	·42	·15	21·64
30–34	·77	·07	·08	·92	30–34	22·83	·62	·31	23·18
35–39	·85	·08	·13	1·06	35–39	23·44	·87	·73	24·17
40–44	·96	·11	·21	1·28	40–44	25·29	1·22	1·11	26·44
45–49	1·13	·17	·35	1·65	45–49	27·33	1·57	1·62	28·97
50–54	1·40	·24	·56	2·20	50–54	30·13	2·44	2·65	32·76
55–59	1·80	·38	·95	3·13	55–59	32·92	3·66	5·21	38·12
60–64	2·38	·62	1·74	4·74	60–64	36·58	5·52	10·28	46·38
65–69	3·16	1·03	3·15	7·34	65–69	38·25	8·49	22·34	59·51
70–74	3·84	1·60	6·06	11·50	70–74	39·14	10·86	36·07	72·58
75–79	4·07	2·12	10·00	16·19	75–79	26·92	10·26	47·44	71·80
80–	...	...	52·00	52·00	80–	14·29	14·29	57·15	71·43

TABLE VI.

Average Sickness (weeks), per Member exposed to risk.

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
		-19					40-44		
1884	·28	...	...	·28	1884	1·19	·14	·36	1·69
1885	·35	...	...	·35	1885	1·21	·13	·38	1·72
1886	·49	...	...	·49	1886	1·24	·16	·42	1·82
1887	·57	...	...	·57	1887	1·24	·12	·45	1·81
1888	·54	...	...	·54	1888	1·21	·12	·45	1·78
1889	·73	...	...	·73	1889	1·11	·11	·49	1·71
1890	·65	...	...	·65	1890	1·31	·13	·46	1·90
1891	·86	...	·02	·88	1891	1·29	·12	·45	1·86
		20-24					45-49		
1884	·64	·02	·01	·67	1884	1·45	·19	·59	2·23
1885	·71	·02	·02	·75	1885	1·39	·20	·59	2·18
1886	·71	·03	·02	·76	1886	1·53	·18	·60	2·31
1887	·64	·02	·02	·68	1887	1·42	·18	·67	2·27
1888	·69	·03	·01	·73	1888	1·42	·20	·66	2·28
1889	·75	·02	·03	·80	1889	1·23	·17	·68	2·06
1890	·87	·03	·03	·93	1890	1·52	·17	·64	2·33
1891	·92	·02	·01	·95	1891	1·53	·16	·64	2·33
		25-29					50-54		
1884	·74	·05	·05	·84	1884	1·96	·34	·87	3·17
1885	·74	·04	·05	·83	1885	1·90	·34	·94	3·18
1886	·75	·05	·04	·84	1886	2·01	·32	1·08	3·41
1887	·75	·04	·04	·83	1887	1·71	·28	·96	2·95
1888	·68	·04	·05	·77	1888	1·98	·30	1·08	3·36
1889	·71	·04	·04	·79	1889	1·68	·30	1·32	3·30
1890	·87	·04	·03	·94	1890	1·80	·23	1·19	3·22
1891	·87	·03	·04	·94	1891	1·79	·24	1·04	3·07
		30-34					55-59		
1884	·91	·07	·10	1·08	1884	2·17	·40	2·25	4·82
1885	·90	·07	·10	1·07	1885	2·32	·46	2·27	5·05
1886	·93	·07	·11	1·11	1886	2·44	·43	2·43	5·30
1887	·86	·07	·12	1·05	1887	2·05	·26	2·40	4·71
1888	·87	·06	·10	1·03	1888	2·43	·56	2·18	5·17
1889	·82	·05	·10	·97	1889	2·24	·45	2·02	4·71
1890	1·01	·06	·10	1·17	1890	2·08	·45	2·11	4·64
1891	·99	·04	·11	1·14	1891	2·26	·36	2·27	4·89
		35-39					60-64		
1884	1·03	·10	·23	1·36	1884	2·90	·64	4·16	7·70
1885	1·03	·10	·25	1·38	1885	2·95	·94	4·65	8·54
1886	1·07	·11	·28	1·46	1886	3·21	·95	4·67	8·83
1887	·97	·11	·28	1·36	1887	2·70	·71	4·63	8·04
1888	·96	·08	·33	1·37	1888	2·82	·80	4·02	7·64
1889	·92	·08	·28	1·28	1889	2·72	·58	4·85	8·15
1890	1·10	·10	·27	1·47	1890	3·00	·58	4·51	8·09
1891	1·13	·09	·27	1·49	1891	2·96	·73	4·14	7·83

TABLE VI—(*continued*).

Average Sickness (weeks), per Member exposed to risk.

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
		65–69					75–79		
1884	3·46	1·08	9·46	14·00	1884	...	...	37·87	37·87
1885	3·18	1·16	9·25	13·59	1885	...	...	15·29	15·29
1886	3·31	·68	10·40	14·39	1886	·55	...	22·72	23·27
1887	4·00	1·38	11·15	16·53	1887	·95	2·48	21·73	25·16
1888	3·42	1·07	11·64	16·13	1888	6·50	3·48	16·92	26·90
1889	3·37	1·32	10·24	14·93	1889	2·93	1·19	21·69	25·81
1890	3·61	1·15	10·08	14·84	1890	2·46	2·07	24·27	28·80
1891	4·17	1·10	9·26	14·53	1891	3·31	1·72	13·78	18·81
		70–74					80–		
1884	2·29	1·23	13·21	16·73	1884	...	...	...	...
1885	3·11	·86	14·90	18·87	1885	...	...	...	...
1886	4·36	·80	12·94	18·10	1886	...	...	...	...
1887	3·52	1·30	12·26	17·08	1887	...	...	...	...
1888	3·67	·96	15·71	20·34	1888	...	...	...	...
1889	3·87	1·74	17·12	22·73	1889	...	...	26·00	26·00
1890	4·51	2·24	17·11	23·86	1890	...	...	26·40	26·40
1891	4·38	1·86	18·33	24·57	1891	10·40	5·80	8·92	25·12

TABLE VII.

Average Sickness (weeks), per Member Sick.

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
–19					**40–44**				
1884	2·81	...	...	2·81	1884	4·95	11·29	36·23	6·75
1885	4·79	...	...	4·79	1885	5·08	11·13	37·14	6·92
1886	3·43	...	...	3·43	1886	5·02	11·34	38·45	7·06
1887	4·95	1·17	...	4·99	1887	4·93	9·38	39·53	6·89
1888	3·14	...	...	3·14	1888	5·11	10·26	40·52	7·16
1889	3·36	...	...	3·36	1889	4·94	9·82	41·98	7·21
1890	2·95	...	...	2·95	1890	4·45	10·64	40·70	6·22
1891	3·36	...	22·20	3·42	1891	4·61	10·85	38·43	6·38
20–24					**45–49**				
1884	3·54	8·15	40·40	3·73	1884	5·50	10·86	38·57	8·02
1885	3·96	9·39	32·49	4·15	1885	5·58	10·54	39·00	8·22
1886	3·71	8·79	28·19	3·94	1886	5·58	11·11	40·88	7·96
1887	3·68	8·01	21·47	3·90	1887	5·61	11·81	39·91	8·40
1888	3·59	11·05	19·03	3·82	1888	5·64	11 90	37·05	8·53
1889	3·40	9·85	33·48	3·62	1889	4·90	10·99	41·58	7·70
1890	3·21	9·15	32·57	3·39	1890	4·92	11·69	39·85	7·15
1891	3·27	7·79	22·53	3·38	1891	5·10	10·52	39·29	7·37
25–29					**50–54**				
1884	3·80	9·54	25·42	4·28	1884	6·73	11·27	34·49	10·12
1885	3·82	10·28	25·57	4·24	1885	6·30	13·82	37·38	9·71
1886	3·76	10·53	27·80	4·18	1886	6·46	10·65	43·50	10·18
1887	3·74	9·82	24·56	4·11	1887	6·00	12·33	41·45	9·55
1888	3·62	9·86	27·85	4·06	1888	6·74	10·88	41·59	10·49
1889	3·63	9·44	27·86	3·98	1889	6·09	10·52	41·87	10·81
1890	3·26	10·39	26·53	3·50	1890	5·61	12·53	41·40	9·27
1891	3·37	9·43	32·81	3·64	1891	5·67	11·60	41·99	9·02
30–34					**55–59**				
1884	4·26	10·36	31·75	4·99	1884	6·65	10·52	43·94	12·73
1885	4·25	10·20	30·89	4·99	1885	6·80	11·38	43·53	12·89
1886	4·22	10·80	29·62	4·95	1886	7·07	11·40	44·45	13·37
1887	4·14	11·03	33·26	4·94	1887	6·87	8·76	47·75	13·28
1888	4·12	10·85	34·89	4·83	1888	7·58	14·61	40·19	13·94
1889	3·99	10·75	35·91	4·65	1889	7·27	10·54	41·11	13·23
1890	3·56	9·76	33·44	4·09	1890	6·08	11·32	40·90	11·58
1891	3·63	8·82	36·17	4·15	1891	6·52	12·14	42·69	12·36
35–39					**60–64**				
1884	4·52	11·22	35·73	5·79	1884	8·05	11·95	47·08	17·18
1885	4·59	10·30	35·93	5·95	1885	8·57	13·50	42·43	19·23
1886	4·74	10·58	38·13	6·26	1886	8·41	13·69	41·09	18·45
1887	4·38	10·75	39·45	5·94	1887	7·70	12·62	45·14	17·76
1888	4·50	10·77	39·27	6·19	1888	8·19	13·82	43·51	17·44
1889	4·34	10·45	37·15	5·84	1889	7·72	13·22	44·95	17·90
1890	4·04	12·04	38·17	5·25	1890	7·62	13·48	41·95	16·31
1891	4·16	11·90	37·25	5·34	1891	7·70	13·00	41·94	16·35

TABLE VII—(*continued*).

Average Sickness (weeks), per Member Sick.

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
		65–69					75–79		
1884	10·57	15·53	43·40	26·33	1884	...	...	50·50	50·50
1885	8·75	14·31	45·07	25·02	1885	...	...	26·75	26·75
1886	9·54	10·86	48·30	26·73	1886	3·80	...	53·00	40·70
1887	9·88	14·03	46·31	26·05	1887	9·33	26·00	45·64	37·74
1888	10·21	10·96	48·05	27·83	1888	19·50	18·25	44·41	37·65
1889	8·72	15·75	44·39	24·71	1889	9·15	7·45	49·29	33·96
1890	9·07	14·38	44·89	24·39	1890	11·53	25·77	43·33	39·99
1891	9·68	11·84	44·43	22·72	1891	10·49	20·37	31·18	25·53
		70–74					80–		
1884	6·87	17·10	39·61	27·01	1884	...	...	...	...
1885	8·12	12·67	47·07	27·62	1885	...	...	...	...
1886	12·12	7·96	49·97	29·25	1886	...	...	...	...
1887	8·93	14·17	43·09	25·86	1887	...	...	...	...
1888	11·31	11·02	46·22	30·98	1888	...	...	...	...
1889	10·09	14·09	45·46	30·98	1889	...	...	52·00	52·00
1890	9·63	16·14	43·53	28·86	1890	...	...	33·00	33·00
1891	11·11	16·06	43·40	31·71	1891	26·90	14·50	22·33	31·41

TABLE VIII.

Numbers Sick out of each 100 *exposed to risk.*

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
–19					**40–44**				
1884	9·97	...	...	9·97	1884	24·02	1·26	·99	25·00
1885	7·40	...	...	7·40	1885	23·75	1·17	1·02	24·81
1886	14·22	...	...	14·22	1886	24·71	1·39	1·10	25·80
1887	11·46	·33	...	11·46	1887	25·04	1·29	1·15	26·26
1888	17·29	...	...	17·29	1888	23·71	1·21	1·10	24·88
1889	21·73	...	...	21·73	1889	22·50	1·16	1·16	23·73
1890	21·90	...	...	21·90	1890	29·34	1·21	1·14	30·52
1891	25·49	...	·07	25·49	1891	28·00	1·12	1·18	29·22
20–24					**45–49**				
1884	18·03	·29	·03	18·09	1884	26·29	1·77	1·54	27·80
1885	17·91	·19	·06	17·95	1885	24·92	1·85	1·52	26·46
1886	19·22	·32	·07	19·28	1886	27·47	1·58	1·48	29·01
1887	17·43	·33	·08	17·55	1887	25·32	1·53	1·67	26·98
1888	19·28	·28	·07	19·33	1888	25·14	1·71	1·78	26·75
1889	21·95	·25	·08	22·03	1889	25·04	1·51	1·63	26·75
1890	27·21	·27	·08	27·32	1890	30·91	1·43	1·60	32·51
1891	28·21	·27	·05	28·27	1891	29·92	1·51	1·64	31·59
25–29					**50–54**				
1884	19·43	·56	·19	19·64	1884	29·09	3·02	2·52	31·30
1885	19·44	·43	·18	19·64	1885	30·12	2·44	2·52	32·73
1886	19·92	·47	·14	20·08	1886	31·12	3·04	2·49	33·56
1887	20·01	·42	·16	20·17	1887	28·50	2·27	2·33	30·93
1888	18·82	·39	·19	19·00	1888	29·30	2·77	2·59	32·00
1889	19·57	·39	·14	19·71	1889	27·49	2·83	3·15	30·64
1890	26·73	·38	·11	26·85	1890	32·00	1·89	2·87	34·73
1891	25·72	·36	·12	25·89	1891	31·56	2·07	2·47	34·02
30–34					**55–59**				
1884	21·35	·72	·31	21·09	1884	32·68	3·78	5·12	37·88
1885	21·12	·73	·33	21·51	1885	34·18	4·01	5·22	39·19
1886	22·03	·69	·36	22·40	1886	34·53	3·80	5·47	39·66
1887	20·84	·63	·34	21·20	1887	29·77	3·01	5·03	35·48
1888	21·04	·56	·30	21·34	1888	32·05	3·83	5·43	37·08
1889	20·56	·51	·27	20·86	1889	30·86	4·22	4·91	35·55
1890	28·31	·67	·29	28·65	1890	34·16	3·94	5·17	40·05
1891	27·26	·50	·31	27·59	1891	34·66	2·94	5·32	39·57
35–39					**60–64**				
1884	22·90	·90	·64	23·55	1884	35·99	5·37	8·84	44·83
1885	22·46	·95	·69	23·18	1885	34·42	6·94	10·96	44·41
1886	22·66	1·06	·74	23·40	1886	38·16	6·94	11·37	47·85
1887	22·09	·98	·72	22·84	1887	34·99	5·62	10·26	45·25
1888	21·36	·79	·83	22·15	1888	34·50	5·82	9·23	43·83
1889	21·11	·77	·75	21·83	1889	35·21	4·39	10·78	45·51
1890	27·31	·78	·72	28·00	1890	39·33	4·28	10·75	49·57
1891	27·08	·78	·71	27·81	1891	38·39	5·59	9·88	47·86

TABLE VIII—(*continued*).

Numbers Sick out of each 100 *exposed to risk.*

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
		65–69					75–79		
1884	32·68	6·97	21·79	53·16	1884	...	...	74·99	74·99
1885	36·33	8·07	20·55	54·32	1885	...	...	57·14	57·14
1886	34·68	6·28	21·53	53·81	1886	14·29	...	42·85	57·14
1887	40·49	9·85	24·08	63·47	1887	28·57	9·52	47·62	66·67
1888	33·50	9·74	24·23	57·97	1888	33·33	19·05	38·10	71·43
1889	38·61	8·40	23·09	60·43	1889	32·00	16·01	44·00	76·02
1890	39·82	7·97	22·45	60·81	1890	21·34	8·00	56·01	72·01
1891	43·09	9·27	20·85	63·95	1891	31·58	8·42	44·21	73·69
		70–74					80–		
1884	33·33	7·14	33·33	61·92	1884	...	...	...	...
1885	38·33	6·67	31·66	68·32	1885	...	...	...	...
1886	35·97	10·07	25·90	61·87	1886	...	...	...	...
1887	39·45	9·18	28·44	66·05	1887	...	...	...	...
1888	32·41	8·70	33·99	65·61	1888	...	...	...	...
1889	38·32	12·34	37·66	73·38	1889	...	...	50·00	50·00
1890	46·82	13·88	39·31	82·66	1890	...	...	80·00	80·00
1891	39·45	11·60	42·23	77·50	1891	40·00	40·00	40·00	80·00

TABLE IX.

Percentage by which the actual Sick Pay was over, *or* **under** *the Expected payments according to the Valuation-provision.*

Ages	Full Pay	Half Pay	Super-annuation	All Claimants	Ages
		1884–87			
–19	**35·77**	**96·77**	**100·00**	**37·21**	–19
20–24	1·71	**39·37**	**63·67**	**3·64**	20–24
25–29	4·18	**7·58**	**54·13**	2·18	25–29
30–34	16·64	8·76	**37·36**	13·93	30–34
35–39	21·07	29·97	**3·11**	19·92	35–39
40–44	27·66	22·23	**·03**	25·12	40–44
45–49	28·73	12·49	**10·09**	**23·37**	45–49
50–54	34·10	30·43	**14·67**	**26·91**	50–54
55–59	24·28	2·49	23·31	22·42	55–59
60–64	23·87	32·57	32·18	26·47	60–64
65–69	11·50	4·52	61·59	24·61	65–69
70–74	**9·35**	**32·89**	9·77	**4·90**	70–74
75–79	**86·00**	**47·99**	2·03	**32·22**	75–79
All ages	17·82	12·98	**4·33**	15·75	All ages

Table IX—(*continued*).

Percentage by which the actual Sick Pay was over, *or* under *the Expected payments according to the Valuation-provision.*

1888–91					
Ages	Full Pay	Half Pay	Super-annuation	All Claimants	Ages
-19	11·94	100·00	42·10	9·90	-19
20–24	20·65	36·18	57·94	18·01	20–24
25–29	10·68	27·08	57·73	7·54	25–29
30–34	19·54	16·83	38·81	15·51	30–34
35–39	21·01	9·59	5·02	19·50	35–39
40–44	28·57	7·54	12·50	26·16	40–44
45–49	26·35	3·58	6·94	21·19	45–49
50–54	29·05	9·58	2·57	23·99	50–54
55–59	24·69	19·23	13·47	22·31	55–59
60–64	20·84	8·04	25·09	20·65	60–64
65–69	16·94	11·56	61·27	28·65	65–69
70–74	8·42	10·31	41·82	20·97	70–74
75–79	14·02	6·76	6·17	9·45	75–79
80–	...	...	80·76	60·95	80–
All ages	21·19	·45	2·78	18·63	All ages

Percentage by which the actual Sick Pay was over, *or* under *the Expected payments according to the Valuation-provision.*

1884–91					
Ages	Full Pay	Half Pay	Super-annuation	All Claimants	Ages
-19	·59	99·10	59·26	2·49	-19
20–24	11·47	37·50	60·34	9·09	20–24
25–29	7·66	18·01	56·06	5·05	25–29
30–34	18·09	4·02	38·09	14·71	30–34
35–39	21·03	19·06	1·26	19·69	35–39
40–44	28·16	14·09	6·95	25·68	40–44
45–49	27·16	6·64	8·01	21·96	45–49
50–54	30·85	16·99	3·60	25·03	50–54
55–59	24·52	12·20	17·61	22·35	55–59
60–64	22·09	18·09	28·00	23·06	60–64
65–69	14·95	9·00	61·37	27·18	65–69
70–74	3·19	2·44	32·47	13·36	70–74
75–79	25·35	12·99	1·99	13·16	75–79
80–	...	...	80·76	60·95	80–
All ages	19·59	5·31	·15	17·36	All ages

TABLE X.

Showing for each of the Eight years 1884–91 *the percentage by which the actual Sick Pay was* greater, *or* less *than the Expected payments according to the Valuation-provision.*

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
		–19					35–39		
1884	56·39	100·00	...	57·21	1884	21·97	25·75	15·86	19·72
1885	45·29	100·00	...	46·50	1885	21·81	21·74	7·69	19·93
1886	24·67	100·00	...	25·68	1886	26·72	40·41	3·91	25·87
1887	12·03	83·30	100·00	13·26	1887	14·17	31·40	5·57	14·34
1888	16·15	100·00	100·00	18·05	1888	13·29	5·47	20·46	13·42
1889	12·37	100·00	100·00	10·02	1889	7·86	·22	2·70	7·21
1890	·50	100·00	100·00	2·37	1890	29·83	16·83	·22	27·38
1891	32·04	100·00	33·33	29·78	1891	32·63	15·57	2·70	29·66
		20–24					40–44		
1884	7·05	40·63	72·95	8·93	1884	24·66	26·65	11·38	21·82
1885	3·27	54·69	61·63	·73	1885	26·54	16·29	6·55	23·30
1886	3·65	28·39	58·04	1·85	1886	29·85	39·92	3·83	28·28
1887	6·78	33·09	62·96	8·32	1887	28·92	7·06	10·87	26·30
1888	·75	20·02	70·45	·85	1888	26·16	9·08	8·07	23·79
1889	8·68	37·20	40·78	6·69	1889	15·82	·21	18·36	15·19
1890	27·56	36·30	40·99	24·86	1890	36·29	13·52	13·12	33·22
1891	34·71	44·75	75·84	31·01	1891	34·91	7·48	10·54	31·52
		25–29					45–49		
1884	3·30	5·24	49·33	1·91	1884	28·22	17·34	14·20	22·79
1885	3·87	13·53	51·06	1·76	1885	23·45	18·81	14·27	18·92
1886	4·80	3·68	57·31	2·80	1886	36·44	7·30	12·02	29·24
1887	4·80	19·19	59·26	2·23	1887	26·22	9·93	3·35	21·93
1888	4·61	24·01	45·56	6·37	1888	25·60	23·55	5·04	22·00
1889	·71	27·36	59·74	3·22	1889	8·47	·09	3·21	6·64
1890	21·92	22·49	68·08	17·99	1890	34·21	·72	9·25	27·25
1891	21·52	33·33	56·28	17·57	1891	34·19	4·78	8·95	26·91
		30–34					50–54		
1884	17·91	11·09	41·08	15·03	1884	38·83	38·37	23·97	29·86
1885	16·37	10·93	39·61	13·66	1885	34·57	37·91	17·79	27·28
1886	20·38	9·85	36·64	17·42	1886	43·32	33·74	4·59	35·88
1887	11·89	3·29	32·23	9·58	1887	22·10	15·24	14·39	16·44
1888	12·15	9·81	38·79	9·00	1888	41·00	24·58	4·39	33·46
1889	6·28	19·23	43·07	3·05	1889	20·05	23·90	17·47	19·95
1890	30·70	2·87	41·27	26·19	1890	28·75	1·30	6·33	23·53
1891	28·42	34·63	32·29	23·25	1891	28·65	·60	6·57	21·79

Table X—(*continued*).

Showing for each of the Eight years 1884–91 *the percentage by which the actual Sick Pay was* greater, *or* less *than the Expected payments according to the Valuation-provision.*

Year	Full Pay	Half Pay	Super-annuation	All Payments	Year	Full Pay	Half Pay	Super-annuation	All Payments
		55–59					70–74		
1884	20·05	6·54	17·25	18·51	1884	39·58	17·25	14·43	17·78
1885	28·55	21·57	19·10	26·37	1885	17·70	47·76	26·05	5·13
1886	35·52	16·46	28·57	32·81	1886	13·85	50·29	5·85	3·91
1887	13·55	29·44	26·90	12·52	1887	7·91	18·11	2·30	5·18
1888	34·77	49·85	14·91	32·53	1888	4·20	39·09	30·91	4·95
1889	24·18	19·01	5·81	20·59	1889	·93	8·87	41·29	16·57
1890	15·03	19·12	10·98	14·62	1890	17·44	40·31	40·96	28·55
1891	25·96	3·58	20·83	22·79	1891	13·94	15·99	49·06	27·16
		60–64					75–79		
1884	22·72	5·75	21·43	20·92	1884	100·00	100·00	106·67	6·06
1885	24·67	53·35	35·35	29·56	1885	100·00	100·00	21·43	62·08
1886	35·61	55·39	36·06	37·52	1886	86·32	100·00	14·28	41·66
1887	13·54	14·74	33·44	18·10	1887	76·19	23·81	9·53	24·72
1888	18·51	29·73	15·09	18·65	1888	60·59	65·90	15·66	27·68
1889	13·84	6·87	38·04	17·39	1889	28·27	43·71	9·09	12·62
1890	25·65	7·10	28·68	24·48	1890	39·68	3·38	21·33	7·41
1891	24·24	17·10	18·96	22·43	1891	19·89	21·28	32·47	25·90
		65–69					80–		
1884	10·36	5·66	48·82	21·39	1884	...	...	..	...
1885	1·15	13·60	49·41	15·67	1885	...	...	...	...
1886	4·37	34·61	62·38	16·56	1886	...	...	...	...
1887	26·44	31·47	75·26	40·60	1887	...	...	...	...
1888	8·36	3·29	84·22	28·74	1888	..	...	...	...
1889	6·94	28·11	62·94	24·57	1889	...	...	76·19	76·19
1890	14·33	10·63	59·22	26·46	1890	100·00	...	75·00	75·01
1891	32·14	5·63	46·40	33·38	1891	...	100·00	92·30	34·61

TABLE XI.

Number and rate of the Secessions and Deaths, with the actual and expected rates of total flux: shown for the two periods 1884–87, 1888–91, *and for the combined period* 1884–91.

1884-87.

Ages	Secessions					Deaths					Total Flux		Proportionate Number of Secessions to each Death
	Number exposed to risk	Actual	Expected	Actual Rate	Actual Per 100 Expected	Number exposed to risk	Actual	Expected	Actual Rate	Actual Per 100 Expected	Average Actual Rate	Average Expected Rate	
–19	1,378	129	94	·0935	137	1,314	1	6	·0008	17	·0943	·0735	129·0
20–24	36,379	2,209	1,964	·0607	113	35,339	131	187	·0037	70	·0644	·0599	16·9
25–29	91,801	4,073	3,161	·0444	129	89,993	456	571	·0051	80	·0495	·0410	8·9
30–34	117,430	3,261	2,904	·0278	112	116,163	728	874	·0063	83	·0341	·0323	4·5
35–39	87,873	1,609	1,332	·0183	121	87,413	688	756	·0079	91	·0262	·0238	2·3
40–44	60,228	810	351	·0135	231	60,138	634	735	·0105	86	·0240	·0180	1·3
45–49	25,979	261	120	·0100	218	26,020	346	439	·0133	79	·0233	·0215	·8
50–54	10,380	85	45	·0082	189	10,414	151	222	·0145	68	·0227	·0257	·6
55–59	5,813	36	28	·0062	129	5,864	140	149	·0239	94	·0301	·0303	·3
60–64	3,126	21	...	·0067	...	3,173	113	119	·0356	95	·0423	·0378	·1
65–69	1,208	12	...	·0099	...	1,230	58	67	·0472	87	·0571	·0549	·2
70–74	282	2	...	·0071	...	291	23	23	·0790	100	·0861	·0801	·1
75–79	25	...	...	...	...	27	4	1	·1481	400	·1481	·1148	...
80–	...	...	...	...	...	...	...	...	...	...	...	...	...
All ages	441,902	12,508	9,999	·0283	125	437,379	3,473	4,149	·0079	84	·0362	·0322	3·6

TABLE XI—(*continued*).

Number and rate of the Secessions and Deaths, with the actual and expected rates of total flux: shown for the two periods 1884–87, 1888–91, *and for the combined period* 1884–91.

1888-91.

Ages	Secessions					Deaths					Total Flux		Proportionate Number of Secessions to each Death
	Number exposed to risk	Actual	Expected	Actual Rate	Percentage of Expected	Number exposed to risk	Actual	Expected	Actual Rate	Percentage of Expected	Average Actual Rate	Average Expected Rate	
–19	3,795	231	256	·0609	90	3,683	7	14	·0019	50	·0628	·0722	33·0
20–24	51,955	2,718	2,832	·0523	96	50,677	164	268	·0032	61	·0555	·0604	16·6
25–29	105,539	4,079	3,659	·0387	111	103,693	386	656	·0037	59	·0424	·0411	10·6
30–34	117,158	3,144	2,900	·0269	108	115,902	631	874	·0054	72	·0323	·0323	5·0
35–39	100,895	1,478	1,512	·0146	98	100,546	779	875	·0078	89	·0224	·0236	1·9
40–44	74,230	753	425	·0101	177	74,242	777	917	·0105	85	·0206	·0181	1·0
45–49	48,950	395	226	·0081	175	49,092	679	843	·0138	81	·0219	·0218	·6
50–54	18,933	126	81	·0067	156	19,047	354	405	·0186	87	·0253	·0256	·4
55–59	8,055	60	40	·0074	150	8,140	229	206	·0281	111	·0355	·0304	·3
60–64	4,453	61	...	·0137	...	4,499	151	168	·0336	90	·0473	·0375	·4
65–69	2,116	38	...	·0179	...	2,163	130	115	·0602	113	·0781	·0537	·3
70–74	674	9	...	·0134	...	704	69	55	·0980	125	·1114	·0798	·1
75–79	131	2	...	·0153	...	141	20	15	·1418	133	·1571	·1148	·1
80–	8	...	...	...	...	9	4	...	·4444	...	·4444	·1600	...
All ages	536,892	13,094	11,931	·0244	110	532,538	4,380	5,411	·0082	81	·0326	·0324	3·0

TABLE XI—(*continued*).

Number and rate of the Secessions and Deaths, with the actual and expected rates of total flux: shown for the two periods 1884–87, 1888–91, *and for the combined period* 1884–91.

1884-91.

Ages	Secessions					Deaths					Total Flux		Proportionate Number of Secessions to each Death
	Number exposed to risk	Actual	Expected	Actual Rate	Percentage of Expected	Number exposed to risk	Actual	Expected	Actual Rate	Percentage of Expected	Average Actual Rate	Average Expected Rate	
–19	5,173	360	350	·0696	103	4,997	8	20	·0016	40	·0712	·0727	45·0
20–24	88,334	4,927	4,796	·0558	103	86,016	295	455	·0034	65	·0592	·0602	16·7
25–29	197,340	8,152	6,820	·0414	120	193,686	842	1,227	·0043	69	·0457	·0411	9·7
30–34	234,588	6,405	5,804	·0273	110	232,065	1,359	1,748	·0059	78	·0332	·0323	4·7
35–39	188,768	3,087	2,844	·0163	109	187,959	1,467	1,631	·0078	90	·0241	·0238	2·1
40–44	134,458	1,563	776	·0117	201	134,380	1,411	1,652	·0105	86	·0222	·0181	1·1
45–49	74,929	656	346	·0088	190	75,112	1,025	1,282	·0136	80	·0224	·0216	·6
50–54	29,313	211	126	·0072	168	29,461	505	627	·0171	81	·0243	·0256	·4
55–59	13,868	96	68	·0069	141	14,004	369	355	·0264	104	·0333	·0304	·3
60–64	7,579	82	...	·0108	...	7,672	264	287	·0344	92	·0452	·0376	·3
65–69	3,324	50	...	·0151	...	3,393	188	182	·0555	103	·0706	·0542	·3
70–74	956	11	...	·0115	...	995	92	78	·0925	118	·1040	·0799	·1
75–79	156	2	...	·0128	...	168	24	16	·1429	167	·1557	·1148	·1
80–	8	...	...	...	...	9	4	...	·4444	...	·4444	·1600	...
All ages	978,794	25,602	21,930	·0262	117	969,917	7,853	9,560	·0081	82	·0343	·0323	3·3

TABLE XII.

Numbers exposed to risk of Secession, and numbers seceding, actual and expected: shown for each of the eight years 1884–91.

Years	Number exposed to risk	No. of Secessions		Actual	
		Actual	Expected	More	Less
–19					
1884	384	47	27	20	...
1885	351	27	24	3	...
1886	327	34	22	12	...
1887	316	21	21	...	...
1888	565	31	38	...	7
1889	798	58	54	4	...
1890	1,023	63	69	...	6
1891	1,409	79	95	...	16
20–24					
1884	9,377	665	507	158	...
1885	9,237	535	500	35	...
1886	9,007	574	485	89	...
1887	8,758	435	472	...	37
1888	9,761	543	528	15	...
1889	11,560	640	628	12	...
1890	13,742	722	750	...	28
1891	16,892	813	926	...	113
25–29					
1884	23,857	1,172	820	352	...
1885	23,078	941	794	147	...
1886	22,659	1,052	781	271	...
1887	22,207	908	766	142	...
1888	23,144	893	797	96	...
1889	24,784	1,002	858	144	...
1890	27,168	1,019	944	75	...
1891	30,443	1,165	1,060	105	...
30–34					
1884	29,333	924	727	197	...
1885	29,609	804	732	72	...
1886	29,302	815	724	91	...
1887	29,186	718	721	...	3
1888	28,747	769	708	61	...
1889	28,874	833	713	120	...
1890	29,255	726	725	1	...
1891	30,282	816	754	62	...
35–39					
1884	20,568	470	309	161	...
1885	21,361	391	323	68	...
1886	22,485	388	343	45	...
1887	23,459	360	357	3	...
1888	24,603	367	371	...	4
1889	25,302	397	379	18	...
1890	25,643	367	383	...	16
1891	25,347	347	379	...	32
40–44					
1884	13,068	202	78	124	...
1885	14,588	186	86	100	...
1886	15,838	210	92	118	...
1887	16,734	212	95	117	...
1888	17,356	185	97	88	...
1889	18,095	207	103	104	...
1890	18,955	187	110	77	...
1891	19,824	174	115	59	...
45–49					
1884	4,709	53	21	32	...
1885	5,759	53	26	27	...
1886	7,020	77	33	44	...
1887	8,491	78	40	38	...
1888	10,039	108	47	61	...
1889	11,613	100	53	47	...
1890	13,039	94	60	34	...
1891	14,259	93	66	27	...
50–54					
1884	2,231	22	10	12	...
1885	2,385	17	10	7	...
1886	2,705	19	12	7	...
1887	3,059	27	13	14	...
1888	3,452	43	15	28	...
1889	4,143	22	18	4	...
1890	5,096	28	21	7	...
1891	6,242	33	27	6	...

TABLE XII—(*continued*).

Numbers exposed to risk of Secession, and numbers seceding, actual and expected; shown for each of the eight years 1884–91.

Years	Number exposed to risk	No. of Secessions		Actual		Years	Number exposed to risk	No. of Secessions		Actual	
		Actual	Expected	More	Less			Actual	Expected	More	Less
55–59						70–74					
1884	1,271	4	6	...	2	1884	42	...	...	...	...
1885	1,401	8	7	1	...	1885	61	1	...	1	...
1886	1,506	11	7	4	...	1886	70	...	...	...	...
1887	1,635	13	8	5	...	1887	109	1	...	1	...
1888	1,767	33	9	24	...	1888	130	6	...	6	...
1889	1,901	10	10	...	...	1889	154	1	...	1	...
1890	2,036	9	10	...	1	1890	174	1	...	1	...
1891	2,351	8	11	...	3	1891	216	1	...	1	...
60–64						75–79					
1884	636	3	...	3	...	1884	4	...	...	...	...
1885	723	6	...	6	...	1885	3	...	...	...	...
1886	838	5	...	5	...	1886	7	...	...	...	...
1887	929	7	...	7	...	1887	11	...	...	...	...
1888	1,014	34	...	34	..	1888	21	1	...	1	...
1889	1,052	9	...	9	...	1889	25	...	...	...	..
1890	1,147	7	...	7	...	1890	37	...	...	...	...
1891	1,240	11	...	11	...	1891	48	1	...	1	...
65–69						80–					
1884	232	5	...	5	...	1884	...	...	...	...	...
1885	273	1	...	1	...	1885	...	...	...	...	...
1886	335	2	...	2	...	1886	...	...	...	...	...
1887	368	4	...	4	...	1887	...	...	...	...	...
1888	431	21	...	21	...	1888	...	...	...	...	...
1889	479	4	...	4	...	1889	2	...	...	...	...
1890	555	5	...	5	...	1890	3	...	...	...	...
1891	651	8	...	8	...	1891	3	..	...	...	...

Table XIII.

Numbers exposed to risk of Death, and numbers dying, actual and expected: shown for each of the eight years 1884–91.

Years	Number exposed to risk	No. of Deaths		Actual	
		Actual	Expected	More	Less
–19					
1884	361	1	2	...	1
1885	338	...	2	...	2
1886	310	...	1	...	1
1887	305	...	1	...	1
1888	550	...	2	...	2
1889	769	1	3	...	2
1890	993	3	4	...	1
1891	1,371	3	5	...	2
20–24					
1884	9,062	36	48	...	12
1885	8,991	44	47	...	3
1886	8,734	28	47	...	19
1887	8,552	23	45	...	22
1888	9,503	29	51	...	22
1889	11,257	33	59	...	26
1890	13,405	48	71	...	23
1891	16,512	54	87	...	33
25–29					
1884	23,344	146	148	...	2
1885	22,664	112	144	...	32
1886	22,176	85	141	...	56
1887	21,809	113	138	...	25
1888	22,738	79	144	...	65
1889	24,331	97	154	...	57
1890	26,704	91	169	...	78
1891	29,920	119	189	...	70
30–34					
1884	28,972	202	218	...	16
1885	29,298	182	221	...	39
1886	28,981	174	218	...	44
1887	28,912	170	217	...	47
1888	28,437	150	215	...	65
1889	28,531	145	215	...	70
1890	28,967	150	219	...	69
1891	29,967	186	225	...	39
35–39					
1884	20,420	173	178	...	5
1885	21,250	170	184	...	14
1886	22,384	186	193	...	7
1887	23,359	159	201	...	42
1888	24,511	183	213	...	30
1889	25,198	188	219	...	31
1890	25,559	197	223	...	26
1891	25,278	211	220	...	9
40–44					
1884	13,033	134	158	...	24
1885	14,574	160	177	...	17
1886	15,813	160	194	...	34
1887	16,718	180	206	...	26
1888	17,343	159	216	...	57
1889	18,087	192	224	...	32
1890	18,952	180	233	...	53
1891	19,860	246	244	2	...
45–49					
1884	4,714	63	80	...	17
1885	5,775	85	97	...	12
1886	7,026	91	118	...	27
1887	8,505	107	144	...	37
1888	10,057	145	171	...	26
1889	11,626	126	199	...	73
1890	13,091	198	225	...	27
1891	14,318	210	248	...	38
50–54					
1884	2,236	33	48	...	15
1885	2,397	41	51	...	10
1886	2,712	31	58	...	27
1887	3,069	46	65	...	19
1888	3,466	69	74	...	5
1889	4,170	76	89	...	13
1890	5,130	97	109	...	12
1891	6,281	112	133	...	21

TABLE XIII—(*continued*).

Numbers exposed to risk of Death, and numbers dying, actual and expected: shown for each of the eight years 1884–91.

Years	Number exposed to risk	No. of Deaths		Actual	
		Actual	Expected	More	Less
55–59					
1884	1,283	29	33	...	4
1885	1,416	37	36	1	...
1886	1,518	37	38	...	1
1887	1,647	37	42	...	5
1888	1,774	48	45	3	...
1889	1,925	59	49	10	...
1890	2,064	63	52	11	...
1891	2,377	59	60	...	1
60–64					
1884	642	18	24	...	6
1885	736	31	28	3	...
1886	855	37	32	5	...
1887	940	27	35	...	8
1888	1,015	36	38	...	2
1889	1,061	28	40	...	12
1890	1,167	46	43	3	...
1891	1,256	41	47	...	6
65–69					
1884	236	14	13	1	...
1885	275	6	14	...	8
1886	343	17	19	...	2
1887	376	21	21	...	...
1888	432	22	23	...	1
1889	488	23	26	...	3
1890	567	28	30	...	2
1891	676	57	36	21	...
70–74					
1884	46	8	3	5	...
1885	62	5	5	...	...
1886	71	3	6	...	3
1887	112	7	9	...	2
1888	132	11	10	1	...
1889	163	17	13	4	...
1890	182	17	14	3	...
1891	227	24	18	6	...
75–79					
1884	5	2	...	2	...
1885	4	1	...	1	...
1886	7	...	...	...	...
1887	11	1	1	...	...
1888	22	3	2	1	...
1889	26	2	3	...	1
1890	40	5	4	1	...
1891	53	10	6	4	...
80–					
1884	...	...	...	...	...
1885	...	...	...	...	...
1886	...	...	...	...	...
1887	...	...	...	...	...
1888	...	...	...	...	...
1889	2	...	...	...	...
1890	3	1	...	1	...
1891	4	3	...	3	...

TABLE XIV.

Showing, for each month of the years 1884–91, *the number of declarations on and off the Sick Fund, and the number remaining Sick at the end of each month.*

Month	Declarations on the Fund	Declarations off the Fund	Number remaining Sick	Month	Declarations on the Fund	Declarations off the Fund	Number remaining Sick	Month	Declarations on the Fund	Declarations off the Fund	Number remaining Sick	Month	Declarations on the Fund	Declarations off the Fund	Number remaining Sick
1884				1886				1888				1890			
Jan.	2,934	2,562	2,433	Jan.	2,943	2,265	3,324	Jan.	3,398	2,875	3,194	Jan.	12,149	9,194	5,862
Feb.	2,068	2,259	2,242	Feb.	2,941	2,935	3,330	Feb.	3,063	2,983	3,274	Feb.	6,447	7,622	4,687
March	2,060	2,158	2,144	March	3,385	3,282	3,373	March	3,437	3,691	3,020	March	3,777	5,252	3,212
April	1,830	2,002	1,972	April	2,838	3,902	2,369	April	2,333	2,783	2,570	April	2,370	2,919	2,663
May	2,333	2,434	1,871	May	1,763	2,143	1,989	May	2,456	2,925	2,101	May	2,793	3,185	2,271
June	1,674	1,715	1,830	June	1,685	1,828	1,846	June	1,927	1,972	2,056	June	2,241	2,289	2,223
July	2,434	2,354	1,910	July	2,380	2,206	2,020	July	1,927	1,945	2,038	July	2,693	2,745	2,171
August	2,112	2,065	1,957	August	1,906	1,916	2,010	August	2,448	2,381	2,105	August	2,299	2,244	2,226
Sept.	1,996	2,080	1,873	Sept.	2,415	2,430	1,995	Sept.	1,938	2,002	2,041	Sept.	2,489	2,432	2,283
Oct.	2,526	2,362	2,037	Oct.	1,915	1,879	2,031	Oct.	2,286	2,135	2,192	Oct.	3,446	3,226	2,503
Nov.	2,103	1,913	2,227	Nov.	2,231	2,042	2,220	Nov.	2,904	2,747	2,349	Nov.	3,066	2,813	2,756
Dec.	2,390	2,090	2,527	Dec.	2,784	2,431	2,573	Dec.	2,166	1,936	2,579	Dec.	3,510	2,624	3,642
	26,460	25,994	...		29,186	29,259	...		30,283	30,375	...		47,280	46,545	...
1885				1887				1889				1891			
Jan.	3,647	3,098	3,076	Jan.	3,260	2,616	3,217	Jan.	4,023	3,416	3,186	Jan.	5,634	4,989	4,287
Feb.	2,353	2,842	2,587	Feb.	2,554	2,788	2,983	Feb.	2,889	2,878	3,197	Feb.	3,425	4,162	3,550
March	2,199	2,395	2,391	March	3,009	3,163	2,829	March	2,674	3,001	2,870	March	3,442	3,693	3,299
April	2,518	2,723	2,186	April	1,991	2,516	2,304	April	2,289	2,614	2,515	April	4,750	4,395	3,644
May	1,707	1,861	2,032	May	1,913	2,160	2,057	May	2,855	3,139	2,231	May	6,663	5,266	5,051
June	1,833	2,027	1,838	June	2,209	2,491	1,775	June	2,026	2,204	2,053	June	5,479	6,700	3,830
July	2,266	2,180	2,124	July	1,998	1,915	1,858	July	2,176	2,219	2,010	July	3,549	4,782	2,597
August	1,736	1,812	1,848	August	1,936	1,961	1,833	August	2,643	2,719	1,934	August	2,389	2,604	2,382
Sept.	1,859	1,822	1,885	Sept.	2,529	2,582	1,780	Sept.	2,386	2,252	2,068	Sept.	2,637	2,548	2,461
Oct.	2,429	2,262	2,052	Oct.	2,309	2,035	2,054	Oct.	3,106	2,885	2,289	Oct.	3,525	3,367	2,619
Nov.	2,206	1,936	2,322	Nov.	2,651	2,278	2,427	Nov.	2,646	2,427	2,508	Nov.	3,671	3,239	3,051
Dec.	2,795	2,471	2,646	Dec.	3,086	2,842	2,671	Dec.	2,931	2,532	2,907	Dec.	5,874	4,453	4,472
	27,548	27,429	...		29,445	29,347	...		32,644	32,316	...		51,028	50,198	...

TABLE XV.

HEARTS OF OAK BENEFIT SOCIETY.

Abstract of Accounts.

Income.

	1884	1885	1886	1887	1888	1889	1890	1891	1884-91
Fund at beginning of year	627,613	696,206	766,350	832,232	907,489	989,371	1,081,245	1,157,518	627,613
Contributions and Levies	208,757	215,356	218,235	223,027	233,791	246,513	262,172	282,983	1,890,834
Entrance and Registration Fees	1,050	993	952	1,105	2,406	2,677	2,786	3,978	15,947
Fines	8,002	8,011	8,284	8,457	9,106	9,775	10,121	11,067	72,823
Interest and Ground Rents	25,923	26,279	29,980	32,408	35,935	36,911	42,304	45,077	274,820
£	871,345	946,845	1,023,801	1,097,229	1,188,727	1,285,250	1,398,628	1,500,623	2,882,037

Outgo.

	1884	1885	1886	1887	1888	1889	1890	1891	1884-91
Sickness and Superannuation	99,908	106,105	116,528	114,913	122,333	124,378	155,316	169,281	1,008,762
Wives' Lyings-in	32,564	31,915	32,784	30,885	30,366	30,755	30,603	33,750	253,622
Funerals (Members and Wives)	23,422	24,647	24,373	24,992	25,682	26,756	31,104	35,761	216,737
Fire Losses	1,439	1,564	1,212	1,663	1,323	1,208	1,246	1,087	10,742
Interests purchased up	1,078	878	916	1,074	1,298	1,299	942	1,137	8,622
Imprisonment for Debt	4	7	5	6	11	5	7	16	61
Total Benefits	158,415	165,116	175,818	173,533	181,013	184,401	219,218	241,032	1,498,546
Expenses of Management	10,982	10,202	10,795	11,145	13,732	13,110	16,130	16,992	103,688
Adjustment—viz., Amounts carried to Revenue not ultimately received	5,742	5,177	4,956	4,762	4,611	6,494	5,462	6,488	43,692
Fund at end of year	696,206	766,350	832,232	907,489	989,371	1,081,245	1,157,518	1,236,111	1,236,111
	871,345	946,845	1,023,801	1,097,229	1,188,727	1,285,250	1,398,628	1,500,623	2,882,037

Table XVI.

HEARTS OF OAK BENEFIT SOCIETY.

Summary of Assets and Liabilities, as per periodical Valuations.

[Funeral, Lying-in, and sundry other Benefits, being provided by levy, not included.]

Year ending	Value of Full Pay *at 18/- per member up to age 80*	Value of Half Pay *at 9/- per member up to age 80*	Value of Super-annuation: Up to age 80, *at 4/- per member, with relief from Contributions, treated as 9/- effective*	Value of Super-annuation: After age 80, *treated as deferred annuity of 4/- weekly*	Total Liabilities	Surplus *not divided*	Value of Contributions *At 28/- per member per annum up to age 80*: Gross	Value of Contributions: Negative Values *included in the foregoing*	Value of Contributions: Net *after deduction of negative values*	Funds: Realized	Funds: Credits	Total Assets	Year ending
31 Dec. 1884	1,847,633	195,353	565,348	108,646	**2,716,980**	35,084	2,043,749	36,483	2,007,266	696,206	48,592	**2,752,064**	31 Dec. 1884
,, 1885	1,926,400	205,783	598,177	115,905	**2,846,265**	40,097	2,106,258	35,943	2,070,315	766,350	49,697	**2,886,362**	,, 1885
,, 1886	1,998,054	215,829	630,367	123,306	**2,967,556**	38,402	2,156,661	34,293	2,122,368	832,232	51,358	**3,005,958**	,, 1886
,, 1887	2,083,543	227,205	666,333	131,513	**3,108,594**	42,598	2,224,916	34,611	2,190,305	907,489	53,398	**3,151,192**	,, 1887
,, 1888	2,213,990	242,086	711,013	141,076	**3,308,165**	53,395	2,358,868	41,928	2,316,940	989,371	55,249	**3,361,560**	,, 1888
,, 1889	2,352,918	258,182	759,762	151,897	**3,522,759**	65,525	2,499,866	48,856	2,450,960	1,081,245	56,079	**3,588,284**	,, 1889
,, 1890	2,527,511	277,421	816,959	164,244	**3,786,135**	60,978	2,689,117	59,173	2,629,944	1,157,518	59,651	**3,847,113**	,, 1890
,, 1891	2,725,081	298,795	880,199	177,946	**4,082,021**	54,464	2,909,029	72,576	2,836,453	1,236,111	63,921	**4,136,485**	,, 1891

Discussion.

Mr. H. W. Manly congratulated the Institute on the valuable addition to its transactions of an essay on a subject of vital importance to the working classes. There was no one more competent to speak on this subject than the author; whose experience was larger than that of anyone else, and to whom it had been almost a life-long study to reduce the values of these various benefits into something like mathematic order. The proper valuation of these sickness funds was a far more difficult problem than the valuation of a life assurance company, and it would be many years before it was satisfactorily solved. Some 15 years ago he had had occasion to study the subject deeply, and he was happy to find that the conclusions he then arrived at had been confirmed by Mr. Hardy. In the first place he found that the valuation balance sheet conveyed no information to the management as to where they were wrong, how they were wrong, or how they were to proceed. Moreover, the size of the deficit which was sometimes exhibited, produced either despair or ridicule—mostly, he thought, the latter—but the result was always the same, they went on just as before. Not only the management, but the members of a society, should be gradually educated up to a point when they could understand something of the necessity of accumulating funds when the society was young, in order to provide for the rapidly accumulating liabilities as the members grew older. In that way Mr. Hardy had done excellent work. He had effectually educated many of the managers of these societies up to appreciate the figures produced by actuaries, and to make them understand what they meant. There were three causes of failure in these societies. In the first place there was the inadequacy of the contributions to provide for the benefits even under the most favourable circumstances. That was a hopeless condition. The first remedy was to increase the contribution or reduce the benefits, both of which the management generally tried to avoid, many of the older members considering, as Mr. Hardy had pointed out, that they had a vested interest which could not be upset. Then there was the absence of proper care with regard to admissions. That he thought of great importance, even of more importance than Mr. Hardy had attached to it. The author had been forced by the statistics to give up the case of the miners, and he would by a similar force be compelled to give up some of the other trades. The third cause of failure was the absence of sufficient supervision. Very many of these societies suffered from the ease with which malingering could be exercised. There was also a fourth cause of failure, which he would leave out of consideration—the excessive expenditure. As to the inadequacy of the contributions, he thought it desirable to make a properly-constructed table based on the actual experience of the society, but the worst feature was that the actuary was only brought in after the mischief was done. With regard to the absence of proper care in admissions and the absence of a sufficiently strict supervision, it was the management which ought to be instructed. How were they to instruct them? He had found, as the author had done, that that

was best effected by properly designed analysis of the actual and expected sickness. If the sickness was extracted according to the age at entry and the length of membership, it became possible to detect the particular point where greater care in selection should be exercised or extra supervision was required. Anyhow a statement that so much more was paid away in sickness than the contributions warranted, was a simple and intelligible statement, and with that before them, the difference in the results of consecutive valuations was often capable of a simple explanation. He had been able not only to trace to its source the introduction of a bad strain, but had been able to show how the difference between the two valuations arose from the difference of interest, and the difference in the rate of sickness. He would ask Mr. Hardy if his attention had been directed to the difference in the rate of sickness according to the length of membership. He had examined that very carefully, and had found some remarkable results. He had found that a large number stated their ages at the exact multiples of 5—that is to say, at 25 or 30 or 35. Whether it arose from the careless way of stating the age, or whether from the rate of contribution increasing at those ages, he could not tell. Then, in tracing the experience according to years of membership, he had found that at all ages of entry an excessive rate of sickness occurred in the fourth year, and a considerable increase in the secessions happened in the fifth year. That effect was not accidental, as it applied to all ages at entry, and the only explanation he could think of was that certain persons who contributed for about four years and had drawn no sick pay, probably thought it was time they got something out of it, and then, having got their money back, they left. Possibly there was a lack of supervision there. Mr. Hardy's statistics showed that the mortality in friendly societies was light; and he believed that the experience was the same in all similar societies. It would be a fortunate thing for the societies sometimes if the mortality was heavier. With regard to the passage from full pay and half pay into the superannuation stage, he found that half pay was little more than the intermediate stage between full pay and superannuation. The explanation seemed to be that genuine sickness was either cured in six months or not cured at all, but in feigned sickness half pay was not good enough, and members declared off the fund until by the rules full-pay allowance commenced again. He did not think it wise to trust entirely to medical certificates, especially in large collecting societies of this kind. The societies were obliged to employ doctors of the humbler class, who were willing to take 3*s*. or 4*s*. per member per year, and for that they were expected to examine them, give certificates, and even provide medicine. The best way, if it were possible, would be to pay the doctors for the number of members they kept off the fund, instead of on the number they actually had to deal with. In conclusion, he wished to ask Mr. Hardy whether he considered the statistics which had been laid before them that evening justified a uniform scale of contributions for all ages between 18 and 30?

Mr. G. F. Hardy said he had been struck with the amount of interest with which the author invested a subject which was usually

considered a very dry one. The Institute was greatly indebted to Mr. Hardy not only for statistics which in themselves were valuable and significant, but also for the generous manner in which he had explained his processes and methods of working, a thing which experts were not always ready to do. One point of great interest had been touched on by Mr. Manly, and that was the question of the death rates. The statistics contained in the paper specially brought out the inadequacy of the Manchester Unity table, both in respect of death rates and as to the amount of continued sickness provided for by that table. The Hearts of Oak represented a mass of members drawn from all parts of the kingdom, some from towns and cities, some from rural districts, and the rate of mortality was nearly 20 per-cent below that provided for in the Manchester Unity table. Anyone who had given attention to the subject knew that that meant a considerable addition to the future liabilities of the society. So important was this question that Mr. Hardy had considered it necessary to use the experience of the society itself in the matter of death rates in his investigations. This feature was commonly met with in friendly societies when they were working with the Manchester Unity table. Quite recently he had had three or four societies under his notice and in every one alike the feature of extremely light rates of mortality had manifested itself, making a very great difference in the expected future liability of the society. This point had some bearing on the interesting experiment of admitting to membership in the Hearts of Oak certain trades, formerly classed as unhealthy; but the mere fact that certain trades such as mining and various other occupations entailed a larger amount of sickness than ordinary occupations was after all not the whole of the case. Much depended on the relation which that additional sickness bore to the relief the society experienced on the other hand from the heavier death rate of such members. The provision in respect of superannuation of miners must be less costly than would be the case for agricultural occupations, where the rate of mortality was very low. He should almost from that point of view say that one of the most dangerous trades for friendly societies to deal with was that of the agricultural labourer. The mortality was extremely light throughout the whole of life, and as soon as anything like advanced age came on the rates of continued sickness were extraordinarily heavy. In reference to the question of uniform contributions, which had been mentioned by Mr. Manly, he would ask Mr. Hardy whether he considered there was any compensating advantage in a greater power of medical selection of members at ages of 25 to 30, as compared with members entering at younger ages. Another point was the matter of secessions. The fact that the limits of age at entry were so narrow as from 18 to 30, practically only a little more than ten years, so that the extreme ages at entry could not differ from the mean age by more than about five years, made it possible to treat the rate of secession as a function of the age alone, and to do so safely. But in societies where the limits of age were wider, when members might enter up to 40 or 50, or even older, it was undesirable to do that, and it would lead to erroneous results. A most interesting and valuable part of the paper was the

appendix, in which it was shown how the liabilities in respect of the benefits might be exhibited according to the ages at which the payments would actually be made. Some of them had no doubt been aware for a long time of the existence of this method as employed by Mr. Hardy, and had frequently used it with advantage, and it was satisfactory to know that it was now to be published in the *Journal*. The great power of this method for tabulating the sickness liabilities, enabling them as it did to pass rapidly from one standard of sickness to another, could only be known to those who had repeatedly used it in practice. One remark in the paper would strike them all as extremely disappointing, where the author stated that in the whole of his experience the Hearts of Oak was the only society which had taken his recommendations as to the adjustment of benefits and contributions. When they thought of the number of societies which had passed under his review, and that upon all, large or small, he had bestowed the same skill and experience as in the case of the Hearts of Oak, it was lamentable to think how generally the results of such experience and skill had been wasted by the refusal of societies to take any notice of the advice tendered. On the other hand there was certainly a brighter side to the picture in the remarkable and encouraging results that had been achieved in the case of the Hearts of Oak, where the recommendations of the actuary had been attended to.

Mr. M. N. Adler said that he was likewise anxious to express his appreciation of Mr. Hardy's courtesy in placing the results of his experience at the disposal of his colleagues. Mr. Hardy had stated that the Hearts of Oak was about the only friendly society which had acted in accordance with the advice he had proffered: this was indeed a sad avowal, and besides being a misfortune for the societies themselves seemed but a poor requital for Mr. Hardy's labours. His own experience had not been quite so lugubrious, and in the case of the larger societies he generally found that the secretary or the committee of management were of fair average intelligence, and it was possible, when brought into personal contact with these representatives, to influence them—and through them the society—to accept timely advice. There were, however, many societies, especially in villages, whose management was in hands utterly unfitted for the task. It was true that valuation balance sheets and abstracts might be incomprehensible to the general mass of the members, but the actuarial report could be worded in plain, simple language, and thus enable the actuary to bring at least the salient features home, especially the fact whether the concern were solvent or not. He believed that comparatively few of the members ever saw the actuarial report, although it was compulsory that such a report should be kept hung up at the registered office of the society. In order to make certain that the result of the actuarial valuation was brought to the notice of the members generally, he suggested that it should be obligatory on the societies to furnish the Central Registry Office with a declaration, signed by the secretary or other principal officer, which should accompany the annual return, to the effect that that particular requirement had been complied with; this would safeguard the members from being kept in practical ignorance

of the condition of their society. He was afraid it must be taken for a fact that there were at the present time numberless societies financially rotten, and something surely should be done to improve the existing system. He thought it might be enacted that when the actuary reported that a deficit existed, it should be compulsory on the society forthwith to appoint a committee who should put itself in communication with the actuary and within a reasonable time submit a scheme under the actuary's advice; and formal resolutions should be come to at a general meeting of the members. It would be necessary that copies of the resolutions passed at such meeting should be sent and filed at the Central Registry Office. The Life Assurance Companies Act of 1870 had undoubtedly proved of inestimable benefit, and he saw no reason why the Friendly Societies Act of 1875, with some amendment, should not prove as beneficial to friendly societies.

Mr. Neison said he was disappointed to see Mr. Hardy take the pessimistic view he did of the results of the labour of the actuary in connection with these organizations. His own experience had been much more favourable. It was to be remembered that any reform which took place in nine out of ten of these societies affected the older members more than the younger, and naturally they felt that they had a vested interest in their funds, and were urged somewhat by selfish considerations. His own experience was that if they could convince members that there was a radical necessity for reform that in course of time the labours of the actuary would not be thrown away. As to the substitution of an annuity at 65 for the sickness pay, members of these societies would not hear of it. Their answer was generally this, "Why, we shall never live to 65." An extraordinary illustration of this came under his knowledge some few years ago, when he had had to address a meeting of members of friendly societies. He asked the chairman if he would kindly put the question, how many of those present thought they would be able to participate in the benefit at 65? There was a large audience of some 2,000, and a man with the reputation of being an intelligible man got up and made the statement that he thought 20 would. Another equally great authority stood up and said he thought perhaps there would be 15. The proper answer was about 1,000. That exemplified the want of grasp which members in the majority of cases had. Again, to pay an annuity of an adequate amount required a larger payment on the part of the member, and it was hopeless to ask the working classes to provide for any pension, when coupled with that was the condition that they would lose all their contributions at their death. Mr. Hardy referred to the necessity he had been under of considerably augmenting the provision of the superannuation allowance in the Manchester Unity experience. He (Mr. Neison) had expected it to be higher, but not quite so high as was shown in the tables before them. As to the rate of mortality and withdrawals in the Hearts of Oak it was remarkably low as compared with other societies, but when they came to the sickness tables they were struck with the higher rate of sickness, and that he had rather expected to find because the Hearts of Oak was totally different in its character to all other friendly societies. It attempted

to deal with sickness spread all over the Kingdom, and consequently there was a very great difficulty in maintaining an efficient supervision, and where supervision was inefficient it would be generally found that there was an excess of short attacks, and a considerable extension of the contracted sorts of illnesses. If they compared the rates which Mr. Hardy had brought before the meeting with the standard of the Manchester Unity or the Foresters, both these features would be very prominently brought forward. He thought the Hearts of Oak were highly to be congratulated on being able to maintain their sickness business at such comparatively easy rates, and also upon having been wise enough to avail themselves of Mr. Hardy's advice.

Mr. Bunn, on behalf of his colleagues on the board of management of the Hearts of Oak, as well as of their members, said that the reception given to the paper would be highly appreciated. Mr. Hardy had rendered a service to a large and important organization which they were scarcely in a position at the present time to estimate. The reference which Mr. Hardy had made in the paper to his own work was a very modest one. The society was started in 1842, at a time when actuarial investigations were things almost unknown. It was started by what had commonly been called the rule of thumb. The rates of contributions were probably copied from a contemporary society, without any consideration as to their adaptability to the special needs of the coming members. It was thought that because one society was conducting a business on a certain basis, therefore it must be safe for them to follow their example, and one of the most astonishing things in connection with that fact was that a society established in that way, without any actuarial advice, could have gone on for a period of something like 33 years without seeking the advice of an actuary. They were then in possession of a sum of £200,000, and they had never experienced a year when their liabilities had exceeded their assets. Surely it was a very pardonable thing that the members thought it was quite time that the founders of the society should get some benefit from that large accumulated capital in the way of increased benefits. A revising committee was appointed, which actually prepared a code of rules providing for increased benefits. It was at that time that Mr. Hardy's advice was sought, and as a result not only was all hope of any increased benefit that might be given to their aged members dashed to the ground, but in its place there was an increased contribution. He thought it spoke well for the then managing body that they were courageous enough to appeal to the great body of members to accept Mr. Hardy's advice. He desired briefly to refer to the policy of publicity which the society had adopted, and which had done much to strengthen public confidence. Sometimes the light of public criticism was directed upon a society in a very unfair and unjustifiable manner, but the Hearts of Oak had adopted a policy of publicity years ago, and were determined to pursue it. They believed that the full publication of their accounts had had a result which was very beneficial to the society. In addition to a separate reprint of the actuarial report, which was furnished to all the managing body, an abstract of that report was

published in their Annual Report, which gave the most important details. The Hearts of Oak was now trying to manage a very large business from one centre, and they were attempting a sickness business, which, under any circumstances, was much more difficult to manage than ordinary life assurance business. Their operations embraced all classes and conditions of men all over the United Kingdom. The result of that experiment could be seen in Mr. Hardy's paper. The first difficulty was certainly that of supervising the sick, and no one except those who had been intimately acquainted with the affairs of a society like this could appreciate the difficulty as it deserved. They had a special committee, whose duty it was to deal with that subject alone. They were also fully alive to the difficulty of the death levy. They had been unable to alter it, but were alive to its possible future consequences. He hoped that in time they would be able to create a death fund reserve that would help them in that direction. Then no doubt many gentlemen had noticed with surprise that they were still valued at 4 per-cent, but they had under their consideration a proposal in the direction of strengthening the valuation in that respect. Some of them hoped that the Legislature would hit upon some plan for compelling societies to put their house in order, but it was a difficult problem to deal with, because coercion might defeat its own ends. The Legislature might compel a society to reconsider its position in the event of an unsatisfactory result of the actuarial investigation, but it would be beyond the power of the Houses of Parliament to compel the members to carry out any specific recommendations. In such cases the members had the remedy in their own hands. If coercion were tried it would be disastrous, because the men who would be the first to leave would be those whose continued membership it was of the utmost importance to retain, namely, the younger members.

Mr. Hardy, in reply, expressed his thanks for the attention they had given him. Mr. Manly had suggested that the proper way of measuring sickness experience was to keep all members of the same age of entry together. That might be desirable in a society where the range of age of entry was considerable, but in a society like the Hearts of Oak where nobody was admitted after age 30, he would ask him to re-consider that recommendation. He had been asked whether he had any objection to a uniform contribution between the ages of 18 and 30? In reply to that he would ask whether the rate for age 30 was adequate? If that was so, the uniform rate would do no damage. He had no objection to a man of 18 paying the assessed risk of a man of 30. Mr. G. F. Hardy had referred to the mortality experience of the agricultural labourer. That, no doubt, was a very difficult liability to measure. He knew of one society in Suffolk which had an unusually large separate pension fund, and when taking out their mortality he found it a great deal better than that shown by the Government tables. As to secessions he never meant that they should be generally applied, but only in the particular case he had in view. He would ask Mr. Adler how the cause of financial soundness could be advanced by requiring a certificate to be forwarded to the Registry that the results of a

valuation had been laid before the members? He would take upon himself to say that there was a great deal of imposition with regard to the superannuation benefit of the Hearts of Oak. He need only add that the labour he had spent over the paper had been more than rewarded by the interest with which it had been received.

CORRESPONDENCE.

ON THE VALUATION OF PREMIUMS.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In a letter in the last number of the *Journal of the Institute*, Mr. James Chatham discusses the question of the average date on which the premiums valued are assumed to fall due. After mentioning various methods of adjusting the account to allow for the unequal distribution of payments throughout the year, he proceeds to point out that, in consequence of the days of grace and the delay of agents in remitting, nearly two months elapse before the premiums are actually received by the office. He then says, "The reserve, "therefore, ought to be increased by, roughly speaking, two months' "premium income." This conclusion appears to be erroneous, as it assumes that the delay in payment at the head office is equivalent to the postponement of the due date for two months. Of course, this is not so, as in the event of death during that period the premium is payable, and the only loss caused by the delay is a small one consisting of interest.

I am, Sir,

Your obedient servant,

T. G. C. BROWNE.

11 *Lombard Street, E.C.*
16 *October* 1893.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—My attention has been drawn to a letter from Mr. C. D. Higham (*J.I.A.*, xxvi, 478), in which he mentions the point I referred to in the letter you were good enough to insert in your last issue. He there suggests "that claim acceleration reserve should be "based on the theoretical instead of on the actual date for payment, "at any rate as long as the fraction combined with the annuity-value "in capitalizing future premiums is dependent on the date of their "falling due rather than of their being received." This method has certainly the merit of consistency, but I am not aware of its ever having been adopted by any office.

I take this opportunity of giving the formula I propose should be used for finding the addition to be made to the reserve to provide for the loss of two months' interest on the premiums, in order to remove any misapprehension my former letter may have created. Assuming the claims are paid on proof of death and title, and that the adjustment for this is an addition of (say) four months' interest to the reserve calculated in the usual way, the formula I propose is

$$\frac{i}{6}\Sigma\mathrm{P}\left(1+\frac{i}{3}\right)(k+a).$$

where k is the fraction used in the valuation. The formula can easily be adjusted to suit any other circumstances. Applying it to the case I supposed, the addition to the reserve would be about £15,000. It will be seen, therefore, that it is not so large as the addition for payment of claims upon proof of death and title, but I think it is sufficiently large to justify special provision being made for it.

I am, Sir,

Your obedient servant,

JAMES CHATHAM.

Edinburgh,
21 *October* 1893.

CONSUMPTIVE FAMILY HISTORY.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—A brief reply seems called for from me to Mr. Macaulay's letter in the last number of the *Journal.* He rightly describes the principle upon which the Mutual cases having a consumptive family history were selected, quoting Mr. Manly's words as follows: "supposing that case were to come before you to-day, would you consider the cause of death thus recorded to be primarily consumption."

In making the selection, I chose only those cases which all medical men would be agreed to consider as consumption, such as decline, chronic pneumonia in a young subject, &c.; so that the results of the Mutual investigation are quite free from any personal judgment. It was not in any case taken for granted that childbirth recorded as the cause of death indicated consumption, nor is it my opinion that in a large proportion of cases it does so, as Mr. Macaulay appears to think.

Childbirth in family history is rightly, I believe, considered as consumption when no further information is forthcoming as a precautionary measure simply; enquiry will usually clear up these cases, and a certain small proportion of them will turn out to be really consumption.

Yours truly,

THOS. GLOVER LYON, M.D.

London,
23 *December* 1893.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1892, published in 1893.]

INCOME	Ordinary Companies	Industrial Companies	Total
	£	£	£
Balance at the beginning of the Year	171,547,169	9,145,113	180,692,282
Adjustment for Balance transferred	−22,585	+22,585	...
	171,524,584	9,167,698	180,692,282
Premiums	14,565,861	5,467,096	20,032,957
Consideration for Annuities	1,096,870	4,330	1,101,200
Interest and Dividends (less Tax)	6,618,912	312,400	6,931,312
Increase in value of Investments	163,092	...	163,092
Fines, Fees, &c.	9,246	473	9,719
Capital Paid-up	65,069	331,878	396,947
Customs Timber Measuring, &c.	2,786	...	2,786
Donations (Itinerant Methodists)	2,190	...	2,190
Transfers from other Accounts	803,061	600	803,661
Miscellaneous	15,873	3,434	19,307
	194,867,544	15,287,909	210,155,453

OUTGO	Ordinary Companies	Industrial Companies	Total
	£	£	£
Claims	12,396,122	2,537,261	14,933,383
Cash Bonuses and Reduction of Premiums	1,045,530	15	1,045,545
Surrenders	823,776	14,113	837,889
Annuities	806,234	1,742	807,976
Commission	752,447	1,501,917	2,254,364
Expenses of Management	1,333,585	974,319	2,307,904
Bad Debts	1,502	93	1,595
Decrease in value of Investments	58,977	395	59,372
Interest on Capital and Dividends and Bonuses to Shareholders	587,042	55,957	642,999
Transfers to other Accounts	775,682	39	775,721
Return to Proprietors (Pelican)	58,475	...	58,475
Miscellaneous	28,248	8	28,256
Balance* at the end of the Year	176,199,924	10,202,050	186,401,974
	194,867,544	15,287,909	210,155,453

* This Balance includes the whole of the Life and Annuity Funds (£180,091,746), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1892).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,343,809	854,916	12,198,725
Life and Annuity Funds	170,499,520	9,592,226	180,091,746
Fire Funds of Companies transacting Life Business	10,362,981	...	10,362,981
Marine Funds of Companies transacting Life Business	630,763	...	630,763
Reserve Funds	4,161,841	...	4,161,841
Other Funds	537,765	264,206	801,971
Profit and Loss Balances	2,736,269	...	2,736,269
Depreciation and Investment Balances	974,534	298	974,832
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims	3,760,612	37,566	3,798,178
Outstanding Accounts	527,662	21,394	549,056
Temporary Loans	116,168	1,083	117,251
	206,754,724	10,771,689	217,526,413

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	83,585,855	309,075	83,894,930
Loans on Policies	9,523,736	27,395	9,551,131
,, Rates	19,383,340	3,793,436	23,176,776
British Government Securities	5,298,096	502,330	5,800,426
Indian and Colonial Government Securities	13,469,021	118,530	13,587,551
Foreign Government Securities	3,734,407	...	3,734,407
Debentures	22,912,170	1,692,555	24,604,725
Shares and Stocks	13,538,470	171,733	13,710,203
Companies' own Shares	582,287	173	582,460
Land and House Property and Ground Rents	13,248,100	3,095,220	16,343,320
Life Interests and Reversions	3,736,742	806	3,737,548
Loans on Personal Security	1,456,968	13,469	1,470,437
Agents' Balances and Outstanding Premiums	4,649,722	423,693	5,073,415
Outstanding Interest	1,985,602	108,729	2,094,331
Cash, Deposits, Stamps, &c.	9,446,152	145,902	9,592,054
Customs Timber Measuring Balances, &c.	1,704	...	1,704
Book-Room Grant (Itinerant Methodists)	62,000	...	62,000
Deficiencies, Preliminary Expenses, &c.	140,352	368,643	508,995
	206,754,724	10,771,689	217,526,413

Increase (+) *or* Decrease (−) *in the Chief Items of this Year's* Summary *compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
Income.*	£	£
Premiums	− 267,498	+ 435,270
Consideration for Annuities . . .	− 87,835	+ 4,255
Interest and Dividends (less Tax) . .	− 125,964	+ 37,838
Net Result of Realization and Re-valuation of Investments	+ 73,251	+ 68
Outgo.*		
Claims	+ 612,232	+ 352,410
Annuities	− 64,836	+ 608
Surrenders	+ 31,703	+ 1,579
Commission	+ 21,035	+ 124,744
Expenses of Management	− 57,212	+ 129,603
Liabilities.		
Paid-up Capital (including sundry Shareholders' Balances)	− 8,330	+ 335,172
Life and Annuity Funds	+4,579,359	+ 719,144
Assets.		
Mortgages (including Loans on Rates) .	+ 766,002	+ 655,890
Life Interests and Reversions . . .	+ 256,423	+ 13
Loans on Policies	+ 306,531	+ 3,600
British Government Securities . . .	− 617,278	− 22,138
Indian and Colonial Government Securities	+ 773,066	+ 4,882
Foreign Government Securities . . .	+ 200,558	...
Debentures	+1,055,369	+ 238,625
Shares and Stocks	+ 607,068	− 5,734
Companies' own Shares	+ 44,677	+ 173
Land and House Property and Ground Rents	+ 396,823	+ 134,499
Loans on Personal Security . . .	+ 140,060	+ 277

* Excluding the figures of the Gresham, as, owing to an alteration in the date of their financial year, no Returns are yet due.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 98, of which 86 have been classed as Ordinary, 8 as Industrial, and 4 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately.

During the year two new names have been added to the official List of Companies, namely, the Clergy Pensions Institution, and the Independent Order of Foresters (Toronto); in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies.*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.		£		£		£	£	£
Whole Term of Life	713,981	349,112,284	88,078	57,450,390	802,059	406,562,674	19,282,668	387,280,006
Limited number of Premiums . .	28,973	16,656,989	5,014	2,379,185	33,987	19,036,174	685,318	18,350,856
	742,954	365,769,273	93,092	59,829,575	836,046	425,598,848	19,967,986	405,630,862
Endowments . .	1,865	419,918	7,007	1,211,289	8,872	1,631,207	6,000	1,625,207
Endowment Assurances . . .	297,694	49,157,283	29,304	8,926,203	326,998	58,083,486	747,263	57,336.223
Joint Lives . .	12,199	2,469,570	2,116	932,449	14,315	3,402,019	308,339	3,093,680
Last Survivor . .	1,183	921,075	1,217	1,295,456	2,400	2,216.531	303,892	1,912.639
Contingent . .	41	42,446	3,041	5,049,292	3,082	5,091,738	1,161,404	3,930,334
Issue . . .	6	10,631	913	3,676,410	919	3,687,041	1,063,842	2,623,199
Miscellaneous . .	197	132,846	4,116	5,103,707	4,313	5,236,553	1,192,367	4,044,186
	1,056,139	418,923,042	140,806	86.024,381	1,996,945	504,947,423	24,751,093	480,196,330
ANNUITIES.								
Immediate . .	...	...	...	...	19,114	905,606	10,244	895,362
Deferred . .	...	...	...	...	4,211	151,052	10,022	141,030
	...	...	...	...	23,325	1,056,658	20,266	1,036,392

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.						£		£
Whole Term of Life	...	...	...	...	12,497,257	117,599,191	...	117,599,191
Limited number of Premiums . .	...	...	...	...	128	3,089	...	3,089
	...	...	...	...	12,497,385	117,602,280	...	117,602,280
Endowments . .	...	...	...	...	71,413	1,039,442	...	1,039,442
Endowment Assurances . . .	...	...	...	...	69,308	1,005,400	...	1,005,400
Joint Lives . .	...	...	...	...	196,036	3,113,509	...	3,113,509
	...	...	...	...	12,834,142	122,760,631	...	122,760,631
ANNUITIES.								
Immediate . .	.	...	..	...	1	15	...	15

The above figures are based on Returns deposited for the most part during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of two companies, namely, the Co-operative and the Northern, the amount of business at a more recent date has been included, but the figures of the six Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

List of Members of the Institute, 1893.

ERRATUM.

WE are requested to state that in this List two asterisks should have been placed against the name of WILLIAM WORTHINGTON, Student, instead of one only, as the List stands—Mr. Worthington having passed the second part of the Examination for the Class of Associate, in April last.—ED. *J.I.A.*

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Methods of deducing the Rate of Mortality from the Experience of Assured Lives; with some mention of a method adopted in investigating the experience of the Clerical, Medical and General Life Assurance Society. *By* William J. H. Whittall, F.I.A., *Assistant Actuary of the Clerical, Medical and General Life Assurance Society.*

[Read before the Institute, 18 December 1893.]

In venturing to bring this subject before the Institute to-night, I am conscious of inviting attention to a matter which has been frequently treated before and by far abler hands than mine. I at once confess that I have little original matter to bring forward; indeed, the main object of this paper is to endeavour to bring into one focus and to a common standard of criticism the various formulas for ascertaining the rate of mortality, which have already been so often discussed in this room. The expression "rate of mortality" is advisedly used here instead of "exposed to risk", because it would seem that attention has hitherto been unduly centred in the latter.

It is now well known that the Council, acting upon the initiative of our President, who has warmly espoused this cause, and in concert with the Faculty of Actuaries, are taking the preliminary steps to collect fresh material for deducing the modern experience of British offices. This being so, the question as to which is the most suitable formula for adoption

will of necessity arise and call for decision. Any contribution, however humble, designed to throw fresh light upon a question which is so difficult and which has so much divided expert opinion in the past, need not, perhaps, be despised; and it is especially my hope that I may this evening succeed in raising such a general discussion amongst members as to the best principles to be adopted as shall materially assist the deliberations of the Council.

We are not obliged to consider that any one formula heretofore devised is necessarily the best. It may not improbably prove to be the case that in a combination of features contained in two or more existing formulas will eventually be found the method which will best commend itself to the Institute as being upon the whole the most suitable for adoption.

It has been usual in discussing the respective formulas already in the field to divide them into two rival categories—though I think that the classification has not always been correctly carried out. The formulas have been arranged into two classes, according as the result of them is to follow "calendar years" or "policy years." To these two divisions a third should, in my opinion, be added for the sake of theoretical completeness, namely, formulas which would result in following the actual years of life of the assured, or "life years."

Of the last-named class there is, I believe, only one prominent instance of its application to assured lives—the method devised by Mr. J. J. Downes for deducing the experience of the Economic Office, and described by him in a pamphlet dated 5 April 1862. Other general examples are the Peerage experience of Messrs. Bailey and Day, and the Upper Class experience of Mr. Ansell. As Mr. Downes's method has been from time to time referred to, but never, I believe, described in the *Journal*, a brief reference to it may not be out of place later on. Additional interest attaches to it from the fact that, as mentioned in the Institute Volume of *Mortality Experience* (p. 9), Mr. Downes was the introducer of the card system.*

The great difficulty in applying to assured lives any "life year" formula, as I propose to call it for purposes of distinction, is that, except in the few cases in which the observation commences on the exact birthday of the life, there is a fractional

* This statement occurs in a Report by the Council of the Faculty of Actuaries. Mr. J. J. Downes himself, in a foot note to his pamphlet (p. 4), states that the suggestion for employing cards originated with Mr. O. G. Downes.

part of a year of exposure to be dealt with before the attainment of the first exact age. This entails a special calculation in each individual case, which must be made with a certain amount of exactitude. These fractional periods may, conceivably, be taken on the average by a process of approximation, as they were, in fact, taken by Mr. Finlaison in his Government Annuitants' Experience 1883, which is also based on the life year method, and which will be referred to later on. For practical purposes, however, we must consider that the method, as applicable to assured lives, entails the calculation of individual fractions in almost every case for both entries and exits.

A further point arises here. When the fractional portions of, say, the current year of age at entry x have been collected, they are necessarily treated as part of the exposed to risk at age $x-1$. This, of course, involves a slight error in the results. The same observation applies to discontinuants; and also to existing, if the experience be carried up to a given point of time. When once, however, this reservation has been made, we have in the life year method, as far as I am able to see, an exact presentment of the mortality experience of the body observed. Dr. Farr described a life table as exhibiting the "march of a generation through time." Here, then, we have the generation of a life office passing not through calendar years, not through policy years, but through years of life—"through time."

Be that as it may, the labour involved in working the life year method and—an even more important objection—its inadaptability to the formation of select tables, have effectually and, I fear, necessarily, barred its entry into favour as a desirable method for assured lives. For our purposes the necessities of modern practice have made it requisite that the material should be in such a form as to be readily manipulated into different shapes and for different purposes; and though I confess to a feeling of regard for its truthfulness, we must consider this method to be unsuitable unless in the case of a homogeneous experience, which is to be thrown solely into the form of a "mixed" or "average" table.

Reverting now to the rival systems of "calendar years" and "policy years", it may be well to seek a clear definition of these expressions, which at present, apparently, are not entirely understood. For instance, I find that Mr. Chatham, who in his prize essay (*J.I.A.*, xxix, 81) gave much attention to the subject now before us, placed Mr. King's formula amongst the calendar year formulas. But Mr. King in the subsequent discussion

disagreed with this view. He considered that the averages were so taken that they really got policy years. Deferring for the moment the further discussion of Mr. King's formula, it is evident that there is here a difference of opinion upon what ought to be determinable as a fact, which is little creditable to our sense of scientific exactitude, and to the terms which have been in common use among us for years past.

At the risk of being tedious, and in the hope of arriving at a clear understanding upon this point, I ask: What is it which we follow through calendar years or policy years as the case may be? Clearly the rate of mortality. What is the rate of mortality? The ratio of the deaths to the exposed to risk. Now this ratio is smallest during the early years of assurance, *i.e.*, during the period when we are most anxious to observe the effect of selection. Hence, the method by which the numerator of the fraction is arrived at is of the utmost importance—far greater, it seems to me, than the method by which the larger denominator is ascertained. It would, therefore, seem to be probable that in the system of determining the ages at death in any particular formula will be found the key to its proper classification.

Coming now to a consideration of the policy year method, the object of which is to observe the rate of mortality through policy years, we are, I think, led naturally to the enunciation of a cardinal principle, namely, that no method deserves the name of a policy year method which does not rigidly locate every death in the exact policy year in which it occurs. And having admitted this as a necessary and vital principle, the question arises: Is anything further necessary to a policy year method? Clearly the method by which the ages at entry are arrived at, though it may affect the general excellence of a formula, does not affect the vital question, which is a matter purely of the relation existent between the age at entry and the age at exit. Where the exit is by death, the duration must, in order to satisfy the principle above stated, be in every case the actual curtate duration, the age at death being consequently obtained by adding such duration to the age at entry.* Where the exit is by discontinuance or by closure of the observations, the actual duration should, for purposes of arriving at the age at exit, be taken; or, where the actual duration is not an integral period, as is usually the case,

* In speaking of the "curtate duration", I adopt the expressive phrase used by Messrs. Hardy and Rothery (*J.I.A.*, xxvii, 165). It represents, of course, the integer of the ascertained duration of the policy.

the best available approximation thereto should be had recourse to. But these latter elements enter only into the exposed to risk—the denominator of the fraction—and being thus of minor importance when compared with the deaths, we may in regard to them well retain, without any sacrifice of consistency or principle, an open mind as to the formula which best determines them. If, however, acceptance is gained for my view that the chief test of a policy year method is to see that the deaths are allocated to the exact years of assurance in which they occur, and that in order to obtain this result the age at death must be the age at entry *plus* the curtate duration, we shall have a simple and easy principle to apply when considering the merits of any particular formula.

Is there any similarly clear principle to be laid down as to the main characteristic of the calendar year method? By this method observation is first made of the calendar year in which the assured enters. If he die or disappear in the same year, a note is made of that. If not, he is observed through each succeeding calendar year, from 1 January to 31 December, until a calendar year arrives in which he disappears, either by death, by discontinuance, or by closure of the observations, and that calendar year is noted. In the calendar year of entry the extent of the exposure is fractional, and that exposure is reckoned at an age one year lower than the first complete calendar year of exposure. Similarly in the year of exit, the fractional period of exposure is reckoned at an age one year older than the last complete calendar year of exposure. There is nothing in this to make it essential how the ages at entry are reckoned, or what assumptions are introduced into the determination of the fractional periods in the years of entry and exit. For instance, the ages at entry might be ages next birthday *minus* ·5, or any other fraction, or nearest ages, or mean ages*; and the assumption as to fractional periods might be, as has usually been the case, that the entrants in any particular calendar year contribute on the average six or some other number of months' exposure in that year, or—more accurately—a system of calculating the actual fractional periods of exposure might be adopted, as in the case of Mr. Downes's life year formula. Then what is the essential feature of a method which, having first assumed that the lives will attain

* I use the term "mean age" at entry, as previous writers have used it, to signify the age obtained by deducting the calendar year of birth from the calendar year of entry. Similarly, the "mean duration" is the calendar year of exit *minus* the calendar year of entry.

certain ages on the 1 January following entry, then proceeds to allocate each death and discontinuance to the proper calendar year in which it occurs? It is neither more nor less than the addition of the mean duration to the age at entry in order to ascertain the age at exit. This conclusion agrees with the proposition before laid down and found to hold good in the case of the policy year method, that in the system of determining the ages at death would be found the key to a proper classification; and, as was to be expected, it agrees with the actual process adopted by the Institute in 1869 in calculating the ages at exit on the cards. It will be noticed that when once the ages at entry and fractional periods of exposure have been determined, approximately or otherwise, only the calendar years of entry and exit are required to be known, in order to determine the age at exit. This reduces the labour of extracting the original facts to its lowest terms—a point which constitutes a strong argument in favour of the calendar year system, and which no doubt secured its adoption by the English Actuaries in 1843 and again by the Institute in 1869.

I have said, in regard to both methods under examination, that the mode of determining the ages at entry is not an essential element. It may, however, be well to point out generally that, inasmuch as in the policy year method the ages are forced as it were to synchronize with the policy anniversaries, that method of determining the ages at entry will be best which most nearly agrees with the actual ages of the assured at entry. Similarly, inasmuch as in the calendar year method, the ages are forced to synchronize with the beginning and end of the calendar year, that method will be best which most perfectly groups the actual ages of the assured around the end of the calendar year of entry.

To summarize then, we see that there are two methods, each of which in one important respect follows the actual facts, namely, the "life year" method, which follows the actual ages of the assured, and the "policy year" method, which follows the policy years. A third method, the "calendar year" method, has found great favour; but there is at first sight little to recommend it beyond the ease with which the original *data* can be extracted. When compared with the other methods, it is seen to follow neither the exact ages of the assured nor the exact years of their assurances. In fact, it may be said to possess the defects of both and the advantages of neither.

I will now proceed to discuss in detail the various methods

which have been either used or suggested; but as the descriptions of the various formulas by the authors themselves are in almost all cases familiar to members, and as similar ground has been so often gone over before, notably by Mr. Chatham in his prize essay already referred to, I will, to avoid repetition, ask that my observations may be read in connection with the statements and explanations of formulas which have previously been given. The dates appended to the titles of the different experiences are the years of publication, and they have been arranged in that chronological order.

I have often thought that the Institute formula would be made clearer to students by assigning different symbols to the completed age, as it appears in the final tables, and to the current year of age, as taken from the card. Mr. King has suggested to me an endeavour to extend this principle to other methods; and as it seemed that such an attempt, if successful, would tend to show more quickly and more clearly the assumptions underlying the various formulas, I have in the following pages assumed that, in the case of subscripts,

x = Exact age, as used in the age column of the final table.

$\{x\}$ = Current year of age, or age next birthday.

(x) = Nearest age.

$|x|$ = Mean age.

t = Exact duration.

$t\rfloor$ = Curtate duration.

(t) = Nearest duration.

$|t|$ = Mean duration.

As regards the entrants and the mode of exit, I will follow previous writers in putting

n = New entrants;

d = Deaths;

w = Withdrawals;

e = Existing.

The endeavour to differentiate by symbols the various modes of arriving at ages at entry and exit requires, in the case of the latter, facilities for exhibiting separately the component parts of the age where it consists of the age at entry *plus* the duration. It is therefore proposed to give the formula for the exposed to risk at age $x+t$ in each case, instead of at age x, as is more usual. It will be understood that in dealing with, for instance,

age at exit $\{x\}+|t|$, each symbol is variable, and the expression includes entrants of different ages and assured of different durations, subject only to the total age attained being correct according to the terms of the formula.

MR. GALLOWAY'S AMICABLE EXPERIENCE, ... 1841.

This method has been very fully described by the author in his work on the experience of the Amicable Society published in 1841; and his account was reprinted by Mr. Chatham (*J.I.A.*, xxix, 94). It has also been brought into prominent notice by Mr. Ryan's paper on methods of deducing Exposed to Risk (*J.I.A.*, xxvi, 256). I need therefore only recapitulate its features briefly.

To begin with, Mr. Galloway avoided the introduction of a double assumption, involving uncertainty both as to the fractional periods of experience in the year of entry and the fractional periods by which the entrants fell short of their next birthdays. He eliminated the former element by adopting the policy year principle and observing the lives through each anniversary of the assurance; but as regards the age, he was obliged to make some assumption, and he adopted the now usual one, that each life fell short of the age next birthday by 6 months. In Mr. Galloway's own words, "The single assumption introduced is, " that the admissions take place, one with another, precisely at the " middle of the current year of age; all the rest is matter of calcula- " tion." In this statement I cannot entirely agree; for when the exposed to risk for ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$ had been ascertained, q_x was found from the formula $q_x=\dfrac{d_{x-\frac{1}{2}}+d_{x+\frac{1}{2}}}{\mathrm{E}_{x-\frac{1}{2}}+\mathrm{E}_{x+\frac{1}{2}}}$.
This point will, however, be further dealt with presently.

The following type of the process followed, after the elementary facts had been collected on preliminary sheets, will, perhaps, make it clear:

Age next Birthday $\{x\}$	Assumed Exact Age $x-\frac{1}{2}$	Number who completed Age $x-\frac{1}{2}$	Living 1841 between Ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$	Discontinued between Ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$	Died between Ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$	Double of Number Exposed to Risk between $x-\frac{1}{2}$ and $x+\frac{1}{2}$	Double of Number Died between $x-\frac{1}{2}$ and $x+\frac{1}{2}$	Four times Number Exposed between x and $x+1$	Four times Number Died between x and $x+1$
21	$20\frac{1}{2}$	40	3	1	...	76	...	188	2
22	$21\frac{1}{2}$	58	1	3	1	112	2	272	4
23	$22\frac{1}{2}$	83	4	2	1	160	2	393	2
24	$23\frac{1}{2}$	122	6	5	...	233	...	570	2
25	$24\frac{1}{2}$	173	4	5	1	337	2	776	4

Mr. Galloway

Assured Exact age	n ω d ε	Policy Years
$x-\frac12$	$n_{\{x\}} = n_{[x-\frac12]}$	
x	$(\varepsilon+\omega)_{\{x\}+\underline{0}|}$	1
$x+\frac12$	$d_{\{x\}+\underline{0}|}$	
$x+1-\frac12$	$n_{\{x+1\}}$	
$x+1$	$\varepsilon_{\{x\}+\underline{1}|}$ — $\omega_{\{x\}+\underline{1}|}$	2
$x+1+\frac12$	$d_{\{x\}+\underline{1}|}$	
$x+2-\frac12$	$n_{\{x+2\}}$	
$x+2$	$\varepsilon_{\{x\}+\underline{2}|}$ $\omega_{\{x\}+\underline{2}|}$	3
$x+2+\frac12$	$d_{\{x\}+\underline{2}|}$	
$x+3-\frac12$	$n_{\{x+3\}}$	
$x+3$	$\varepsilon_{\{x\}+\underline{3}|}$ $\omega_{\{x\}+\underline{3}|}$	4
$x+3+\frac12$	$d_{\{x\}+\underline{3}|}$	
&c	&c	&c

For Aggregate Table

$$E_{x+t-\frac12} = \sum_{\{x+3\}} n - \sum_{\{x\}+\underline{t}|-1}(\varepsilon+\omega+d) - \tfrac12(\varepsilon+\omega)_{\{x\}+\underline{t}|}$$

among whom $d_{\{x\}+\underline{t}|}$ died.

Here $\{x\}+\underline{t}|$ includes all values of $\{x+n\}+\underline{t-n}|$

For Select Table.

$$E_{[x-\frac12]+t} = n_{\{x\}} - \sum_{\{x\}+\underline{t}|-1}(\varepsilon+\omega+d) - \tfrac12(\varepsilon+\omega)_{\{x\}+\underline{t}|}$$

among whom $d_{\{x\}+\underline{t}|}$ died

Here $\{x\}+\underline{t}|$ does not include any values of $\{x+n\}+\underline{t-n}|$ except the value when $n=0$.

Mr. Galloway reached integral ages by assuming $q_x = \dfrac{d_{x-\frac12}+d_{x+\frac12}}{E_{x-\frac12}+E_{x+\frac12}}$

The third column contains the number who "completed", or attained, the age $x-\frac{1}{2}$; and the number in it corresponding to, say, the age $x+t-\frac{1}{2}$ is represented by the expression $\Sigma_{\{x+t\}}n-\Sigma_{\{x\}+\underline{t}|-1}(d+w+e)$. It will be seen from the table that the exposed to risk at age $x+t-\frac{1}{2}$ consists of the above expression, less one-half of the withdrawals and existing at age $\{x\}+\underline{t}|$. The object of the device of doubling and quadrupling the exposed to risk as the table progresses is, of course, merely to avoid fractions.

We are now able to construct the formula for the exposed to risk, which is,

$$\mathrm{E}_{x+t-\frac{1}{2}}=\Sigma_{\{x+t\}}\,.n-\Sigma_{\{x\}+\underline{t}|-1}.(d+w+e)-\tfrac{1}{2}(w_{\{x\}+\underline{t}|}+e_{\{x\}+\underline{t}|});$$

and

$$q_{x+t-\frac{1}{2}}=\frac{d_{x+t-\frac{1}{2}}}{\mathrm{E}_{x+t-\frac{1}{2}}}=\frac{d_{\{x\}+\underline{t}|}}{\mathrm{E}_{x+t-\frac{1}{2}}}.$$

This agrees in form with Mr. Chatham's expression,

$$\mathrm{E}_{x-\frac{1}{2}}=\Sigma n_x-\Sigma(d_{x-1}+w_{x-1}+e_{x-1})-\tfrac{1}{2}(w_x+e_x),$$

assuming, as he no doubt intends, that n_x denotes the new entrants at age x next birthday; and that an analogous signification is attached to d_x, &c.

It also agrees, *mutatis mutandis*, with the improved formula which Mr. Ryan gives late in his paper,

$$\mathrm{E}_{x+\frac{1}{2}}=\Sigma^{x-\frac{1}{2}}(n-f)+n_{x+\frac{1}{2}}-cw_{x+\frac{1}{2}},$$

where $f=d+w+e$.

The improvement here suggested consists partly in a better method of performing the arithmetical work and partly in the substitution of the expression $cw_{x+\frac{1}{2}}$ for $\frac{1}{2}(w+e)_{x+\frac{1}{2}}$. The e is eliminated entirely by only observing the experience up to the last anniversary of the granting of the policy, and thus getting rid of the fractional period in the last calendar year of exposure. The withdrawals cannot, however, be thus eliminated, and Mr. Ryan gives good reasons for preferring the deduction of $cw_{x+\frac{1}{2}}$, representing the sum of the fractional periods by which the withdrawals fall short of the full 12 months in their last years of exposure, to Mr. Galloway's deduction based on the assumption of an equal distribution of the withdrawals.

Reverting to Mr. Galloway's fundamental assumption that the "completed" or exact age was in each case 6 months less than the age next birthday, it is to be noted that he found the actual period to be 4·15 months—a result roughly confirmed by

Mr. Jellicoe in the Eagle Experience. Mr. Galloway saw that in the subsequent use of his table the error would be on the safe side, and disregarded the difference. Though, as I shall presently remark, the facts to which I have personally had access do not exhibit any marked modern tendency in this interval to diminish, yet, with the increase of competition for business, coupled, possibly, with an increasing practice amongst offices of "dating back" proposals to secure business, the interval referred to is not improbably decreasing rather than increasing. At any rate, the error, whatever it may be, runs entirely through the table. This is the penalty exacted by the simplicity of the process; for having once settled the age at entry, policy durations alone remain to be taken account of, and an initial error in the age at entry is repeated age after age to the close of the observation.

This, however, is not the only, nor, indeed, the most serious defect in the method under review. Reference to the table before given shows that the final q_x was derived from the formula $\dfrac{d_{x-\frac{1}{2}}+d_{x+\frac{1}{2}}}{\mathrm{E}_{x-\frac{1}{2}}+\mathrm{E}_{x+\frac{1}{2}}}$. Having traced the best mode of ascertaining the exposed to risk for all half-age values of x, Mr. Ryan concludes as follows: "The values of the function at integral "ages could be arrived at, either by Galloway's expedient "of taking the arithmetical mean of each two adjacent "terms of the series, or, as would under many circumstances be "preferable, by employing some more accurate method of "interpolation." I suspect from Mr. Ryan suggesting a more accurate mode of interpolation that he is dissatisfied with Galloway's arithmetical mean, and, indeed, he proceeded to suggest that it might be better to deal direct with the function q_x, in which I quite agree.

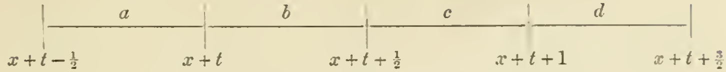
Galloway's own plan appears likely to destroy many of the characteristics of the original facts, especially at the young or old ages where the numbers are rapidly increasing or decreasing. It may be shown that

$$\mathrm{E}_{x+t}=\mathrm{E}_{x+t-\frac{1}{2}}+\tfrac{1}{2}n_{\{x+t+1\}}-\tfrac{1}{2}(d+w+e)_{\{x\}+\underline{t|}}$$
$$+\tfrac{1}{4}(w+e)_{\{x\}+\underline{t|}}-\tfrac{1}{4}(w+e)_{\{x\}+\underline{t|}+1}$$

That is, adopting Mr. Galloway's assumption,

$$\mathrm{E}_{x+t}=\mathrm{E}_{x+t-\frac{1}{2}}+\tfrac{1}{2}n_{x+t+\frac{1}{2}}-\tfrac{1}{2}(d+w+e)_{x+t-\frac{1}{2}}$$
$$+\tfrac{1}{4}(w+e)_{x+t-\frac{1}{2}}-\tfrac{1}{4}(w+e)_{x+t+\frac{1}{2}}.$$

This permits of a graphic illustration as follows:



Here we see that the number exposed to risk for the space $b+c$ is derived from that for the space $a+b$ by

(1) adding half the new entrants exposed over the space $c+d$,

(2) subtracting half the deaths, withdrawals and existing for the space $a+b$,

and (3) making a small adjustment in respect of the terminal years of the withdrawals and existing.

Then as regards the deaths—a much more important point even than the exposed to risk—we have d_{x+t} made equal to the arithmetical mean between $d_{x+t-\frac{1}{2}}$ and $d_{x+t+\frac{1}{2}}$. That is, in the above graphic figure, the deaths for the space $b+c$ consist of one-half of the deaths for the space $a+b$, added to one-half of those for the space $c+d$.

Mr. Ryan's suggested correction, based on the actual exposure in the year of withdrawal, gets rid of one cause of objection as regards the exposed to risk, and leaves the case *pro* and *con* somewhat thus:

(1) The method provides for the very important point of observing exact policy years. But

(2) The ages are based on an assumption which is slightly, though indeterminately erroneous, and which affects the whole of the table. And

(3) It brings out the deaths and the exposed to risk in half ages, further assumptions being necessary in both cases to pass to integral ages.

Mr. J. J. Downes's Economic Experience, ... 1862.

The dates of birth, as well as those of entry and exit, were all ascertained by means of prepared tables to four places of decimals.

The various particulars were then entered on a card of the following form. (Script characters are used to represent the manuscript entries):

No. 1316

Life Luton, Charles

£. 1000 Table 1

ENTRY.	EXIT by Death
Date 29 . 6932	55 . 8685
Born 82 . 1699	82 . 1699
Age 47 . 5233	73 . 6986
48 Minus . 4767	

Cause of death

Thus was obtained both the exact age at entry and the exact age at exit, and, consequently, the exact number of years and days that the society was on the risk. Taking the above case as an illustration: ·4767 represents the fractional period of the year of entry, which is to count as part of the exposed to risk at age 47; while ·6986 represents the fractional period of the year of exit and counts as exposed to risk at age 73. The years of exposure between ages 48 and 73 coincide exactly with the actual life years of the assured.

The cards had first to be divided into deaths, discontinued and existing; and then each of these was divided into ages at entry. The particulars were then transferred to sheets in the following manner:

Deaths—Age at Entry, 48.

No. of Policy	47	48	49	50		72	73	74
1,316	·4767	1·0000	1·0000	&c. &c.		1·0000	·6986	

When all the sheets for a particular age at entry were completed a summary was made. The summaries thus obtained for each year of age at entry were then again collected into one summary of all the ages in each class, and, finally, the addition together of all the classes—the existing, dead, purchased, &c.—gave the required arrangement of facts to be dealt with in the ordinary way.

It is to be observed that, as is inseparable from the nature of the subject, a certain assumption is made here as in other methods, but it is neither far-reaching nor in itself unreasonable. The total of the fractions shown in the above specimen by which the entrants fell short of age 48 is of course classified as exposed to risk at age 47. A similar principle applies to the exits. Thus, suppose four exits after age 73 to furnish fractional periods of exposure in the final year amounting in all to two years, the assumption made is that two lives are exposed throughout the year of age from 73 to 74. The actual specimen given above is copied from Mr. Downes's pamphlet, so that he would appear not to have counted the year of death as a full year of exposure; he does not give a formula for the exposed to risk.

The method briefly sketched above is very laborious, but modern experience would suggest improvements in it. First, two places of decimals may very well be substituted for four. Next, a re-arrangement of Mr. Downes's card, as annexed, has suggested itself to me. The date of birth is only entered once and then underneath the date of exit, from which it has to be subtracted. The decimal part of the age at entry on Mr. Downes's card has, when ascertained, to be subtracted from unity. To save the double operation, I therefore place the date of entry below the date of birth. The operator would make the subtraction of the decimal portion and then shift the mental operation to see that the integral portion of the date of birth deducted from that of the date of entry produced the office age at entry. This age, as suggested by Mr. Downes, could be independently inserted on the card from the records, as a check.

No. £		
Life		
	Date	Age
Exit . .	55 . 87	73 . 70
Birth . .	82 . 17	
Entry . .	29 . 69	48—. 48
Mode of Exit..		

Another obvious improvement, calculated to save much trouble without any corresponding disadvantage, would be to terminate all the observations on the latest birthdays of the assured in the final calendar year of the experience. By this means, all terminal fractions in the case of the existing, would be avoided.

The sorting would then be arranged to throw the facts into the following form.

Age	New Entrants n		Discontinued w		Existing e	Deaths d
	No.	Sum of Fractions $=\phi n$	No.	Sum of Fractions $=\phi w$		

The summations for the fractional columns must, of course, be made on subsidiary sheets, but this would not entail any great amount of extra labour; and when once the facts have been got

into the above shape, the remaining processes are almost as simple as by any other method.

For a working formula we have

$$\begin{aligned} \mathrm{E}_{x+t} &= (\Sigma_{\{x+t\}}\, n + \phi n_{\{x+t+1\}}) - (\Sigma_{\{x+t+1\}}\, w - \phi w_{\{x+t+1\}}) \\ &\quad - \Sigma_{x+t}\, e - \Sigma_{\{x+t\}}\, d \\ &= \Sigma_{\{x+t\}}\,(n-d) - \Sigma_{\{x+t+1\}}\, w - \Sigma_{x+t}\, e + (\phi n + \phi w)_{\{x+t+1\}} \end{aligned}$$

and

$$q_{x+t} = \frac{d_{\{x+t+1\}}}{\mathrm{E}_{x+t}},$$

which is, I think, an almost self-evident expression. If I am right in thinking that Mr. Downes did not allot a full year of exposure to the deaths, $\phi d_{\{x+t+1\}}$ would need to be substituted for $d_{\{x+t+1\}}$ in the exposed to risk were it desired that the formula should exactly represent Mr. Downes's own process.

By this system one would get down to solid ground, and the advantage of the confidence thus inspired in the results, so far as they go, would be great; but, as I remarked earlier in the paper, its want of flexibility and the virtual impossibility of adapting it to the select method, appear to be insurmountable objections.

The Institute of Actuaries' Mortality Experience, ... 1869.

It would have been more strictly accurate to treat of this method as that adopted for the Seventeen Offices Experience in 1843, of which it is an exact reproduction, so far as the formula involved is concerned. It is, however, now so generally known as the "Institute Method", that I have thought it better to follow the general phrase. A full description of it has been given by its author, Mr. Woolhouse, in Vol. xiii of the *Journal* (p. 75); but the copious notation there employed has now quite fallen into disuse.

The formula itself has been stated in different ways. Our present object being to investigate the assumptions underlying it, I have chosen that form which appeared to show best the true characteristics of the method, and that is the formula adopted by Mr. Meikle (*J.I.A.*, xiii, 267) for the Scottish Experience, where the expression for the exposed to risk, altered to the foregoing notation, is as follows:

$$\mathrm{E}_{x+t} = \Sigma_{\{x+t\}}\, n - \Sigma_{\{x\}+|t|}(d + w + e) + \tfrac{1}{2}(n_{\{x+t+1\}} - w_{\{x\}+|t|+1}),$$

and

$$q_{x+t} = \frac{d_{x+t}}{\mathrm{E}_{x+t}} = \frac{d_{\{x\}+|t|+1}}{\mathrm{E}_{x+t}}.$$

The age at exit was ascertained by adding to the age at entry the "mean duration"—*i.e.*, the difference between the calendar years of exit and entry; and this method, therefore, fulfils what I regard as the first condition of the Calendar Year method. The Institute method has been discussed with the utmost fulness. The entrants of any calendar year are assumed to enter in the middle of the year, and to complete their current year of age at the end of that year—*i.e.*, they are assumed, in entering, to be evenly spread over the calendar year, and to fall, on the average, six months short of their next birthdays. As a matter of fact, the average interval has usually been found to be about four months. It may be replied that the new business of most offices tends to enter towards the end of the year, and that that would justify the combined assumptions. On the other hand, many offices close their books at the beginning or middle of a calendar year, in which case the assumptions involved in this method would combine to cause increased error. In this way much is left to the action of averages, and much, therefore, is left to uncertainty.

It will tend to clearness of perception to note that two different questions are here involved, namely, the method of determining the ages at entry and that of dealing with the fractional periods of the calendar year of entry. As regards the determination of the age at entry, it may be noted that a person aged $\{x\}$, who entered immediately after the beginning of the calendar year, and whose birthday was close at hand, would actually be nearly $x+1$ at the end of it; and one who entered just before the end of the year, and whose birthday was nearly a year later, would actually be little over $x-1$ at the end of it. Hence the real age of a person appearing in the table at completed age x may lie anywhere between $x-1$ and $x+1$. Whether, even on the average, the completed ages are correct, depends entirely upon whether the entrants on the average fall short of their next birthdays by the same number of months and days as that by which the date of entry falls short of the end of the calendar year. The entrants in any calendar year having, by hypothesis, attained their next birthdays at the end of that year, they then commence their first full year of exposure. From that time forward they are observed through calendar years, and hence all entrants up to and including age $\{x+t\}$ enter into the formula for the exposed to risk at age $x+t$. In respect of the fractional periods of the calendar year of entry, the entrants are relegated back to the beginning of it, where the fractional

Assumed exact ages	Institute Method				Calendar Years
	n	ω	d	ε	
$x-1$					
$x-\frac{1}{2}$	$n_{\{x\}}$	$\omega_{\{x\}+101}$			1
x	$d_{\{x\}+101}$	$\varepsilon_{\{x\}+101}$			
$x+1-\frac{1}{2}$	$n_{\{x+1\}}$	$\omega_{\{x\}+111}$			2
$x+1$	$d_{\{x\}+111}$	$\varepsilon_{\{x\}+111}$			
$x+2-\frac{1}{2}$	$n_{\{x+2\}}$	$\omega_{\{x\}+121}$			3
$x+2$	$d_{\{x\}+121}$	$\varepsilon_{\{x\}+121}$			
$x+3-\frac{1}{2}$	$n_{\{x+3\}}$	$\omega_{\{x\}+131}$			4
$x+3$	$d_{\{x\}+131}$	$\varepsilon_{\{x\}+131}$			

For an aggregate Table.

$$E_{x+t} = \sum_{\{x+t\}} n - \sum_{\{x\}+|t|} (\omega + d + \varepsilon) + \tfrac{1}{2}\left(n_{\{x+t+1\}} - \omega_{\{x\}+|t|+1}\right.$$

among whom $d_{\{x\}+|t|+1}$ died

$$E_{x+t-1} = \sum_{\{x+t-1\}} n - \sum_{\{x\}+|t-1|} (\omega + d + \varepsilon) + \tfrac{1}{2}\left(n_{\{x+t\}} - \omega_{\{x\}+|t-1|+1}\right)$$

$$\therefore E_{x+t} = E_{x+t-1} + n_{\{x+t\}} - (\omega + d + \varepsilon)_{\{x\}+|t|} + \tfrac{1}{2}\left(n_{\{x+t+1\}} - \omega_{\{x\}+|t|+1}\right.$$

$$\left. - n_{\{x+t\}} + \omega_{\{x\}+|t|}\right)$$

$$= E_{x+t-1} - (d+\varepsilon)_{\{x\}+|t|} + \tfrac{1}{2}\left(n_{\{x+t+1\}} - \omega_{\{x\}+|t|+1}\right)$$

$$+ \tfrac{1}{2}\left(n_{\{x+t\}} - \omega_{\{x\}+|t|}\right)$$

portion, namely, one-half, of those, for instance, aged $\{x+t\}$, figure in the final results as aged $x+t-1$. Hence one half of the entrants at age $\{x+t+1\}$ are included, as already shown, in the formula for E_{x+t}. Whether these fractional periods are correct on the average depends, of course, on whether the entrants are equally distributed throughout the year.

Reverting now to the ages at exit, which were derived from the ages at entry by adding the mean duration, let us consider separately the different modes of exit, taking first the deaths. The effect of adding a mean duration $|t|+1$ to age at entry $\{x\}$ is to cause a death at age $\{x\}+|t|+1$, that is, in the $[t+1]$th calendar year after entry. But the completed age at the beginning of that year is $x+t$. Hence the deaths in that year $d_{\{x\}+|t|+1}$ (*i.e.*, by hypothesis, d_{x+t}) form the numerator for obtaining the ratio q_{x+t}, and must be left to be observed up to the end of the year for purposes of the denominator. Thus, in deducing E_{x+t}, the deaths up to and including age $\{x\}+|t|$ must be eliminated—as in the formula.

Taking next the withdrawals, if one occur in the tth calendar year after entry, the age at exit is $\{x\}+|t|$. Now as the complete age $x+t$ is not attained by the withdrawer until the end of that year, it is clear that all withdrawals at ages up to and including $\{x\}+|t|$, must have disappeared before attainment of age $x+t$, and must therefore be deducted in ascertaining E_{x+t}. But this is not all. On the assumption of the equal distribution of withdrawals, one half of those withdrawing in the calendar year covered by E_{x+t} (namely, $\frac{1}{2}w_{\{x\}+|t|+1}$), must also be deducted, as is shown in the formula above.

Lastly, taking the existing, which were observed to the end of a calendar year, the position is different again. Those assured who are existing at the end of the tth calendar year after entry, will go off at the age $\{x\}+|t|$; but as they all run to the end of the year, the age at exit $\{x\}+|t|$ will by hypothesis synchronize with the completed age $x+t$. Thus in deducing E_{x+t} we must eliminate all existing who attained ages not exceeding $\{x\}+|t|$, as shown in the formula; and there is of course no further deduction to be made, as in the case of withdrawals, on account of fractional periods of exposure lost in the year covered by E_{x+t}. It at first sight appears to be erroneous to regard the existing as all running to the end of the year, but it must be remembered that, in ascertaining in these cases the mean duration on the

cards, no weight was given to the fact that, in every case, the exposure ran to the end of the year of exit. The real duration was not $\lfloor t|$, but, on the average, $\lfloor t| + \frac{1}{2}$; and it might, perhaps, have been better to write $e_{\{x\}+\lfloor t|+\frac{1}{2}}$ instead of $e_{\{x\}+\lfloor t|}$ in the formula, but I was afraid of further complicating it.

This endeavour to reconstruct by general reasoning the formula for E_{x+t} has, I think, the advantage of bringing out in clear relief the special characteristics of the method and the assumptions involved in it. We see that the manner in which the fractional periods of the years of entry and withdrawal enter into the exposed to risk involves the assumption of equal distribution throughout the year. Apart from the influence of these fractional periods upon the exposed to risk, the character of the Table depends upon the manner of ascertaining the ages at entry and exit, and in regard to these there are to be considered the following facts:

1. That the ages at entry,* even when taken on the average, are not improbably incorrect, and may be as much as a year wrong either way in any individual case; and
2. That the determination of the ages at exit by the addition of the mean duration results in the maintenance of any original error in the ages at entry. It is my opinion that the cases where the original age is overstated are just those where the duration likewise is overstated; and that under-statements are similarly exaggerated in the final age at exit.

Although these latter causes of error might, on the average, counterbalance one another, there is no certainty that this is so, and the net result of the method is, it will be seen, to entirely lose touch with the actual age. We have already seen that the effect of ascertaining the age at exit by the addition of the mean duration is, unlike that obtained by the use of the actual duration, to entirely lose touch with the year of assurance also; and in adding that an experience deduced by

* Mr. W. Oscar Nash has since suggested to me that for "ages at entry" it would be clearer to substitute here "assumed ages at the assumed date of entry." He adds, "there are two assumptions both varying from the truth in individual "cases—(1) that entrants are aged exactly $x-\frac{1}{2}$ at entry; and (2) that all enter "in the middle of the calendar year; and it is only by taking the sum of the "errors here possible that a variation of 'a year either way' arises."

this formula is in one of the worst forms for constructing select tables, I am only saying what applies, as before stated, to the calendar year method generally.

Mr. A. J. Finlaison's Government Annuitants' Experience, ... 1883.

Mr. Finlaison in his report on this experience tells us that the ages in the tables are either the exact ages, or the number of completed years of age, the ages of the new entrants being stated according to their last birthdays. By making trial in a large number of cases, taken haphazard from the total experience, it was found that on the average a period of four months had, at the date of entry, elapsed since the last birthday. The assumption that one-third of a year had passed between the last birthday of each nominee and the date of entrance was, therefore, adopted as a basis of the investigation, and was, Mr. Finlaison stated, "the "only supposition necessary with regard to fractional periods of "years of age." The first period of exposure thus consisted of eight months only. In the calculation of the rate of mortality for the whole of the current year of age at entry, it was assumed that the rate for the four months preceding the purchase was identical with that for the eight months which was on the average actually observed. It is, I think, clear from this that Mr. Finlaison's assumption is, as he states, the single supposition made with regard to fractional periods of years of age, and, further, that it only affects the first year of age under observation. From the day that the nominee attained his next birthday after entry, exact years of life were observed; and in accordance with the principle of observing life years the existing were taken only until their last exact birthdays and—the most important point—the deaths were taken at their last birthdays preceding decease.

I have already suggested that the complete application of the life year principle involves the actual calculation of the fractional periods of the current year of age in every instance. It must, however, be admitted that the case of annuitants differs considerably from that of assured. In regard to the latter, the fractional periods will be small and there will always be doubt, in view of the changing conditions of the business, as to whether the trial cases are a fair average of the whole; but in regard to the former the fractional periods form a considerable proportion of the first

year of age and the conditions of the business are less subject to variation.

The question of the treatment of the fractional periods of the years of age is, however, of minor interest compared with that raised by Mr. Finlaison's application of the life year method to the formation of select tables, where the problem is not how to deal with the years of age, but how to follow the policy years, or rather annuity years. Here, again, the fact that the fractional period of exposure in the opening year of age consisted of no less than two-thirds of a year differentiated the experience from one relating to assured lives and probably suggested the assumption that was adopted, namely, that the rate of mortality found to be existent during that eight months should be considered to apply to the first year of exposure. Regarding the experience, therefore, from the point of view of policy years, the first nominal year of exposure consists of the annuity period 0 to $\frac{2}{3}$; the second nominal year of the actual annuity year $\frac{2}{3}$ to $1\frac{2}{3}$; and so on. The $\frac{1}{3}$ year of actual observation which is thus at every select age thrown forward, as it were, into the next nominal year of exposure is itself only an average quantity. It will thus, I think, be seen that the method under examination, closely as it follows the actual ages, does not closely follow the annuity years.

It is, indeed, obvious that no method can follow both exact life years and exact policy years at the same time; and if the opinion should be arrived at that it is better to follow exact policy years and to forsake exact ages, we shall all feel that we have been much helped towards this view by Mr. Finlaison himself, who in his now classical report has shown how more important even than age is selection in its effect on early mortality, and generally what a rich field for investigation the question of selection presents to the future enquirer.

Mr. Finlaison's experience, like the next which will be noticed, is published in the form of select tables. It may, however, be worth while, for the sake of reproducing the salient features of the method and of comparison with other methods, to give a formula for it adapted to the usual aggregate table. The general life year formula as applicable to annuitants, where the existing are only observed to their last birthdays, would be

$$E_{x+t} = (\Sigma_{\{x+t\}}n + \phi n_{\{x+t+1\}}) - \Sigma_{x+t}e - \Sigma_{\{x+t\}}d.$$

Adopting Mr. Finlaison's assumption as to the fractional periods in the life year of entry, we have

$$E_{x+t} = (\Sigma_{\{x+t\}} n + \tfrac{2}{3} n_{\{x+t+1\}}) - \Sigma_{x+t} e - \Sigma_{\{x+t\}} d$$
$$= \Sigma_{\{x+t\}} (n-d) - \Sigma_{x+t} e + \tfrac{2}{3} n_{\{x+t+1\}}$$

and $$q_{x+t} = \frac{d_{\{x+t+1\}}}{E_{x+t}}$$

Mr. D. H. Wells's Connecticut Mutual Experience, ... 1884.

In this case I think it best to quote the author's own description of the method adopted, which is as follows: "The "age when insured, to the nearest year, was obtained by "deducting the date of birth from the date of issue of the policy, "and checked by comparison with the age as entered on the "company's registers. The duration of the policy was obtained "by a comparison of the dates of issue and expiry, if expired, and "by subtracting the year of issue from 1878 if still in force. In "case of policies which had expired by the death of the insured, "any fraction of time in excess of whole years was counted as a "year. . . . . In all other cases the duration was taken to "the nearest year." Policies which lapsed midway in a policy year were averaged by alternately adding and dropping a year.

Taking in turn the different elements which enter into the formula for the exposed to risk, it is first seen that the ages at entry are the nearest ages, as naturally followed from the custom of charging premiums at nearest ages, and from the fact that the nearest ages constituted the office ages standing in the company's registers. Mr. Wells does not deal with ages at exit, the whole of the experience being arranged according to duration years. It is, however, I think, clear from the details given that if we are to construct a formula which will compare with the other formulas for E_{x+t}, the age at exit must always be taken as the age at entry *plus* the duration. Then, in the case of withdrawals, Mr. Wells tells us that the duration was the nearest duration; in fact, he explains in another place that almost every discontinued policy ran naturally to the end of a policy year. Hence the nearest duration was virtually the exact duration, and was thus clearly indicated for adoption. In the case of existing policies, the duration was found by deducting the year of issue from 1878, the year which closed the observations. As we are informed elsewhere that the observations were carried to the anniversaries of the policies, the above method, which was

in fact the adoption of a *mean* duration, virtually resulted in obtaining an exact duration. Finally, in the case of the deaths, the duration was the curtate duration *plus* unity. This at first sight might suggest that the deaths are thus made to go off the table at the end of the year of death; namely, a year later than would properly be the case. As I have before stated, however, the exposed to risk and deaths are in the tables arranged under each year of policy duration, commencing with the first, and I can find no reason to suppose that the deaths were entered under any other than the correct policy years. I regard Mr. Wells's statement above quoted as referring generally to the method of calculating the exposures, for which purpose the deaths would properly be observed to the end of the year. Thus the deaths were scheduled under what may be called their "current duration"; and if Mr. Wells had proceeded to calculate his ages at exit, the deaths forming the numerator for deducing q_{x+t} would appear as d_{x+t+1}. Changing our point of view, however, from the end of the year of death to its beginning, in accordance with custom, it would appear to be the proper construction of the method to take, as the duration to be used for ascertaining the age at death, the curtate duration, and not the curtate duration increased by unity.

No formula for the exposed to risk was stated by the author, and it does not seem to be practicable to state one giving exact effect to quite all the influences above mentioned. The following formula adapted to the aggregate table would, however, seem to indicate every important feature of the method:

$$\mathrm{E}_{x+t} = \Sigma_{(x+t)} n - \Sigma_{(x)+(t)} w - \Sigma_{(x)+t} e - \Sigma_{(x)+\underline{t}-1} d.$$

and
$$q_{x+t} = \frac{d_{(x)+\underline{t}}}{\mathrm{E}_{x+t}}.$$

This formula then is strictly a policy year one, and is further, in its method of determining the ages at entry and in most other respects, based upon principles which subsequent experiments and experience have, as I hope to show presently, led me to regard as the best.

MESSRS. HARDY AND ROTHERY'S BARBADOS MUTUAL EXPERIENCE, ... 1888.

This formula was one which was adopted by Messrs. G. F. Hardy and H. J. Rothery prior to gathering material for their paper (*J.I.A.*, xxvii, 165) on the mortality of the Barbados

Mutual Society. In this case, the formula for the exposed to risk is as follows:

$$E_{x+t} = \Sigma_{x+\lfloor t} \cdot n - \Sigma_{x+\lfloor t} \cdot w - \Sigma_{(x+t)} \cdot e - \Sigma_{x\lfloor + \underline{t}\rfloor -1} \cdot d$$

and

$$q_{x+t} = \frac{d_{x+\underline{t}}}{E_{x+t}}$$

It will here be seen that the age at entry and the age at discontinuance is in each case the mean age. The age at the close of the observations is the nearest age,* while the age at death is the age at entry increased by the curtate duration of the policy.

In the discussion, Mr. G. F. Hardy remarked that the formula had been devised for simplicity and for obtaining the deaths accurately in the assurance years in which they fell, in order to test the mortality by policy years; and I believe I am right in saying that, in some of its features, it was so framed as to be adapted to the materials which happened to be easily accessible. Notwithstanding the fact that theoretical perfection was not claimed for this formula by its authors, the method is, if it be judged by the standard which we are applying, one which eminently fulfils its important object of correctly locating the deaths as regards policy years. It is, therefore, in the strictest sense a policy year method. With regard, however, to the elements which enter into the exposed to risk, it does not appear to be entirely consistent. The ages at exit of the discontinuants and the existing, unlike the deaths, are not derived from the age at entry by the addition of a correct duration. They are independently derived from the original facts, the former by the system of mean ages, which I think must be regarded as unsuitable for the purpose, and the latter by that of nearest ages. Hence we see that the ages at exit of the three different classes are derived by three different methods. The effect on the exposed to risk of any inaccuracies introduced by this diversity of method is no doubt small, but the general result is to give the formula a somewhat unsymmetrical appearance, and I expect that Mr. Hardy himself would be the first to admit the desirability of avoiding any such diversity, unless the nature of the materials compels it.

* The authors also call this a mean age, but in the sense in which they themselves apply this term in regard to the ages at entry and discontinuance, and in which I have also used the term, they appear to be wrong in using it so.

MR. KING'S SUGGESTED METHOD FOR OBSERVING THE EXPERIENCE OF AN INTER-VALUATION PERIOD, ... 1888.

Here Mr. King's own formula for the exposed to risk is

$$E_x = \Sigma(n_x - w_x - d_{x-1} - e_x),$$

where:

n_x = entrants during period of observation, where x is the age nearest birthday on nearest 31 December to date of entry.

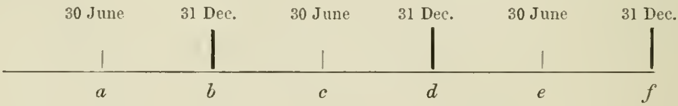
w_x = discontinuants, where x is age nearest birthday on nearest 31 December to date of discontinuance.

d_x = deaths, where x is age nearest birthday on 31 December preceding death.

e_x = remaining on books on 31 December at close of the observations, x being age nearest birthday at that date.

and $q_x = \frac{d_x}{E_x}$. [*J.I.A.*, xxvii, 218.]

This, at first glance, looks a little puzzling; so that with the idea of ascertaining exactly the assumptions which are involved in the formula, I have looked at it in the following graphic method:



Now an entrant in the period *bc* is relegated to *b*, and his nearest birthday at the calendar point *b* must lie between *a* and *c*. Similarly an entrant in the period *cd* is relegated to *d* and his nearest birthday at the calendar point *d* must lie between *c* and *e*. Hence it appears to me that we get results exactly similar to those obtained by taking mean ages, only we are working with calendar years from June to June, instead of December to December. To establish this, assume the office to close its books on the 30 June. An entrant in period *cd* would be relegated to nearest 30 June, *i.e.*, *c*. His nearest birthday at that point of time *must* lie in the calendar year *bd*; in other words, we simply take his *mean* age at entry. Similarly with an entrant in the latter half of the office year, *de*; his nearest birthday at the point *e* must lie in the calendar year *df*, and his age thus derived is again the *mean* age. Discontinuances are likewise seen to be determined by their mean ages according to date of

discontinuance. The same principle also applies to the existing and the deaths, with this distinction—evidently a proper and consistent distinction—that in the former case weight is given to the fact that their incidence is at the end of the year, and in the latter at the commencement.

Mr. King's formula is thus clearly a consistent system of mean ages—mean ages which, in the case of an office closing its year in June, would be determined by the use of the ordinary calendar years—and may, therefore, according to my present notation, be expressed as follows:

$$\mathrm{E}_{x+t} = \Sigma_{x+t}(n-w-e) - \Sigma_{x+t-1|}.d$$

and $$q_{x+t} = \frac{d_{|x+t|}}{\mathrm{E}_{x+t}}.$$

Viewed in this light and brought to the test of the principles before stated, is it a "policy year" formula or a "calendar year" formula? Brief consideration shows that by it every death does not necessarily occur in the correct policy year. If a man born in December 1830 assured in January 1860, he would—still supposing the office year to close in June—enter at 30; but if he died, say, in the following April, he would die at 29. This proves that it is not a policy year formula in the strict sense laid down at the outset, although Mr. King himself considered that "the averages were so arranged as to make the policy years harmonize with the financial years of the company", and that they "really got policy years" (*J.I.A.*, xxix, 178). Neither, on the other hand, is it a calendar year formula, if there be any truth in the principle that the addition of the mean duration to the age at entry in ascertaining the age at exit is the test of that method: for Mr. King rejects durations of all sorts, either actual or mean. Finally, it certainly does not follow the actual years of life. In fact, Mr. King's formula appears to defy classification with any of the categories which at the outset we thought to be complete.

The formula is entirely consistent in the method of adopting mean ages throughout, and is boldly conceived in its special characteristic of leaving almost everything—within, of course, certain well-defined limits—to the doctrine of averages to work smoothly. For this reason, however, I should consider it unsuitable for a small experience, or even for a large one which is to be divided up and analyzed. Indeed, I doubt whether a select table could be constructed by a method under which a man might

even die at a younger age than that at which he entered. This latter point is, probably, not so important as it sounds. As regards the exposed to risk, its effect is not to abstract a year from the total years of exposure, but only to cause the end of the year of death to coincide with the entry, and thus prevent the occurrence of any exposure.

The assumption involved in the use of mean ages is that the policies and birthdays are evenly distributed throughout the year. Where the weight of the observation occurs at the beginning or end of the year, an error is on the average introduced by the use of mean ages. In Mr. King's formula, however, the calendar years used in arriving at the mean ages are always so taken that the weight of the facts falls in the centres of those years. It is an important point in favour of the formula that the disadvantages attaching to the use of mean ages are by this provision reduced to a minimum.

Though I have subjected this formula to a general criticism, it is right I should remind members that Mr. King's special object in constructing it was to secure an easy means of working direct from valuation class books already classified according to nearest ages as at the end of the office year, and thus observing the current mortality of the office. The formula appears to be exceedingly well designed for the particular purpose which its author had in view.

METHOD ADOPTED IN INVESTIGATING THE EXPERIENCE OF THE CLERICAL, MEDICAL AND GENERAL LIFE ASSURANCE SOCIETY.

In the year 1888, the Directors of the Society with which I am connected resolved that its mortality experience should be investigated. Some attention was devoted to the characteristics of existing formulas, as brought out in the foregoing enquiry—which, indeed, has now been put into shape partly from notes made at that time. In particular it was noticed that, as the financial year of the office in question closed on 30 June, around which date annually the weight of the observations would occur, Mr. King's formula would enable one to work with ordinary calendar years, which would have been a great advantage, and would have given great simplicity to the processes, provided always that the system of mean ages could be shown to be suitable.

Some consideration was therefore given generally to the subject of mean ages and mean durations, as contrasted with the system based on nearest ages and nearest durations. As before

stated, the assumption involved in mean ages is that the policies and birthdays are evenly distributed throughout the year. In the case of any particular mean age, however, there is room for a possible error of one year, whereas the nearest age could never be more than six months out. With the object of testing practically, as well as theoretically, the relative merits of mean ages and nearest ages, an investigation was made into all the new entrants in the office in the year 1853–4, and also those in the year 1886–7. The results are summarized in the following table:

	1853-4 577 Entrants		1886-7 593 Entrants	
	Totals	Averages	Totals	Averages
Next Birthdays . .	20,848	36·132	20,159	33·995
Nearest Ages . .	20,648	35·785	19,939	33·624
Mean Ages . . .	20,646	35·782	19,963	33·664
Exact Ages . . .	20,640	35·771	19,931	33·610

It will be noticed, as a matter of general interest, that the average period elapsing between the date of the assurance and the next birthday in the year 1853–4 was 4·3 months; and in the year 1886–7 it was 4·6 months. Mr. Galloway, in the Amicable Experience, found the period to be 4·15 months—a result which was confirmed by Mr. Jellicoe's investigation of the Eagle mortality. The above facts do not, therefore, confirm the usual theory that in the present day the interval is decreasing, but rather the contrary. They are, however, already seven years old; and the conditions of business are rapidly changing. Quite recently I have been informed that offices have offered, not only to "date back" a proposal some three or four months, but to allow the lower premium thus chargeable to run only from the date of completion! If true, this points strongly to a future diminution in the average interval between assurance and attainment of the office age.

Reverting to the general question, the above table is sufficient to show that in practice the nearest ages and mean ages are each very near the truth when taken on the average. For the reasons before stated, however, the nearest ages would appear to be preferable for obtaining ages at entry, as they can never be more than six months out, and therefore follow the facts better in individual cases.

The further question then arose as to the mode of obtaining

the ages at exit, as to which two alternatives presented themselves. First, to ascertain the nearest age at exit by a similar process to that by which the nearest age at entry was obtained, and to consider the difference between the two to be the duration; or, secondly, to ascertain from the facts the nearest integral duration of the policy, and to obtain the age at exit by adding this to the age at entry. Though the nearest duration, like the nearest age at entry, could never be more than six months out, it is no doubt an objection to the latter mode that, if the error in the age at entry and duration happened to run in the same direction, an accumulated error of nearly one year would result in the age at exit. On the other hand, the nearest duration, unlike the duration obtained under the former alternative, possesses the important advantage of closely observing the policy years. This is especially so in the case of deaths, where we only had to take account of the integral portion of the duration instead of the nearest duration to ensure that the weight of every death would be allocated to the exact policy year in which the mortality occurred. As it was expected that considerable importance would be attached, in the course of the investigation, to observing the mortality for different durations of policies, the last-named consideration was a very material one. Notwithstanding the fact that the system of nearest ages and nearest durations, which necessitates reference to the exact dates both of birth, entry and exit when writing cards, was likely to prove somewhat more laborious in working, the balance of advantage was, after careful investigation, considered by the Actuary, Mr. Newbatt, to be in favour of this system, and it was accordingly determined to adopt it.

The following was the general formula for use:

$$\mathrm{E}_x = \Sigma_x n - \Sigma_x (w+e) - \Sigma_{x-1} d; \text{ and } q_x = d_x \div \mathrm{E}_x.$$

where

n_x = new entrants	(x = nearest age)
d_x = dying	(x = age at entry + curtate duration)
w_x = withdrawals	(x = age at entry + nearest duration)
e_x = existing	(x = age at entry + nearest duration).

Translated into our present notation this becomes

$$\mathrm{E}_{x+t} = \Sigma_{(x+t)} . n - \Sigma_{(x)+(t)} . (w+e) - \Sigma_{(x)+t|-1} . d$$

and

$$q_{x+t} = \frac{d_{(x)+t|}}{\mathrm{E}_{x+t}}.$$

In its main feature, apart from the treatment of the deaths, namely, the adoption of nearest ages at entry, this formula is similar

to that which I have given earlier as representing, so far as I could interpret it, the method adopted for the Connecticut Mutual experience. It is of interest to note that the ages at entry thus adopted as the result of an independent process of reasoning, and after starting from the point of view of a British office, are the same as the ordinary office ages on the American plan.

The following form of card was used as being suited to this formula, and as containing all the other facts likely to be useful in the investigation:

No. .... Amount £ ....

Description of Policy ....

Life ....

Occupation ....

	D. M.	YEAR.	
Date of Birth	...–...	....	
Date of Entry	...–...	....	Ages. [Healthy.]
„ Exit	...–...	....	
Duration	...–...	....	
Age at Entry	...–...	....	....
„ Exit	...–...	....	....
Mode of Exit	....		

CAUSE OF DEATH.	

REMARKS.

Male and female lives were separated, a card of different tint being employed for each class. A modified card was used for rated-up cases, and an entirely separate investigation was made into their mortality at actual and rated-up ages.

In filling up the above particulars, the date of birth was, in all cases, given completely; the necessary approximations being made where the information was incomplete. But for the date of entry and the date of exit, only the month and year were inserted, except where the months so fell that for the purpose of calculating the nearest age at entry, and the nearest or curtate duration, it became necessary to enter the day of the month also. It is well, however, to state that the experience gained by this experimental curtailment of the dates, did not tend to prove that trouble was in the end saved by it. Where a mass of cards have to be placed in the hands of many different persons, it is better to have all the spaces filled than to give the writer intricate instructions for dealing with exceptional cases.

When the cards had been completely written, as far as the office records were concerned, the next step was to pick out all deaths, as the duration in these cases was not to be calculated in the same way as in the others. The insertion of the duration was then proceeded with. This was done with care, and was checked by an independent computer in every case as a separate process. In the case of the deaths the duration inserted was, as already explained, the number of years completed at the time of exit, in other words, the curtate duration; while in the case of the withdrawals (*i.e.*, forfeited, lapsed and purchased policies) and also of the existing assurances, the duration was the nearest number of completed years. The deaths were completed and checked before the durations were inserted for the withdrawals and existing. Still keeping the two sets of cards separate, the nearest age at entry was then inserted and at once checked by a separate computer. The age at exit was then inserted and independently checked, this step being a matter of simple addition.

The cards being then fully written up and the durations and ages checked, the *d* cards were replaced, and the entire number were sorted according to dates of birth, in order to bring together all those relating to the same life.

The next step was to eliminate duplicates. Broadly regarded, the method of treating these was an endeavour, as far as the rated-up policies were concerned, to include every duplicate which

had individual characteristics of its own, and to exclude the others. Thus, policies effected at different dates and at substantially varying additions, and also policies which, though on the same life, were not concurrent, were retained by separate cards. On the other hand, policies effected at the same date or even at different dates, when the addition was the same or nearly the same, were treated as duplicates, and one card only was retained, the policy of longer duration being preferred. The latter also afforded the general rule in healthy cases, but where either healthy or invalid policies which at some time were both in force expired at different dates, the earlier policy was treated as discontinued at the date of the second policy. In the latter event, the date of exit was altered, and a corresponding modification of the duration, age at exit, and, where necessary, of the mode of exit, had, of course, to be made.

All extra payments for climate risks were next examined. A lenient view was taken of certain minor risks for which modern practice would not require an extra, such cases being included and the extra ignored: the others were only included up to the time when the extra became payable. The date of exit, &c., in such cases, were altered in the same way as already described in the case of duplicate policies.

The cards were now ready for separation into "existing", "withdrawals", and "deaths", and for sorting into ages at entry and the subsequent processes in the usual way.

Conclusion.

Having now had under consideration both the general principles underlying the different classes of formulas, and also the peculiar characteristics of the principal formulas which have been in actual use, we are fairly in a position to arrive at some general conclusions on the subject.

The first point upon which it is desirable to arrive at an understanding is as to the fundamental principle to be adopted. I think I am only giving expression to what is present in most minds when I say that for our purposes a policy year method is now almost indispensable. For ascertaining the effect of selection, for the calculation of premiums—notably term premiums—for the valuation of recently effected policies, for testing the mortality experience of a rapidly growing business and for many other purposes, that method alone will satisfactorily supply our needs.

It is only necessary, I think, to study the monumental labours of Mr. Sprague and many others in deducing Select Tables from the existing Institute data, to be converted to the opinion that any new experience should be taken in such a form as to supply us at first hand with Select Tables by years of assurance; and this can only be done by adopting some form of the policy year method in determining the ages at death and in deducing the exposed to risk.

On the other hand, it must be remembered that the calendar year method has not been without champions, and it is interesting to record the following expressions of opinion. They are, I believe, the leading apologies for the calendar year method which we have upon record, and I am anxious that the most that can be said for that method should be recalled to the minds of members. In 1879, Mr. Sutton, speaking in the discussion after Mr. Sprague's paper on Select Mortality Tables, said, "There is another more "important and practical objection, which is that for ordinary "valuation purposes we do not want complete years of assurance. "When an office closes its books for valuation, the policies effected "in the year previous are generally considered as having been on "the average half a year in existence; similarly, those effected in "the second year previous to date of valuation are considered as "being on the average one and a half years old at date of "valuation, and so on. Completed years of assurance do not "represent the actual facts, but the Institute Mortality Tables "give us what we require. We have there the year of assurance, "the year's rate of mortality amongst policies half a year old at "the commencement of the year, one and a half years old at the "commencement of the year, and so on. It seems to me that "the Institute Tables have scarcely as yet been properly "appreciated" (*J.I.A.*, xxi, 255). It would be very interesting to know whether Mr. Sutton is still of the same opinion, or whether, on further consideration, he does not think the policy year method preferable. In 1887, Mr. H. W. Manly, in the discussion upon Mr. Ryan's paper advocating a policy year method, made a speech from which the following is an extract: "He remained of the opinion that the proper way in which to "make a table of Mortality was by taking calendar and not "policy years; his reason was that the advantage was in favour "of the method which conformed most closely to the conditions "under which they wanted to apply their tables. The first use "of a Mortality Table was the calculation of premiums. For

"that object, if they wished extreme accuracy, it would be "advisable to use policy years instead of calendar years, but "the difference would be very small. . . . . The principal "use of a Mortality Table after they had calculated their premiums "was for valuation purposes; and what they wanted was a table "that would conform to the actual conditions they made their "valuations on, say on 31 December, and he held that to "calculate their table of Mortality as from that date was a "better plan; it could be applied more easily and would bring "out truer results than if they adopted policy years, which were "more properly applicable to the calculations of premiums. ". . . . They calculated the exact mortality at the end of a "calendar year, not at the end of a policy year; then, again, they "wanted their table framed to comply with the same conditions "as to time as the facts which they were going to deal with. "The advantages, he thought, were overwhelmingly in favour "of following the observations through the calendar year." (*J.I.A.*, xxvi, 272.) Mr. Ryan subsequently remarked that Mr. Manly had ably expounded the view that calendar years formed the better basis, but he (Mr. Ryan) had never heard it suggested that the Institute Tables were less applicable to the case of a company valuing as at 30 June than to that of a company valuing as at 31 December, and if Mr. Manly admitted that, the foundation of his argument would disappear.

In Mr. Ryan's view of this matter I personally entirely concur. I have thought much over the above statements, and still cannot see the force of the advantages claimed for the calendar year method. Mr. Chatham has pointed out (*J.I.A.*, xxix, 99) that, not only do the valuation dates of offices differ, but also the methods by which they determine the valuation ages of the assured. Indeed it is, I think, evident on consideration that the suitability of a table for valuation purposes depends more upon homogeneity in the respective manners of arriving at the ages than any other feature. For instance, suppose the existence of a policy year Table in which the ages at entry were the "nearest ages", and that the valuation ages of an office are also "nearest ages" as at the date of valuation; then surely such policy year Table is a more correct one to use than, say, the Institute Table, where the ages were so determined that the exact age of a person aged x in the exposed to risk might lie anywhere between the limits $x-1$ and $x+1$. And if so little can be said in favour of the calendar year method for the purposes of

constructing a valuation standard, still less can be said for it if the object of the table be the calculation of premiums, or the testing of the mortality experience of an office in policy years, or the observation of its rate of discontinuance.

In one practical respect the calendar year method has an undoubted advantage, namely, the simplicity of the original data required. The three calendar years of birth, entry and exit, or else the office age at entry with the calendar years of entry and exit, are all that are required; whereas for a policy year method we require the three exact dates, or at any rate two, as in Galloway's method. With access to carefully kept modern records, there is, however, no reason to suppose difficulty in this.

Supposing, then, that it were determined to adopt a policy year method, we proceed to enquire what further questions must be settled. On one point only, namely, that the duration in the case of deaths shall be the actual curtate duration, our principle is fixed and immovable. In regard to other points, important as many of them are, there is none which may not legitimately be the subject of some difference of opinion, or which ought not to be discussed in the fullest manner. In order to narrow this discussion, we must remember that, except where, as in the Institute or Galloway's method, fractional periods of the years of entry and exit enter into the exposed to risk, only two facts enter in a mortality table, by whatever name its method goes, namely, the age at entry and the age at exit. On the manner in which these, and in particular the age at death, are determined, depend entirely the special characteristics of the table.

Taking, first, the age at entry, I have already remarked that that method will be best which best makes the assumed ages coincide with the actual ages of the assured on the policy anniversaries. Now, the Council have decided that the new experience shall commence with the year 1863; and if a policy year method is to be followed, it is, I think, obvious that, in the case of policies effected before that year, each observation must necessarily commence with the policy anniversary in 1863, and in the case of more recently effected policies, with the grant of the assurance. With regard, however, to an entrant before 1863, it will not be sufficient to know the age at the 1863 anniversary only; but this age must be resolved into its component parts of original age at entry and past duration in order that the observation may enter properly into the select table. Hence the question we have to solve in every case is the simple one: By what method

will the assumed ages most nearly approximate to the actual ages at the actual date of entry?

For this purpose we must, I think, dismiss the office age at entry as a possible basis. There is always a feeling of uncertainty as to whether the average of the fractional periods of assurance in the current year of age tends to alter, and a consequent difficulty in devising a satisfactory means of ascertaining an average fractional age at entry. Even if this were possible, the necessity of subsequently passing from fractional to integral ages for the final table must be held, for reasons stated in discussing Galloway's formula, to be a strong further objection unless the circumstances be such as to present no alternative. We must, therefore, fall back upon either mean ages or nearest ages, and for the reasons stated in the discussion of the "Clerical" formula, the nearest ages appear to me to be far preferable to any others.

Passing on to the ages at exit, it has already been premised that in the case of deaths, the age is in every case the age at entry *plus* the curtate duration. But in regard to discontinuants, we have some option as to the method of calculating the duration. We may take the nearest durations, or the mean durations, or we may even ascertain the age at exit directly from the facts by some independent process without seriously vitiating the results. Either of the latter alternatives would, however, sacrifice the symmetry of the formula without resulting in any corresponding gain; and, personally, I prefer, for the sake of consistency with the system of nearest ages at entry, and for other reasons which have already been pretty fully stated, the method of ascertaining the age at discontinuance by adding the nearest duration of the assurance to the age at entry. With regard to the existing, some different considerations arise. Are the observations to be carried up to the end of the year 1893, or are they to terminate with the policy anniversaries in that year? If the former, then an assumption must be introduced as to the fractional periods of exposure in the year 1893. For instance, we could in this case also work with the nearest durations, knowing that they can never be more than a half-year wrong in any individual case; and this might possibly be found to be the best method of dealing with that point. On renewed consideration, however, it appears to me that the plan of terminating the observations with the policy anniversaries in the year 1893, would give results which would not only follow more closely the actual facts, but would be much more consistent with the spirit of the policy year method generally. If this plan

were to be adopted, the age at exit of the existing would be the age at entry, *plus* the exact duration; and such withdrawals and deaths as occurred subsequently to the policy anniversaries in the final year 1893 would, of course, be treated as existing, and their ages at exit determined by the same rule.

The formula for the exposed to risk would then be

$$E_{x+t}=\Sigma_{(x+t)}.n-\Sigma_{(x)+(t)}.w-\Sigma_{(x)+t}.e-\Sigma_{(x)+\underline{t|}-1}.d$$

and
$$q_{x+t}=\frac{d_{(x)+\underline{t|}}}{E_{x+t}}.$$

In this formula, which, like all the others previously given, applies to the aggregate table, the age at entry in the case of old policies is of course the "age at entry in 1863", and not the original age at entry.

DISCUSSION.

Mr. A. J. FINLAISON said the paper was very opportune and he congratulated Mr. Whittall on his method of exhibiting the formulas which had been used from time to time for obtaining the numbers exposed to risk. The advantage of Mr. Whittall's formulas lay in the power they afforded for the comparison of different systems of construction, and it was from this point of view that they should be considered rather than as working formulas. As regards Mr. Whittall's notation some of the symbols had reference to fixed ages and fixed points of time; some to ages and periods of time taken between limits according to one set of conventions, and other symbols to ages and periods of time in reference to a different class of convention. The several classes of symbols were then bracketed together in a manner which, while it illustrated the operations which had been gone through to obtain the result, did not always seem to permit of interchange, *e.g.*, Mr. Whittall's $|x+t|$ could not be substituted for his $(x)+(t)$ in the formulas, although in one sense they were an equality. Again the numbers dying, the number of withdrawals, and the numbers existing at the close of observation were grouped together, and a set of subscript symbols placed with a symbol of summation put before a bracket as applying to the whole group. But some of these subscripts conveyed a different meaning in regard to the deaths and withdrawals than in relation to the numbers existing at the close of observation; because the numbers existing had reference to occurrences at a fixed point of time, while the deaths and withdrawals had reference to occurrences between limits of time. He suggested that Mr. Whittall should amplify the definitions attached to the general list of symbols. Mr. Whittall had gone a long way to establish that an arrangement based upon "a policy

year method" was the most desirable system to apply to the mortality records of life assurance societies. The examination of the Twenty Offices experience and the Government Annuitants experience seemed to establish that at every period of life, for the first four or five years from the date of a contract, an increase of a year in the duration of a policy had a greater influence on the rate of mortality than an advance of a year in the age of the life concerned. The arrangement of the records should therefore be made with reference to the more potent force rather than the less important force. If, however, the materials were arranged according to policy years a classification should be made of the withdrawals. One of the greatest wants in life assurance finance was a standard table of net premiums, and such a table was only to be derived from a mortality table in which due weight was given to the force of selection upon the mortality of recently assured lives. Mr. Whittall had remarked that Mr. Downes was the introducer of the card system. That statement was contained in the report of the Faculty of Actuaries in Scotland. But Mr. Downes was anticipated by more than ten years in the use of cards, and also in a published description of the use of the card system in the construction of tables of mortality. Cards were used in the National Debt Office by Mr. A. G. Finlaison in the latter part of the year 1852 in the observation then made by him on mortality among the members of friendly societies. A description of the cards and of their use was given in the report of Mr. Finlaison, dated 1 August 1853 (page 4) in the following terms. . . . . . "This object was accomplished by having the transcript made on sheets of cardboard on the following plan. . . ." Then a diagram of the card was given, and it was stated that the cards were sorted and re-sorted by different assistants, and the description concludes as follows. . . . "The plan thus followed had not, it is believed, been previously devised. It is of wide application and may be a great use on other occasions in dealing with statistics, and for this reason it has been mentioned in unusual detail." This card system of Mr. A. G. Finlaison was further described in a paper read by Mr. H. Tompkins on 26 June 1854 (*J.I.A.*, v, 15), in which the following words would be found: "The collection of facts relating to each individual was contained on a slip of cardboard. These facts remained together throughout the whole procedure and were used over and over again in different combinations."

Mr. F. B. Wyatt said the first point in the paper which occurred to him was the attention which Mr. Whittall called to the fraction which expressed the rate of mortality. He rightly there pointed out that the numerator did not receive sufficient elucidation. He very clearly defined the two methods of procedure, the calendar year and the policy year. He (Mr. Wyatt) did not know that that had been accomplished before. The author had said: "We are, I think, led naturally to the enunciation of a cardinal principle, namely, that no method deserved the name of a policy year method which does not rigidly locate every death in the exact policy year in which it occurs." With that statement he (Mr. Wyatt) entirely agreed, but any method to be satisfactory should also locate every discontinuance in its exact policy year. As to the discontinuances, it was

necessary to distinguish whether they were voluntary or compulsory. There were, no doubt, considerable difficulties in doing that, but it was so important a matter that the attempt should be made. Then as to the age of entry: to get at the nearest age meant that they must have the date of birth and the date of the policy, and a calculation would have to be made in each case. It was important for them to consider whether it was worth while to go through the labour involved in that. He thought the mean age was quite near enough, and would probably be found to err by a few days only. Absolute precision was not attainable, as in the majority of cases no proof had been given of the ages declared in the application for assurance. As to the Institute method, there was a great deal to be said in favour of it. They must not forget the difficulties that had to be contended with, because they now found after an experience of 30 years that the method was not all that could be desired. The difficulties at the time were many. There was the impossibility of getting at the nearest or mean age in all cases, as in several of the older official records nothing was stated except the age next birthday. It was important to study the mortality experiences that had been extracted since 1869. There had, he believed, been no large experiences examined in England, but a great many abroad: for instance, the Connecticut Mutual and the Gotha Life Offices experiences, which were both extracted on the policy year plan. Perhaps he might also mention his own office's experience. He was firmly convinced that the "policy year" method was the only one appropriate for an investigation of the kind now contemplated. At one time he was wedded to the Institute method, and in getting out the experience of the society with which he was connected he adopted that method; but before he had finished he came to the conclusion that if he ever had to perform a similar work again, he certainly should not do it by the Institute method.

Mr. A. H. Bailey, referring to the Institute Experience, said he knew something of the difficulties that prevailed. One of the first things they had to consider was that they were not endeavouring, like De Moivre or Gompertz, to determine a scientific law of mortality. The tables were wanted for pecuniary results and to decide what were sufficient rates of premium. For this purpose no attempt had been made to arrive at any results for a less interval than a year; and indeed it would suffice to determine the rate of mortality at the quinquennial periods of age. The results were always subjected to process of graduation, which knocked the calculations about in a most remarkable way. It therefore appeared to him that as with all the various methods suggested the difference in the results was very small, the process to be adopted was the one which necessitated the least labour. On that point he thought that the easiest method was clearly the best. When the Institute Experience was got out, it was found that in a number of cases the old register contained no records of the date of births at all. Therefore they had to be content with the ages next birthday. That certainly would not be the case in the last 30 years. He believed the dates of birth were invariably now recorded everywhere, and it seemed to him that the materials they wanted were the date of birth, the year of entry,

and the year of exit. For the date of birth it should be assumed that everyone born between 1 July in one year and 30 June in the following year was born on 31 December. And then the calendar year method would answer every purpose. The discontinuances would be uniformly distributed throughout the year, and be exposed to risk for half a year. In that way they would get the best results, and results which would be sufficient for every purpose. He thought it would be advisable to exclude the current year of entry altogether, as the dates of entry in these days were altered so much, and the results would be much more accurate if they were excluded. By such means he believed that they would get all they required in the simplest form.

Mr. G. H. Ryan said that Mr. Whittall was chiefly concerned in discussing the best available methods for arriving at a mortality experience from the rough observations to be collected by insurance companies. But at the outset of his paper he seemed unnecessarily to restrict the scope and object of the enquiry. He said "What are we concerned with? Clearly the rate of mortality." But there were other forces which appeared to him (Mr. Ryan) to be of great importance, and one certainly was the rate of discontinuance. They had had elaborate essays dealing with the subject, and he hoped they would have more in the future, because he could not help thinking that they had as much to learn regarding the rate of discontinuance from the new experience as they had regarding the rate of mortality. As regards the relative merits of the systems of policy years and calendar years, he (Mr. Ryan) was cordially in unison with the author. It appeared to him, that the case for policy years as distinguished from calendar years, was clearly made out. Some time ago he had placed on record his opinion that the "policy year" system was the best for general purposes, and since then he certainly had discovered no ground for changing his views. With regard to the age at entry, he agreed that the office age at entry must for all practical purposes be abandoned. If they were only concerned with the rates of premium for new entrants, the office age would appear to give them exactly what they required free from any assumption whatever, but for valuation and miscellaneous purposes it clearly was unsuitable. It exaggerated the mortality throughout, and its place could easily be supplied by a more exact measure. He agreed with Mr. Wyatt that Mr. Whittall had succeeded in showing that there was practically no difference between the mean ages and the nearest ages, and seeing that the mean ages could be so much more conveniently obtained by many companies, he would suggest that they should be content with mean ages at entry. With regard to the age of exit, Mr. Whittall had adopted the obviously appropriate system in reference to the deaths. But as to the discontinuances, he (Mr. Ryan) could not fall in with his suggestion. Somewhere in the paper it was written that in the Institute system where averages were adopted freely that "much was left to averages, and therefore much was left to uncertainty." Better words could not be used to describe Mr. Whittall's plan in its effect upon the rate of discontinuance, if his suggestion of using the nearest duration was adopted in getting out the age of exit. With that exception he approved of the method

proposed for adoption, but he could not see how they could get over their difficulty as regards the withdrawals, otherwise than by calculating the exact exposure in the final year of exit, and making the full correction (referred to in the paper as cw_x) in the "exposed to risk." In view of the great advantages which would follow from having their results as accurate as possible, it might be quite worth while to incur the considerable labour of closely tracing the discontinuance in the final year of exposure for the purposes he had indicated. He would suggest to Mr. Whittall a slight addition to his symbolical and highly ingenious notation, namely, a definition of "exposed to risk", because he thought there was likely to be some confusion in the minds of students owing to certain remarks of the author. In one place he had spoken of observations taking place towards the end of the year of exposure as being "relegated back to the beginning of the year", and elsewhere he referred to the same circumstance as involving a slight error. With deference he (Mr. Ryan) submitted that there was no error at all, and this, he thought, would be obvious if a definition of the term had been given. The "exposed to risk" were the total years of life observed between two ages. They did not strictly relate to the "beginning of the year" or any specific point of time, and thus all observations during the year fell naturally and properly into their place. Towards the close of his paper, Mr. Whittall had said that they required the system of policy years "for ascertaining the effect of selection for the calculation of term premiums", but he (Mr. Ryan) would suggest that if they wished to get a scientific measure of the risk under term policies they should have the experience of term policies alone. He did not think that any select tables based on the observation of whole-life and endowment assurances would give them a sufficiently accurate assessment of these risks. The mere fact that people chose the lowest premiums that an office offered was bound to have some influence upon the mortality, and other considerations told in the same direction.

Mr. R. P. Hardy said that when they considered the particular applications in view, he would submit that it was not the method that was apparently most perfect in technique that should be rigidly insisted upon. What they rather required, as Mr. Bailey had said, was to find a measure of probable receipts and payments in respect of life assurance purposes. They were not seeking that illusory thing, an exhibition of the physical law of mortality. Also they had to apply this measure to groups of lives where there existed undetected numerous errors in ages. The basis of these experiences rested upon a mass of facts, many of which were wholly unverified, and most of which had been derived under circumstances that were largely affected by the past social influences, and had been obtained under varying standards of selection. The exactness which Mr. Whittall sought in this particular case was unattainable either in the measure or in the valuation groups. What he (Mr. Hardy) feared was that by insisting upon too great minuteness in the data the volume of facts tendered to them for tabulation would be lessened, and he would therefore say that the equally safe and older and simpler form of schedule was

sufficient. He held that they should trust to graduation to restore the technical uncompleteness. He would ask Mr. Ryan what real difference in the rate of mortality it would make whichever method was selected. Would it affect the pence or farthings of premiums? Then as to the law of discontinuances, he would ask how it was possible to derive a law that was to be relied upon from the working of a multiplicity of offices, in some of which he believed there was only one lapse in the year while others were doing more or less a forced and consequently unstable business. The law of discontinuances, as derived from facts harking back 30 years, was very different from that which prevailed at the present day, and would sensibly vary from that which was likely to prevail in the future. He was desirous of standing fast in the old paths.

Mr. George King submitted that the question was not what was the best method of deducing the rate of mortality, taking it generally, but what was the best under given circumstances. He agreed with Mr. Bailey that the best for any particular purpose was that which was the easiest with the facts before them, because after all simplicity and facility of calculation were extremely important. Not only did they get out their experience more rapidly, but it was more likely to be accurate, and it was less costly. He hoped that they would be able to arrive at some very simple method of operation. Their object would be to form select tables, because they were of special importance at the present day, and by getting the actual date of birth they could get those select tables. He was not sure what would be the best method of getting the duration or the ages of exit, but he believed they could also get them with great exactness. He would advocate such a method as would do away with fractions. He thought the ·5 or any other decimal in their results was objectionable. It very much increased the labour and risk of error, and added to the bulk of the printing and to the cost. He hoped they would have several sets of tables for different classes of policies. His own formula, which Mr. Whittall had brought into prominence, had been put forward, not for the purpose of bringing out a great mortality table such as was now in contemplation, but merely for getting out the mortality experience of a company from one valuation to another, and for that purpose, he believed, it was the best at present promulgated, because it seemed to be the simplest and most accurate. It fitted in with the office books, and the only trouble was to get out cards for the entrants, discontinuants and deaths during the short period between two valuations. Mr. Whittall had found a difficulty in classifying the formula. He (Mr. King) found that difficulty too. He could not, however, help thinking that the author was not quite accurate in his remarks upon this point. He (Mr. King) submitted that he did not deal with mean ages at all—he dealt with the nearest age. The author had stated that by mean ages they could get as much as a year's error in a particular case, but he (Mr. King) maintained that in the method he had adopted they could never get more than six months' error. Then as to the deaths, it was quite true that the particular case used as an example by Mr. Whittall was not brought under observation at 29 but at 30, but he was really at risk at age 29, and the fraction of life of that individual who died while at

risk, but not under observation, was compensated for by others to whom more life was attributed than they actually passed through. One of the merits of the formula was that both in the case of new entrants and discontinuants correction was made for the periods of the year at which the policies entered or lapsed. It would be easier for students to understand, if before each formula for the aggregate exposed to risk Mr. Whittall had dealt with one particular age at entry and traced those policies to their terminations. They would thus see more exactly the position, in each of the methods, of the entrants, the discontinuants, the existing and the deaths.

Mr. H. W. Manly said he would first relate the feelings he experienced when constructing some years ago the mortality experience of an office with which he was connected. He was desirous of being minutely accurate, and proceeded to the extent of ascertaining to two decimal places the exact age of each of the policyholders. He represented the years of observation by two decimal places short of the exact age and the previous age, and when the policy went out of observation he carried to two decimal places the completed age, and so on. Eventually he produced a table which he considered represented the true mortality of the lives that had been assured. But when completed, he came to the conclusion that though it certainly produced very pretty results, he could not agree that it was the proper way of producing a mortality experience. For what purpose were they going to extract this experience? If it was desired to obtain the effect of selection from the year of entrance through successive years, he held it was no use to take out the exact age of entry, because it did not matter whether the life was six months older or younger, especially at the early ages. The great potential factor in such an investigation was the time during which the lives were under observation. By assuming the age next birthday as the exact age at entrance, and then tracing the experience through the exact policy year, they would be dealing with facts as they came before them. If all the policies were paid by annual premiums, then discontinuants through lapses would go out of observation at the end of the policy year, and there would be no necessity for adjustment on that account, but in practice the liability was extended for an extra month, and they would have to add a month to the next year of observation for each lapse. [Mr. Finlaison—That would balance the half-yearly premiums.] But there came disturbing elements in the half-yearly premiums and the quarterly premiums, which should be treated separately. The deaths should be accurately recorded in the policy year in which they occurred. When they had got such an experience, what could they do with it? They would have the means of calculating very exact risk premiums; but he did not believe in the calculation of very exact premiums, especially for term assurances, because when ascertained it was necessary to load them heavily in order to secure themselves against contingencies and expenses. How were they going to apply these tables to one of the most important duties they had to perform—the valuation of their policies at the end of their valuation periods? They wanted tables that would exactly represent the condition of affairs at the time when they required to apply them;

and for the purpose of valuations he considered that a method similar to the Institute method was the best. The Institute tables more closely represented the conditions they had to deal with on that date than any other; but he quite admitted that there was a great deal to be said for the tracing of the observations through policy years as had been suggested. It had been suggested by Mr. Ryan, that a table made up according to calendar years, that was from 1 January to 31 December, could not apply to an office which valued at 30 June. But he (Mr. Manly) considered that the conditions of an office that valued at 30 June were the same as those of an office which valued on 31 December, and that in valuing at 30 June it was equally well suited by the Institute Tables, based on the calendar-year system.

The PRESIDENT (Mr. A. Hendriks) said that it was within the knowledge of all present that the Council appointed a Committee over which he presided, so that the selection of the best method was to some extent *sub judice*, and he would therefore not venture upon the expression of any strong opinion on the respective merits of these methods. He would only say that he hoped there would not be any material or great variation from the method adopted upon previous occasions. His reason for saying that was mainly based upon the idea that they ought to have *continuity*. They would necessarily in the future have to make comparisons between the existing tables and the coming tables, and the less they changed the systems of the compilation of those tables the better it would be. He was far from saying that after an interval of 30 years there were not certain improvements to be made, or matters they might wish to eliminate, but still he hoped that they would change as little as they could the method which had been adopted in the compilation of the existing tables.

Mr. WHITTALL, in reply, said that the great object they had in view was that the new experience should be the best possible; and that which was, in the opinion of those who were the best able to judge, the best method ought to be adopted irrespective of personal considerations. Mr. Finlaison's observations with reference to the origin of the card system were very interesting. He was surprised that the statement in the official volume of the Institute should have remained so long unchallenged. Mr. Finlaison also criticized the system of notation introduced, and on that, he (Mr. Whittall) need only say that the formulas were designed to show the ages at which the various classes of discontinuants went off the books. If his notation did not represent the ages at which the assured went off the books then of course the criticism which had been made was a fair one; but Mr. Finlaison had not proved that the facts were misrepresented in the formulas. [Mr. FINLAISON—I objected to grouping the three classes of discontinuants within one bracket]. For the moment, he could not see where the incorrectness existed. With Mr. Wyatt's suggestion as to the adoption of the mean age at entry, in which Mr. Ryan had concurred, he had considerable sympathy. He had given reasons in the paper for preferring the nearest age, but he had also said that the age at entry was not the most important thing, and that if the facts were not available for ascertaining the nearest age, it was desirable to fall back upon the mean age. Reference

had been made by Mr. Wyatt and also by Mr. Ryan to the subject of discontinuance. The rate of discontinuance was a very important question, and it would be very desirable that their tables should be so arranged as to exhibit it directly. Mr. Ryan proposed to bring in the fractional periods of exposed to risk in the year of discontinuance by exactly calculating the fractions for that final year, but it would be sufficiently close to take a half of the discontinuants, as the question only affected the exposed to risk. As regards age at exit they would have to tabulate the discontinuants according to the method suggested in the paper for the deaths, and instead of taking the discontinuants to the nearest duration would simply have to take them to the curtate duration. That would mean that the fractional periods of the final policy years of the discontinuants would have to be brought in when calculating the exposed to risk of death; and, on the other hand, when calculating the exposed to risk of discontinuance, they would have to bring in the fractional periods of the terminal years of the deaths. Mr. Ryan appeared to say that the exposed to risk being simply the number of years' exposure between two ages, out of which certain deaths occur, that that involved the assumption that the rate of mortality would be constant during the period, and that therefore there was no difficulty in spreading the fractional periods of an initial or terminal policy year over a whole year of the exposed to risk. He (Mr. Whittall) disagreed with him there. It was one thing, after they had arranged their facts properly, to ascertain the rate of mortality in such a form that it was shown in annual steps, but it was another thing to arrange the actual data beforehand on the preconceived assumption that the rate of mortality was constant during the year. That appeared to be begging the question. As to Mr. King's formula, he agreed that it was the best for observing the rate of mortality in an office between the two valuation years, subject, however, to the office in question adopting nearest ages at the valuation—a practice which was by no means universal. He disagreed, however, with Mr. King, when he said that under the formula the ages at entry could not be more than six months out. There was room in Mr. King's formula for a possible error of one year in ascertaining the ages at entry. This limit of possible error was inseparable from a system of mean ages; and he felt obliged to retain his opinion that Mr. King's formula was essentially based on mean ages, for reasons given in the paper. Mr. Manly did not seem to be now quite so much in favour of the calendar year method as before, and he had admitted that after all it would be very useful if the mortality could be shown for the policy years. But he (Mr. Whittall) disagreed with the speaker who said that they might get that result by graduation. Dr. Sprague and many eminent men in the profession had devoted themselves to graduating the old experience in order to construct select tables, but they were the very men who were most anxious to see the new experience got out properly from the beginning according to policy years. Mr. Bailey's method of arriving at the age at entry was, he thought, exactly the same as Mr. King's. There, however, the similarity ended. Whereas Mr. King dealt only with final ages, Mr. Bailey would follow the calendar years. Mr. Bailey's method would be a variation of the

Institute method with the fractional periods in the first calendar year eliminated, and with an improved system of grouping the ages upon the first 31 December after entry. Mr. Manly misunderstood the bearing of Mr. Ryan's remarks respecting the applicability of a calendar year table to a valuation at 30 June. Mr. Manly first stated that a calendar year table was most applicable to valuations, because they coincided with the calendar years according to which the table was constructed. He now says it would apply at 30 June if that be the close of the office's financial year. But these arguments were mutually destructive. It appeared to him that the principle of which Mr. Manly was in search was, that an office could best use, in its valuations, not a calendar year table, but a table constructed according to the financial years of offices.

On the Tabulation of the facts extracted from the Records of a Life Office for the purpose of investigating its Mortality Experience. *By* T. B. Sprague, M.A., LL.D., *Manager and Actuary of the Scottish Equitable Life Assurance Society.*

In the course of an investigation I have lately made, into the rate of mortality that has prevailed among the female lives insured in the *Scottish Equitable Life Assurance Society,* I have tried various methods of arranging the facts furnisht by the books of the office; and I think it may be useful to persons engaged in similar investigations, if I describe these methods, and state the conclusions at which I have arrived as to the best course of procedure.

The observations were, of course, limited to insured lives, no account being taken of the lives on which endowments were purchast, for sums payable only in the event of the life attaining a certain age. Furthermore, all the lives charged an extra premium for impaired health or bad family history, were excluded from observation. This is the course that was adopted by the *Institute of Actuaries* in forming the H^M and H^F tables; but a very different course is sometimes adopted when the Institute Method has been professedly followed. It appears, in fact, that in the compilation of several of the mortality experiences of life insurance companies, which have been publisht during recent years, every life that was under observation,—whether insured or simply endowed; and whether accepted at the ordinary rate, or charged an extra premium, either for impaired health, or for bad family history, or for foreign residence;—has been included

without distinction. Results thus obtained seem to me to be of very doubtful value; and there can, I think, be no doubt that, if our investigations are to be of any real use, all lives charged an extra premium for any reason, should be dealt with separately. Our principal object in making a mortality investigation, is to ascertain, for our future guidance, what has been the mortality among the lives which we classed as "average", and accepted at the tabular rate of premium. Then, by comparing the mortality experienced with that which was to be expected, according to the tables on which our premiums are based and our valuations made, we learn whether these tables are trustworthy, or whether it would be desirable to replace them by others which agree more closely with the observed facts.

Although all rated-up lives were excluded, I have included those who were charged the now usual female extra, and also the few who were charged a further extra for first pregnancy; as all such should, I think, be regarded as normal female lives. As regards foreign residence, I have not only excluded the cases in which an extra was charged from the outset; but, when any life was, some years after entry, charged an extra premium for licence to reside abroad, I have terminated the observation of the life at that time; thus treating the policy precisely as if the insurance had ceased at that date. In a few cases, lives insured at the ordinary rate of premium have also been excluded from observation. For instance, when a policy is granted on the life of the last survivor of three or four daughters against that of their mother, if one or two of the daughters die before the mother and the others survive her, no claim would arise under the policy, and the assured would gain nothing by reporting the death or deaths to the Office. It is, therefore, not unlikely that deaths may have occurred among lives in this position, without the fact coming to the knowledge of the office; and if the years of life pertaining to such insurances were included in the observations, the corresponding deaths would be omitted, and the results to that extent disturbed. I therefore excluded policies of this kind altogether from the observations. But all policies payable simply on the death of the last survivor of two or more lives, have been included in the observations. It has been the custom of my office in such cases to make enquiry periodically, as to whether all the lives were still in existence; and it is believed that in this way we have been kept advised of all the deaths that have occurred in connection with these policies.

Duplicate Policies.—When there have been two or more policies on the same life, they have been carefully examined, in order to determine whether the risk has been continuous or not. If one of them came to an end before another was taken, so that there was an interval during which the life was not insured, the policies have been treated as if they were on different lives; but if the later policy was taken out before the earlier one ceased, the two have been thrown together, and treated as a single insurance. Special precautions were taken to secure that the same life was not counted twice, in consequence of having changed her surname through marriage, without the fact being recorded in the Registers of the Society. For this purpose all the cards were arranged according to date of birth, and it was thus found that in 7 cases a change of name had taken place without being noted in the Register.

Revived Policies.—When a policy had been forfeited, and subsequently revived, it was treated as if there had been no forfeiture. The number of policies revived more than a year after the date of forfeiture (whether for the full amount or for a reduced amount as paid-up policies) was ascertained to be only 25, upon 21 lives; and the aggregate period for which the Society was off the risk as regards these lives, amounted to about 40 years. The strictly correct method of dealing with these cases would, perhaps, have been to treat the policies as discontinued at the date of forfeiture; and to suppose that fresh policies had been effected at the date of revival, at the increased age of the assured. This would, however, be open to the objections, (1) that the evidence of health furnished on revival of a policy, is not so strict as that on first acceptance; and (2) that the conditions of a revived policy, as regards foreign residence, are often not the same as those of a new policy issued at the date of revival. The 25 policies were also far too few to be treated as a separate class; and they were therefore treated in the same way as if there had been no forfeiture and revival, but the risk had been continuous.

The date on which it was originally intended to close the observations was 1 March 1890, and all entrants up to that date were included; but, as the investigation occupied a longer time than was anticipated, it was subsequently decided to bring the observations down to 1 March 1891, without, however, including any fresh entrants. The 1st of March was chosen, because that is the day on which the office year closes. The number of lives included

in the observations was 2,205. Of these, 668 died, 807 (called "Existing") were on the books of the office at the close of the observations, and the remaining 730 had passed out of observation, and were, for a reason which will be presently explained, divided into 659 who voluntarily "withdrew", and 71 whose discontinuance was involuntary.

Nearest Duration Method.

The method of tabulating the material facts which I finally adopted, may be conveniently called the "Nearest Duration Method". The particulars of the policies were, as usual, written on cards; and those which are made use of in the final tabulation of the facts according to this method, are:—

(1) Age at entry or "commencing age".

(2) A letter (or mark), indicating whether the life was still insured at the close of the observations; and, if not, in what way she had passed out of observation; in other words, whether the life is to be entered as Existing; and if not, the mode of exit.

(3) The duration, or time during which the life was under observation.

Commencing Age.—In these observations the assumed age of each life at entry, is neither the office age, x, nor $x-\frac{1}{2}$, but is the age at the nearest birthday; and I have adopted the new term "Commencing Age" instead of Age at Entry, in order to indicate this departure from the ordinary usage of this country. The Commencing Age can never differ by more than 6 months from the real age. Supposing the exact age at the date of the assurance to be 25 years and 4 months, British Life Offices would take the office age to be $25\frac{1}{2}$, or 26, according as it is their practice to calculate the premium by half-years or by years of age; but in the present investigation the "Commencing Age" will be 25. If, again, the exact age is 25 years and 10 months, the "commencing age" will be 26, which is also, I believe, the office age according to the universal practice—at all events, of British Offices. I believe that in the United States and some other countries, the age at entry is the age at nearest birthday, and so coincides with my Commencing Age. The ordinary phrase "age at entry" is therefore ambiguous; and it seemed to me desirable

to adopt a new term, in order to prevent the possibility of any misunderstanding that might otherwise have arisen in consequence of the varying practice of offices. When the date of the assurance happened to be exactly equidistant from two birthdays, so that the exact age was $x+\frac{1}{2}$ years, the commencing age was taken as the difference between the calendar years of birth and entry. When the year of birth only was given in the papers, it was assumed that the birthday was 1 July, so that in this case also the commencing age was the difference between the years of birth and of entry.

I have found the average difference between the commencing age and real age to be extremely small; in fact, adding together the commencing ages of all the 2205 lives included in the observations, the total differs from the total of the real ages at entry, calculated to the nearest month, by only 20 months in excess.

Mode of Exit.—It is usual in mortality observations of this kind to say that a life may pass out of observation in any one of three ways; (1) by being "existing", or still on the books of the office, at the close of the observations; (2) by death; (3) by the discontinuance or other termination of the policy during life. In order, however, to obtain the means of tracing with greater accuracy the effect of withdrawals on the mortality, I have thought it desirable to divide the third of these classes into two, containing respectively those who voluntarily "withdrew", by the lapse or surrender of their policies; and those who exercised no option in withdrawing, the withdrawal being, in fact, compulsory. The latter included term policies, which expired; endowment assurances, which matured; policies upon joint lives, when one life died and the survivor was a female,—for instance, policies on the joint lives of husband and wife, when the wife passed out of observation upon the death of her husband before her; survivorship policies, where a female life was insured against another life, and passed out of observation upon the death of that other; and, lastly, the lives that were taken out of observation, as mentioned above, in consequence of being charged an extra for foreign residence. It is difficult to find a suitable word to include all these classes; and, for want of a better word, I include them all under the heading "Matured".

We thus have the four classes, Existing, Matured, Withdrew, Died; and these were indicated by writing on the cards the letters E, M, W, D, respectively.

Duration.—The manner of calculating the duration of the policy, forms the special feature of this method. In all cases the duration has been taken to be an integral number of years. In the case of the Existing, if $n+\delta$ is the time for which a policy has been in force, where n is the number of complete years and δ a fraction of a year, the duration has been taken as n, if $\delta<\frac{1}{2}$; and as $n+1$, if $\delta>\frac{1}{2}$; while, if $\delta=\frac{1}{2}$, the duration has been taken as n and $(n+1)$ alternately. When a policy has been in force less than a year, so that $n=0$, precisely the same rule has been followed. A similar course of procedure was adopted in the case of the Matured and the Withdrawn. But the Died have been dealt with in a different way, the duration having always been taken as $n+1$; thus counting, as is usual, the year of death as a complete year.

(The Method would apply equally well, or perhaps even better, if the observation on each existing policy closed on its anniversary in a given year—say 1893—instead of on a fixed date—say 31 December 1893. In this case all the Existing would really have been under observation for an integral number of years; and, of course, any deaths that occurred in the closing year, 1893, between the anniversary of the policy and 31 December, would have to be disregarded.)

Tabulation.—The cards having been completed, the necessary particulars have to be ascertained from them, and entered on schedules of the following form; and the number of schedules will be equal to the number of the commencing ages which occur in the observations.

Duration	Commencing Age......					
	Existing	Matured	Withdrew	Died	Total	At Risk
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Years						
0						
1						
2						
3						
...						
...						
...						
...						

Sorting.—For this purpose several successive sortings of the cards are necessary. The cards may be first sorted so as to bring together in one group, all those in which the commencing age is the same. The cards in each of these groups may then be sorted into four sub-groups, containing respectively the Existing, Matured, Withdrew, and Died; and, lastly, the cards in each of these sub-groups must be sorted into final groups according to the duration. Then the cards in each of these final groups have to be counted, and their numbers entered in columns (2), (3), (4), (5), of the proper schedule.

In order that the whole of the process may be clearly understood, I now give the actual figures for Commencing Age 30. It will be observed that all who are registered opposite any duration, n, in any one of the columns (2), (3), (4), (5), are held to have been under observation during the whole of the nth insurance year, this being a necessary consequence of the plan we have adopted, of calculating the duration to the nearest year. It follows that all the persons so registered, pass out of observation at the end of the nth year, and their total number is therefore set down in column (6) opposite the same duration n. In the present observations there were in all 72 entrants of the age 30; but one of these withdrew at the end of 6 months, and is therefore held not to have been under observation at all, and is registered against the duration 0. There were thus 71 persons under observation (or "at risk") during the first insurance year; and of these 1 died, while 5 withdrew, and 1 was existing (or still insured) at the close of the observations; so that the total decrement of the year was 7. Deducting these, the other 64 were under observation during the second insurance year, with the result that 2 died and 4 withdrew; together 6; leaving 58 who were under observation in the third insurance year; and so on. We thus see that the numbers in columns (2), (3), (4), (5), opposite each duration (n), are to be added together, and the total put in column (6), headed "Total Decrement"; and subtraction of this decrement from the number in column (7), gives the number who pass on to the next year, and are under observation during the $(n+1)$th insurance year. In practice, if the numbers are large, it will be found more convenient to fill up column (7) by adding from the bottom upwards, the formula being

$$(7)_n = (7)_{n+1} + (6)_n.$$

Duration (n) (1)	Commencing Age 30 Existing (2)	Matured (3)	Withdrew (4)	Died (5)	Total Decrement (2)+(3)+(4)+(5) (6)	At Risk in nth year (7)
Years 0	...	...	1	...	1	72
1	1	...	5	1	7	71
2	...	...	4	2	6	64
3	...	...	9	...	9	58
4	3	...	1	...	4	49
5	1	...	2	1	4	45
6	...	1	3	...	4	41
7	2	...	1	2	5	37
8	...	...	1	...	1	32
9	...	...	...	...	...	31
10	1	...	...	...	1	31
11	1	...	...	...	1	30
12	...	...	1	...	1	29
13	...	...	1	1	2	28
14	1	...	...	...	1	26
15	...	...	...	...	...	25
16	1	...	...	...	1	25
17	...	...	1	...	1	24
18	...	...	...	...	...	23
19	...	...	...	...	...	23
20	...	...	...	2	2	23
21	...	...	...	...	...	21
22	...	...	...	...	...	21
23	1	1	1	...	3	21
24	1	...	...	...	1	18
25	2	1	...	...	3	17
26	1	...	...	1	2	14
27	...	...	...	1	1	12
28	...	...	...	...	...	11
29	...	...	...	...	...	11
30	1	...	...	...	1	11
31	...	...	...	...	...	10
32	...	...	...	...	...	10
33	2	...	...	...	2	10
34	...	...	...	2	2	8
35	...	...	...	...	...	6
36	1	...	...	...	1	6
37	...	...	...	...	...	5
38	1	...	...	...	1	5
39	1	...	...	...	1	4
40	...	...	...	...	...	3
41	...	...	...	1	1	3
42	...	...	...	...	...	2
43	...	1	...	...	1	2
44	...	...	...	1	1	1
	22	4	31	15	72	947

Exact Duration Method.

Before deciding on the above method I tried another, which aims at more minute accuracy. The facts noted on the cards are the same as in the former method, with the single exception that the duration of each of the Existing, Matured, and Withdrawn policies, is calculated to the nearest month, instead of to the nearest year. When a policy has been in force for n years and r months, it is found convenient, instead of stating the duration as n years $+ r$ months, to state it as $(n+1)$ years $-(12-r)$ months; so that, for instance, if the duration is 15 years and 8 months, this is written on the card 16^{-4}. The schedule on which the particulars furnished by the cards are to be entered, will be seen from the following table [p. 214], which gives particulars of the 72 entrants at 30.

Here the year of duration of column (1), (which, for brevity, I will call Duration simply) is the current insurance year, or the year which was entered upon; for instance, a life which had been under observation for 15 years and 8 months, so that the duration is marked on the card as 16^{-4}, is entered opposite "Duration 16". The numbers to be inserted in columns (2), (4), (6), (8), of the schedule, opposite "Duration n", are therefore got by simply counting the cards in the final groups, arranged according to this duration.

The numbers in column (9) are the totals of those in columns (2), (4), (6), (8), opposite the same duration; and, the total number of entrants being written in column (10) opposite "Duration 1", successive subtraction of the figures in column (9) gives the numbers who entered on each year of duration. The lives entered opposite any "Duration n", were not all under observation during the whole of the nth year; and, in order to get the aggregate time during which they were under observation in the year, it is necessary to subtract from the number in column (10), who entered on the year, the totals of the deductions in columns (3), (5), (7). These deductions were got by adding mentally the months noted on the cards as deductions from the integral years of duration. All this will be better understood by an examination of the figures in the table.

This method has the advantage of giving with great accuracy the time during which the lives were under observation; but it is obviously much more troublesome than the "Nearest Duration" Method, and would become very laborious if the numbers

Year of duration (n)	Commencing Age 30												
	Existing			Matured			Withdrew			Died	(2)+(4)+(6)+(8)	Entered on Year	No. at Risk (10) − { (3)+(5)+(7) }
	No.	Deduction		No.	Deduction		No.	Deduction					
(1)	(2)	(3)		(4)	(5)		(6)	(7)		(8)	(9)	(10)	(11)
		Years	Mo's.		Years	Mo's.		Years	Mo's.				
1	...	...	...	...	...	...	6	...	6	1	7	72	$71\frac{6}{12}$
2	1	...	11	..	...	...	4	...	...	2	7	65	$64\frac{1}{12}$
3	...	...	...	...	...	...	8	...	9	...	8	58	$57\frac{3}{12}$
4	2	...	3	...	...	...	2	...	11	...	4	50	$48\frac{10}{12}$
5	2	1	4	.	...	...	1	...	...	1	4	46	$44\frac{8}{12}$
6	...	...	...	...	...	...	3	...	9	...	3	42	$41\frac{3}{12}$
7	1	...	...	1	...	7	2	...	11	2	6	39	$37\frac{6}{12}$
8	1	...	10	...	...	...	1	...	...	...	2	33	$32\frac{2}{12}$
9	...	...	...	...	...	...	...	...	...	...	...	31	31
10	1	...	3	...	...	...	...	...	...	...	1	31	$30\frac{9}{12}$
11	...	...	...	...	...	...	...	...	...	...	...	30	30
12	1	...	11	...	..	...	...	...	...	...	1	30	$29\frac{1}{12}$
13	...	...	.	..	...	...	2	...	8	1	3	29	$28\frac{4}{12}$
14	1	...	1	...	...	...	...	...	...	...	1	26	$25\frac{11}{12}$
15	...	...	...	...	...	...	...	...	...	...	...	25	25
16	1	...	4	..	...	...	...	...	...	...	1	25	$24\frac{8}{12}$
17	...	...	...	...	...	...	1	...	...	...	1	24	24
18	...	...	...	...	...	...	...	...	...	...	...	23	23
19	..	...	...	...	...	...	...	...	...	...	...	23	23
20	...	...	...	...	...	...	...	...	...	2	2	23	23
21	...	...	...	...	...	...	...	...	...	...	...	21	21
22	...	...	...	...	...	...	...	...	...	...	...	21	21
23	1	...	...	1	...	3	1	...	4	...	3	21	$20\frac{5}{12}$
24	1	...	...	.	...	...	...	...	...	...	1	18	18
25	1	...	...	1	...	4	...	...	...	...	2	17	$16\frac{8}{12}$
26	2	...	11	...	...	...	...	...	...	1	3	15	$14\frac{1}{12}$
27	...	...	...	...	...	...	...	...	...	1	1	12	12
28	...	...	...	...	...	...	...	...	...	...	...	11	11
29	...	...	...	...	...	...	...	...	...	...	...	11	11
30	1	...	4	...	...	...	...	...	...	...	1	11	$10\frac{8}{12}$
31	...	...	...	...	...	...	...	...	...	...	...	10	10
32	...	...	...	...	...	...	...	...	...	...	...	10	10
33	1	...	...	...	...	...	...	...	...	...	1	10	10
34	1	...	11	..	...	...	...	...	...	2	3	9	$8\frac{1}{12}$
35	...	...	...	...	...	...	..	...	...	...	...	6	6
36	1	...	...	...	...	...	...	...	...	...	1	6	6
37	...	...	...	...	...	...	...	...	...	...	...	5	5
38	1	..	...	...	...	...	...	...	...	...	1	5	5
39	1	...	...	...	..	...	...	...	...	...	1	4	4
40	...	...	...	...	...	...	...	...	...	...	...	3	3
41	...	...	...	...	...	...	...	...	...	1	1	3	3
42	...	...	...	...	...	...	..	...	...	...	...	2	2
43	...	...	...	1	...	...	...	...	...	...	1	2	2
44	...	...	...	...	...	...	...	...	...	1	1	1	1
	22	7	1	4	1	2	31	4	10	15	72	959	$945\frac{11}{12}$

involved were large. On comparing the figures in the two tables it will be seen that this method makes the total time for which entrants of 30 were under observation, 945 years 11 months,

while the other method makes it 947 years. Taking entrants of all ages, the total times of observation according to the two methods, were 30,531 years 3 months, and 30,521 years, respectively. These figures prove that there is practically no loss of accuracy by adopting the method which I have called the Nearest Duration Method.

A certain amount of labour would be saved if, instead of closing the observations on the various policies on a certain day, as 1 March 1891, the observation in respect of each policy had been closed on its anniversary in a certain year; for instance, in the office year ending on 1 March 1891. In this way each of the Existing would have been under observation for an integral number of years. This is really the course which I adopted in the first instance; but, for the purpose of comparing the figures with those got by the other methods, the observations were brought down to 1 March, as already stated.

Final Age Method.

This is a third method of which I made trial. In this case, instead of the duration of each policy being noted on the corresponding card, the age at exit (or at the close of the observation) was so noted. For each existing policy, this age, which (for brevity) I call the Final age, was taken to be the age on the birthday that fell nearest to 1 March 1891. If the birthday was 1 September, so that the exact age on 1 March 1891 was $y+\frac{1}{2}$ years, where y is an integer, the age was taken alternately as y and $y+1$. In the case of the Matured and Discontinued, the Final Age at the date of discontinuance, was calculated upon similar principles; so that each of them was assumed to be under observation during the whole of the year of discontinuance. If the exact age at entry is $x\pm\delta$, and the exact age at exit $y\pm\epsilon$, where x and y are integers, and δ, ϵ, both $<\frac{1}{2}$, the duration is taken as $y-x$; and as the exact duration is $y-x\pm\epsilon\mp\delta$, the assumed duration may differ from the real by $+\epsilon+\delta$, $+\epsilon-\delta$, $-\epsilon+\delta$, or $-\epsilon-\delta$; that is to say, by almost a year either in excess or defect. In this respect, therefore, the approximation to the truth is not so close as could be desired; and I think the method inferior to those I have already described.

Died.—In order to reckon the year of death as a complete year in the usual way, it was necessary to adopt a different rule for calculating the final ages of those who died. In these cases, the final ages were got by adding to the commencing age one

more than the complete number of years for which the life had been under observation. The following table gives particulars of the entrants at 30.

Final Age x	Commencing Age 30					
	Existing	Matured	Withdrew	Died	(2)+(3)+(4)+(5)	At risk from Age $x-1$ to x
(1)	(2)	(3)	(4)	(5)	(6)	(7)
30	...	...	1	...	1	72
31	1	...	5	1	7	71
32	...	...	4	2	6	64
33	...	...	9	...	9	58
34	2	...	1	...	3	49
35	2	...	1	1	4	46
36	...	...	4	...	4	42
37	1	1	1	2	5	38
38	1	...	1	...	2	33
39	...	...	...	...	...	31
40	1	...	...	...	1	31
41	1	...	...	...	1	30
42	...	...	..	...	...	29
43	1	...	2	1	4	29
44	...	...	...	...	...	25
45	...	...	...	...	...	25
46	1	...	...	...	1	25
47	...	...	1	...	1	24
48	...	...	...	...	...	23
49	...	...	...	...	...	23
50	...	...	...	2	2	23
51	...	...	...	...	...	21
52	...	1	...	...	1	21
53	1	...	1	...	2	20
54	1	...	...	...	1	18
55	2	1	...	...	3	17
56	1	...	...	1	2	14
57	...	...	...	1	1	12
58	...	...	...	...	...	11
59	...	...	...	...	...	11
60	1	...	...	...	1	11
61	...	...	...	...	...	10
62	...	...	...	...	...	10
63	2	...	...	...	2	10
64	...	...	...	2	2	8
65	...	...	...	...	...	6
66	1	...	...	...	1	6
67	...	...	...	...	...	5
68	1	...	...	...	1	5
69	1	...	...	...	1	4
70	...	...	...	...	...	3
71	...	...	...	1	1	3
72	...	...	...	...	...	2
73	...	1	...	...	1	2
74	...	...	...	1	1	1
	22	4	31	15	72	950

Here the number in column (6) is the sum of those in columns (2), (3), (4), and (5); and successive subtraction of the numbers in column (6) from those in column (7) opposite the same Final Age, x, gives us the numbers who pass on to the next year of age, and are assumed to have been under observation from age x to $x+1$. Thus, the entrants at age 30 being 72, 1 withdrew at the same age, and is not held to have been under observation at all: so that we reckon 71 as under observation in the first insurance year from age 30 to 31, of whom 1 died. There was also 1 existing at the close of the observations, and 5 withdrew; or 7 in all went off, as entered in column (6). Deducting these, there remain 64 who passed on to the next year, and were under observation from age 31 to 32; and so on. This method gives the total years of life for entrants of 30, as 950; against 947 by the Nearest Duration Method, and $945\frac{11}{12}$ by the Exact Duration Method. The total years of life at all ages, according to this method, were 30,538 against 30,521 and $30{,}531\frac{3}{12}$.

On comparing the tabulations according to the 3 methods I have described, it will be seen that there are slight differences in the distribution of the Existing, Matured, and Withdrew; but the distribution of the Deaths is identical in all three. The Exact Duration Method is obviously more exact than the other two, but a great deal more laborious.

The Institute Method.

The three methods of tabulating the facts which I have described, have certain features in common, in which they differ from the method adopted in compiling the H^M Table, which I call the "Institute Method".

In order to compare the working of those methods as completely as possible with that of the Institute Method, I arranged that the *Scottish Equitable* Female Experience should be taken out according to the latter, as well as according to the other three; but, when the work was finisht, I found that it gave 31,141·5 years of life, whereas the number according to the Exact Duration Method, was only 30,531·3. The difference, 610·2, was too great to be due to any accidental cause, and clearly indicated that there must be an error of principle somewhere.

On investigating the point, I found that my assistant, not having correctly apprehended the principle of the Institute

Method, had calculated the ages of the Existing (on 1 March 1891) by an incorrect process.

Age of the "Existing" in the Institute Method.—In the Mortality Experience Volume* the facts are tabulated in the

Current Age at Entry 30			
Current Age at exit	Number of Entrants 5,791		
	Existing	Discontinued	Died
30	319	75	4
31	252	365	28
32	230	220	35
33	235	153	49
34	198.	147	51

form shown in the above extract. The phrase "current age" is used as an abbreviation of "current year of age", which occurs frequently in the preface. I have elsewhere (*J.I.A.* xxix, 478) stated my reasons for thinking the phrase an objectionable one, and I will not repeat them here, but will only say that it seems to be exactly equivalent to the more usual and more correct one —age next birthday.

On reading the heading in this extract the natural inference is that the Existing at the close of the observations, are to be tabulated under their ages next birthday; and accordingly my assistant tabulated the 807 Existing in that way. But the instructions for filling up the cards state that the age at exit is to be found by adding to the age at entry, the difference between the years of entry and of exit; and this applies to the Existing, as well as the Discontinued and Died, and will give, in the case of the Existing, not their ages next birthday, but, on the average, their true ages. The same thing appears still more clearly, as I shall presently show, from the example given in the preface, (page 18), of the manner in which the numbers at risk at each age are got from the observed numbers. Before considering that example, it seems desirable to examine fully the principles on which the Institute experience was compiled, and to compare them with those adopted on some other occasions.

* The Mortality Experience of Life Assurance Companies collected by the Institute of Actuaries. London, C. & E. Layton, 1869.

In all investigations into the mortality of insured lives, annuitants, &c., the object is to obtain data for the construction of a mortality table; and, for this purpose, to ascertain the probability, q_x, of dying in a year, at each integral age x.* The insurances, however, very seldom date from the birthdays of the assured; so that the real age at the date of insurance is almost always fractional, say $x+\delta$, where x is an integer and δ a fraction. The fractional part of a year, between entry and the next birthday, may be dealt with in various ways.

(*a*) If we proceed on the assumption that the mortality depends only on the age, the obvious course is to include the observations relating to the fractional time from age $x+\delta$ to age $x+1$, in the year of age x to $x+1$.

(*b*) In some cases this fractional period and the corresponding deaths, have been left out of observation altogether; and, bearing in mind that the force of mortality is generally increasing throughout each year of age, so as to be greater towards the end of the year than at the beginning, this method is perhaps better than (*a*).

(*c*) Another method is the one adopted by Mr. A. J. Finlaison in his Report (1883) upon the Mortality of Government Annuitants. He states that it has been found that, on the average, "4 months had passed from the birthdays of the nominees at the date of the grants of the life annuities"; and he therefore proceeds as if each life entering between the ages x and $x+1$, was of the age $x+\frac{1}{3}$, and was therefore under observation for $\frac{2}{3}$rds of a year before attaining the age $x+1$.

(*d*) In the three methods I have employed, as described above, it is assumed that all entrants whose real ages lie between $x-\frac{1}{2}$ and $x+\frac{1}{2}$, are of the exact integral age x.

(*e*) In the Institute Method the entrants are treated as if they had all been born on a 31 December, and had entered on a 30 June, so that those whose ages lie between x and $x+1$, are assumed to attain the age $x+1$ on the 31 December following entry, and to have been then under observation for exactly half a year. The reasons for this are stated in the following extracts from the Mortality Experience Volume. "In so large a " number of cases, to estimate the precise age in parts of a year, " was thought to involve more trouble and labour than such

* Several of the questions here considered were dealt with in my paper *On the premiums for the insurance of recently selected lives.* (*J.I.A.* xx, 95: See particularly pages 98–101 and 111.)

"minute accuracy would repay. . . . But, as in all cases "the office age on entry is the age next birthday, it was decided "that on the whole the current year of age, or office age, at the "date of assurance, would afford the means of approximating very "closely to the actual age, by the single assumption that the "assured attained that age at the end of the year of entry." (Page 3.) "The date of entry and of exit, like the age at entry, "it was considered would in very large numbers be so spread over "the whole of the year, that for all practical purposes the average "period of observation in each may be taken at half a year; and "the date of entry and of exit would consequently correspond "with the middle of the current year of age." (Page 4.)

Objections to the Institute Method.—I believe that neither of the two assumptions here made, as to (1) the age of entry and (2) the date of entry, is correct. When the premium is computed by years of age, the average time from entry to next birthday is certainly less than 6 months, and may perhaps be more correctly taken as 3 months. When the premiums are computed by half-years of age, the assumption as to the age at entry will be nearer to the truth; and still more when they are computed by quarter-years; but these practices are comparatively modern, and I am not aware of any statistics showing their influence on the entry age. Again, it is the general, if not the universal, experience of Life Offices, that the number of policies effected towards the close of the books for the year, greatly exceeds the number effected in a corresponding time at the beginning of the year; or, rather, the number credited to the beginning of the year; since it is usual for many of the policies then effected to be dated back into the old year. From some approximate estimates made with regard to the entrants in the *Scottish Equitable Life Assurance Society,* I infer that the average period between entry and the following 1 March is about $\frac{1}{3}$rd of a year. If the average interval between the date of entry and the following 31 December, is the same, say 4 months, as the average interval between the date of entry and the next birthday, the assumption that the exact age $x+1$ will, on the average, be attained on the 31 December following entry, will be correct; but the Institute method would still be incorrect; for it treats the experience of those 4 months as if it were the experience of 6 months, and thus underestimates the mortality of the "year 0".

The "year 0" will in what follows be called the "year $\frac{1}{2}$". In the Institute Tables showing "Number exposed to risk, and

number who died in each year of assurance, for each age of entry", the observed portion of the calendar year of entry, is called the year "0"; and the subsequent calendar years are called the years of assurance, 1, 2, 3, &c. But in the preface to the tables, the year "0" is called the first year of assurance; the year 1 is called the second year of assurance; and so on. This is an inconsistency which is likely to cause confusion and misunderstanding; especially when we compare the Institute observations with others that are compiled according to a method which, like those above described, shows the mortality during the first 12 months after entry, and in each subsequent year. In the latter case, we must necessarily number our years 1, 2, 3, &c.; and, in order to keep the difference of procedure constantly in view, I shall in what follows speak of the Institute year "0" as the year $\frac{1}{2}$; and the following Institute years as the years $1\frac{1}{2}$, $2\frac{1}{2}$, &c.

It will now be convenient to consider the example given in the Institute volume, showing how the numbers at risk are got from the above quoted figures relating to the 5791 Entrants at "Current Age 30".

					Exposed to risk	
The number of entrants was		5791,	half of which	2895·5		
Discontinued . . .		75,	,, ,,	37·5		
					2858	Age 29, or 1st year
Entered on the first year .		5791				
Deduct Existing .	319					
Discontinued	75					
Died . .	4					
		398				
Entered on the second year		5393	less half of the Discontinued	182·5	5210·5	Age 30, or 2nd year
Deduct Existing .	252					
Discontinued	365					
Died . .	28					
		645				
Entered on the third year		4748	less half of the Discontinued	110	4638	Age 31, or 3rd year
&c. &c.						

From this it is clear that the Existing are considered as having been under observation during the whole of the year, at the end of which the observations closed (1862 or 3.) According to the principles on which the facts have been extracted, the 5791 entrants at 30 next birthday, are supposed to have been $29\frac{1}{2}$ at entry; and deducting the 4 who died and the 75 who discontinued in the six months after entry, which is called in the example "1st year", but which we call the "year $\frac{1}{2}$", all the others (including the 319 existing) attained the age 30 on the 31

December following entry. Thus 5393 persons of the age 30 entered on the year $1\frac{1}{2}$; and the 252 of these who were Existing at the end of it, then attained the age 31; and so on. We see, therefore, that the heading "current age at exit" is incorrect and misleading, as regards the Existing; but it is correct as regards the Discontinued and the Died.

Having regard to all the circumstances, it seems that, for the correct and convenient application of the Institute Method, it is necessary that the observations should close on a 31 December. It would therefore not have been practicable to compare the results obtained by the Institute Method with those obtained by the other methods, unless the observations in all were closed on a 31 December; and as this would have involved doing all the work over again, I have not thought it necessary to pursue the comparison further.

For the sake of completeness, and in order to facilitate comparison with other observations, it will be convenient now to arrange the above quoted Institute figures for entrants of 30, in the usual tabular form.

Age next birthday at 30 (*say* $29\frac{1}{2}$) *Entrants* 5791.

Tabulated age at exit	Existing	Discontinued	Died	(2)+(3)+(4)	Enter on year	Half the Discontinued	At risk	Year of Assurance	Age at beginning of year
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)=(6)−(7)	(9)	(10)
30	319	75	4	398	5791	37·5	*	0 (or $\frac{1}{2}$)	$29\frac{1}{2}$
31	252	365	28	645	5393	182·5	5210·5	1 (or $1\frac{1}{2}$)	30
32	230	220	35	485	4748	110	4638	2 (or $2\frac{1}{2}$)	31
33	235	153	49	437	4263	76·5	4186·5	3 (or $3\frac{1}{2}$)	32
34	198	147	51	396	3826	73·5	3752·5	4 (or $4\frac{1}{2}$)	33

* The number at risk in the year $\frac{1}{2}$ must be got by a different process and is therefore left blank here.

The Rate of Mortality depends on the time since entry.

Notwithstanding the above objections to the Institute method, I believe that it would give very good results, if the rate of mortality depended only on the age. But it is well known that the rate of mortality among insured lives, depends upon the time that has elapst since entry, as well as upon the age; and that, for recent entrants, it depends far more upon the former than upon the latter.

As the effect of selection is very much the same in the case of annuitants as among insured lives, I have taken the following figures from Mr. Finlaison's above mentioned report, in order to illustrate this point.

Female Life Annuitants.—Adjusted rate of mortality for the year in which life annuities are purchast, and for each of the three subsequent years.

Age	Years elapst from Purchase			
	0	1	2	3
x	Rate of Mortality $q_{[x]}$	Rate of Mortality $q_{[x-1]+1}$	Rate of Mortality $q_{[x-2]+2}$	Rate of Mortality $q_{[x-3]+3}$
55	·00881	·01180	·01471	·01563
56	903	1246	1559	1663
57	925	1315	1652	1771
58	948	1388	1751	1884
59	971	1466	1855	2006
60	995	1547	1966	2135

The following tables show the differences of these rates, corresponding to an increase of a year (1) in the age, and (2) in the duration.

Ages	Differences corresponding to an increase of a year of age, the duration remaining the same.			
55–6	·00022	·00066	·00088	·00100
56–7	22	69	93	108
57–8	23	73	99	113
58–9	23	78	104	122
59–60	24	81	111	129

Age	Differences corresponding to an increase of a year of duration, the age remaining the same.		
55	·00299	·00291	·00092
56	343	313	104
57	390	337	119
58	440	363	133
59	495	389	151
60	552	419	169

On comparing these, it will be seen that the differences between the rates of mortality in the 1st and 2nd years after

purchase, are many times greater than the differences between the rates of mortality at successive ages in the 1st year; and several times greater than the differences in the 2nd, 3rd, and 4th years.

This point will be further exemplified by the following figures, which show the rates of mortality that prevailed in each of the first five insurance years, among the whole of the 2205 females insured with the *Scottish Equitable.*

Insurance year	Number at risk	Deaths	Rate of Mortality	
1	2187	17	·00772	
2	1962	18	·00918	
3	1802	19	·01054	·00986
4	1646	15	·00911	
5	1525	18	·01180	

The corresponding figures relating to the H^F lives of the Institute experience, are

Insurance year	Number at risk	Deaths	Rate of Mortality.	
0 (or ½)	8261·5	53	·00642	
1 („ 1½)	15178	176	·01159	
2 („ 2½)	13402·5	191	·01425	
3 („ 3½)	11919	180	·01511	·01467
4 („ 4½)	10704·5	152	·01420	
5 („ 5½)	9662·5	158	·01635	

In both these cases it is obvious that the increase in the rate of mortality, in passing from one insurance year to the next, is very much greater than could be caused by an increase of a year in the ages of the lives under observation. We thus see that it is much more important to group together the lives which have been under observation for the same time, than to group together those that are of exactly the same age.

General Conclusions.

The conclusions I draw from the foregoing figures are:—

(1) That, in our investigations into the rate of mortality among assured lives, we shall not arrive at trustworthy results unless we take into account the time that has elapst since entry, as well as the age.

(2) That, as the effect of selection is greatest immediately after entry, and rapidly diminishes during the early insurance years, it is important to determine, as accurately as possible, the rate of mortality in each of those years: that is to say, in the 12 months immediately after entry, which we may conveniently call the 1st policy year; and in the following years, namely the 2nd, 3rd, &c., policy years, so long as the effect of selection is traceable.

(3) That each of the methods (*a*), (*b*), (*c*), above described, is unsuitable for the purpose; because in them the observations relating to the 1st policy year for entrants of the age $x+\delta$, are divided into two parts, which are placed respectively under the years of age, x to $x+1$, and $x+1$ to $x+2$; and similarly the observations in the 2nd policy year are divided between two years of age, $x+1$ to $x+2$, and $x+2$ to $x+3$; and so on.

(4) That the Institute Method is unsuitable, because it divides in a similar way the observations of each policy year between two calendar years.

If the Institute Method of procedure were strictly correct, it would give us, for entrants of any age, $x+\frac{1}{2}$, the rate of mortality during the six months after entry, and during each of the following years. On a similar assumption Mr. Finlaison's method would give us, for entrants of the age $x+\frac{2}{3}$, the rate of mortality during the four months after entry, and in each of the following years. In order to deduce from such results the rate of mortality in the 1st, 2nd, &c. years after entry, for entrants of the age x, a long series of troublesome calculations will be necessary; and as the force of mortality, which is zero at the instant of entry, increases very rapidly during the years immediately after entry, the results of our calculations would probably differ materially from the truth. But, among the entrants at age $x+\frac{1}{2}$, there will be included persons who have been insured at the close of the calendar year, for all periods from one day to one year; and all these are grouped together by the Institute Method, with the result that it is not possible to say whether the probability of death given by it for the year $\frac{1}{2}$, is even a fair approximation to the probability of dying in the six months after entry.

(5) That no method will be satisfactory which does not arrange the observations according to policy years.

Several sets of mortality observations have been published in which this course has been adopted, among which may be mentioned the experience of the *Connecticut* Mutual Life Office to

1878, containing 780,353 years of life and 8,746 deaths, and that of the *Gotha* Life Office to 1878, containing 1,021,456·5 years of life and 21,538 deaths.

The question, whether the experience about to be collected shall or shall not be taken out according to policy years, seems to me to be the most important point to be settled; and it may be settled without coming to any decision on the other points that will have to be considered; such as, how the age at entry is to be reckoned; whether the observations on the Existing are to be closed on the anniversary of the policy, or at the end of a calendar year; how the ages at exit (or the durations) are to be reckoned, &c. This question should, I think, be considered and settled before the other questions are taken up; and these other questions should all be considered and settled before the work of filling up the cards is commenced, in fact, before the form of card is decided on. Judging from my own experience, I should say that the only satisfactory way of coming to a conclusion with regard to such points, is to make trial of the different methods, with some selected body of facts; and I have no hesitation in saying that the experience I have had, while superintending the compilation of the *Scottish Equitable* Female Experience, has been of the greatest assistance to me in forming an opinion on the various points discust above.

On the Mortality among the Healthy Female Lives insured with the Scottish Equitable Life Assurance Society. By Thomas Bond Sprague, M.A., LL.D., *&c.*

As stated in the foregoing paper, the number of persons to whom the observations relate was 2,205, and the total years of life 30,521. The average time for which the lives were under observation was therefore 13·84 years. The number of deaths was 668; the number of discontinuances 730; and the number of existing 807. The experience may therefore be described as an unusually mature one. The numbers at risk and the deaths in each insurance year have been calculated separately for each age at entry; but, as the numbers at individual ages are small, it is probably not worth while to publish them. The following are the figures irrespective of the age at entry.

Age	At risk	Deaths		Age	At risk	Deaths	
		Actual	Expected			Actual	Expected
				Brought forward	16,274	174	158·6
3	1	...		50	724	7	11·6
4	2	...		51	723	10	12·0
5	2	...		52	722	15	12·6
6	6	...	·4	53	709	9	13·1
7	11	...		54	704	17	13·9
8	17	...		55	693	13	14·6
9	26	...		56	693	14	15·6
10	35	...	·2	57	677	18	16·3
11	45	...	·2	58	652	13	16·8
12	53	...	·2	59	644	11	17·7
13	67	...	·2	60	634	15	18·8
14	84	...	·2	61	609	14	19·5
15	100	...	·3	62	574	12	19·8
16	106	...	·3	63	543	14	20·3
17	120	1	·5	64	519	18	21·0
18	132	...	·6	65	477	23	20·7
19	148	1	·8	66	436	14	20·3
20	168	1	1·0	67	412	11	20·5
21	178	1	1·2	68	383	19	20·4
22	200	...	1·4	69	353	10	20·3
23	228	1	1·5	70	326	15	20·3
24	250	3	1·7	71	300	11	20·4
25	277	1	1·8	72	272	22	20·3
26	309	3	2·1	73	236	22	19·6
27	318	1	2·2	74	204	17	18·6
28	344	4	2·4	75	181	19	17·8
29	372	1	2·6	76	151	16	15·9
30	409	4	3·2	77	129	12	14·8
31	439	6	3·5	78	113	9	13·9
32	485	9	3·8	79	97	10	12·9
33	521	5	4·3	80	83	10	12·0
34	546	5	4·6	81	64	9	10·1
35	567	8	5·0	82	54	13	9·3
36	605	8	5·5	83	40	8	7·4
37	632	8	6·0	84	28	5	5·6
38	649	3	6·3	85	22	2	4·6
39	678	8	6·8	86	20	4	4·4
40	688	5	7·1	87	15	3	3·5
41	697	5	7·3	88	10	4	2·4
42	706	11	7·6	89	6	2	1·5
43	716	9	8·0	90	4	1	1·1
44	719	7	8·3	91	3	1	·9
45	732	12	8·9	92	2	...	·7
46	726	11	9·5	93	2	...	·8
47	718	9	9·8	94	2	1	1·0
48	720	10	10·4	95	1	...	·6
49	722	13	10·9	96	1	1	·8
Carried forward	16,274	174	158·6	Total	30,521	668	745·6

The expected deaths have been calculated according to the H^M table, as it was considered that in this way we should best be able to ascertain whether the mortality among these female lives differs so much from that which generally prevails among male insured lives, as to justify the extra premium of 5*s.* per-cent per annum up to the age of 50, which is now generally charged. In order to get rid of the irregularities due to paucity of facts, it is necessary to arrange the figures in groups of ages. This is accordingly done in the following table. The groups are mostly quinquennial; but in a few cases, where such groups did not give sufficient regularity, a different grouping was adopted.

Ages	Years of Life	Deaths		Rate of Mortality per-cent	Ratio of Actual to Expected Deaths
		Actual	Expected		
3–21	1,301	4	6·1	·31	·66
22–26	1,264	8	8·5	·63	·99
27–31	1,882	16	13·9	·85	1·15
32–35	2,119	27	17·7	1·27	1·53
36–40	3,252	32	31·7	·98	1·01
41–45	3,570	44	40·1	1·23	1·10
46–50	3,610	50	52·2	1·39	·96
51–55	3,551	64	66·2	1·80	·97
56–60	3,300	71	85·2	2·15	·83
61–65	2,722	81	101·3	2·98	·80
66–75	3,103	160	198·5	5·16	·81
76–80	573	57	69·5	9·95	·82
81–96	274	54	54·7	19·71	·99
All Ages	30,521	668	745·6	2·19	·90

It will be seen that at ages 27–45, the actual deaths exceed the expected; and, considering these ages alone, the years of life are 10,823, the actual deaths 119, and the expected 103·4. The excess, therefore, is 15·6; and, comparing this with the years of life, we have $\frac{15\cdot6}{10{,}823} = \cdot0014$, which indicates that the higher mortality would be met by an annual charge of 2*s.* 10*d.* per £100 assured at the ages 27–45. Under the age of 27, the experience has been favourable, but it is small in extent. From ages 46 to 55, it may be said that the mortality among the females was practically identical with the H^M mortality; at ages 56–80 it was very much less; and above 80, where the numbers are again small, it closely approximated to the H^M mortality.

On a Mode of tabulating the facts, for the purpose of ascertaining the Numbers exposed to Risk, and calculating the Rate of Mortality experienced by Assurance Companies. By James Meikle, F.I.A., F.F.A.

As there is a prospect of the Scottish Insurance Companies again collecting the statistics of their mortality experience, it is desirable to consider anew the mode in which the facts should be tabulated, in order to ascertain the rate of mortality experienced among assured lives. The previous collection was drawn from the experience of 10 offices, and consisted of the total lives assured from the beginning of the several offices, brought down to 31 December 1863. Thirty years having elapsed, it is proposed to begin with the facts then existing—tabulating the deaths arising among these; and also to tabulate new assurances, dealing with them separately. It is proposed also to discuss whole-life assurances and endowment assurances. It is also proposed to collect the experience of male and female annuitants. With so large an undertaking in prospect, it is expedient to re-consider the modes previously adopted, and in some detail to state the manner in which these experiences should be now collected.

Various modes have hitherto been followed. It is not my present purpose to re-capitulate these. My present intention is merely to state the system that appears to me most suitable. And inasmuch as Dr. Sprague is also considering the subject, and has given me an early perusal of his paper "On the Tabulation of the Facts", &c., which embodies his idea of the most suitable method, and as he has kindly furnished me with the details of his calculations, I shall exemplify my method by making use of the same materials.

Mortality during Years of Life.

The leading features of my method may be comprised in three words. I base my calculations upon the actual ages attained—measured by months—on entering the observations, derived from the date of birth and date of assurance; and the actual ages attained on leaving the observations, whether by discontinuance or by survivance. The deaths are assumed to die at the *end* of the year of life in which they died; and assurance claims are thus payable six months after death, earlier payment being the subject of adjustment. The surviving and the discontinuing lives are thus under observation for the exact periods of the duration

of the policies; and the deceasing lives are under observation, on an average, for (say) six months after date of death, assuming the deaths to be equally spread over the twelve months. I tabulate these facts for the several sections of the risks—the discontinued, existing, and the died: that is to say, for each of these sections there is ascertained separately, the number of years of life under observation between every two ages, by those who have discontinued their assurances, those who are in existence at the close of the observations, and those who died. These, when added together, give the total number under observation in each year of life; and the ratio of the number dying in that year to the number under observation furnishes the annual rate of mortality.

By dealing separately with each class in this way, great advantage is derived over the method which slumps all classes in one group. The general weight of the observations is ascertained. An experience derived from a mass of facts of discontinued policies seems to me to be possessed of very slight significance, compared with an experience in which there are few discontinuances, or where fewer of the lives pass out of observation otherwise than by death. Thus, the years of life of the discontinued observations in

the H^M experience were $13\frac{1}{2}$ per-cent of the whole,
and in the American experience $22\frac{1}{2}$ „ „

This method further facilitates a modification of the total general results by making any arbitrary assumptions in regard to these discontinuances, and of considering how far the tendency of such assumptions affects the rate of mortality that would then result. If surrendered policies were separately treated in this way, some interesting views may be had of the effect on the rate of mortality of voluntarily passing out of observation by surrendering their assurances.

Such is my general mode of dealing with observations when called upon to ascertain one general table of the rate of mortality from the youngest to the oldest age. If the rates of mortality were desired of persons effecting insurances at each age, some slight modification would be required in respect of the first year of the assurances; but this point will be afterwards referred to.

Dr. Sprague in his paper has examined three modes of tabulating the facts, but he inclines to the method which he has denominated the "Nearest Duration Method." He appears to make use of the same calculations as I make. For my method the actual duration of the risk is inherent in the difference

between the exact age at entry and the exact age at exit—the only two factors adopted. Dr. Sprague ascertains the actual duration by deducting the date of entry from the date of exit, and deals with the nearest integral number of years of that duration. He has thus obtained several results before he lands at the factor adopted. If the fractional period of the year is less than six months, he cuts off that portion. If greater than six months, he assumes the duration to be of the next integral number of years. But he further applies this assumed term of "Nearest Duration" to the nearest integral age at entry, and he thus deals entirely with integral ages and integral durations. To this mode of amending or of abbreviating the facts there might be no objection. The loss in one case may be made up by gain in another case. The ultimate results of the two methods of tabulation nearly coincide by the assistance of a balance of small differences. I have preferred, however, to adhere to the facts, and neither to add to nor to deduct from them. Dealing with large numbers does not appal me. Expedients flow in under such circumstances which seem to lessen the labour in place of increasing it. In dealing with small numbers, I require to make mental calculations, whereas in dealing with large numbers, printed tables of multiplication at once relieve me of that labour, and the result is even more easily attained than when dealing with small or with a medium number of facts. But information is derived from the tabulation of these details which otherwise would be lost. I am able to show the number of assurances effected in the same month as their birthday, in the previous month, and so on. These may not be of much use, but they are interesting. Thus, in the 2,205 assurances which Dr. Sprague has employed for illustrating his views, I may inform him that, while the average number assuring in each month of their year of life is $\frac{2205}{12}=184$

404 of these effected their Insurance in the same month as their birthday.						
213	"	"	1	month previous to the month of their birth.		
188	"	"	2	"	"	"
165	"	"	3	"	"	"
175	"	"	4	"	"	"
149	"	"	5	"	"	"
172	"	"	6	"	"	"
170	"	"	7	"	"	"
169	"	"	8	"	"	"
165	"	"	9	"	"	"
136	"	"	10	"	"	"
99	"	"	11	"	"	"
Total, 2205						

From these figures we may also find the average number of months the lives assured were younger than their office ages next birthday was 4·55 months.

$$
\begin{array}{rcl}
404 \times 0 &=& 0 \\
213 \times 1 &=& 213 \\
188 \times 2 &=& 376 \\
165 \times 3 &=& 495 \\
175 \times 4 &=& 700 \\
149 \times 5 &=& 745 \\
172 \times 6 &=& 1032 \\
170 \times 7 &=& 1190 \\
169 \times 8 &=& 1352 \\
165 \times 9 &=& 1485 \\
136 \times 10 &=& 1360 \\
99 \times 11 &=& 1089 \\
\hline
2205 & & 10{,}037
\end{array}
$$

$$\frac{10{,}037}{2205} = 4{\cdot}55$$

These results can also be shown for each of the three classes—the discontinued, the existing and the died; and it can be pointed out that the ratio of divergence from the average is greater in the existing policies than in the discontinued and the died—thus probably illustrating the growing tendency of dating back the proposal to a date previous to the birthday.

To enable anyone to comprehend the full force of the difference between Dr. Sprague's "Nearest Duration Method" and my method, I have re-cast his material, so as to enable me to give his results according to my method; that is to say, while he makes no separation between the several sections of the lives, but deals with them as if they belonged to one group, my practice has been to calculate the years of life which each section separately contributes to the total number.

Thus Dr. Sprague's method is, taking an example from his manuscript of the entrants at age 31:

Total number entering at nearest age 31	59
Number passing out of observation, 1st year . .	5
Leaving number entering upon age 32 . .	54
Number passing out of observation, 2nd year .	3
Leaving number entering upon age 33 . .	51
&c.	&c.

If, however, the number of lives who eventually die, discontinue, mature, or remain, are similarly dealt with, we obtain the portions of the total number belonging to the separate sections, thus:

Age		Die	Discontinue	Mature	Existing	Total
31	Total number entering .	19	18	2	20	59
	Pass out . . .	0	5	0	0	5
32	Enter on next year . .	19	13	2	20	54
	Pass out . . .	1	2	0	0	3
33	Enter on next year . .	18	11	2	20	51
	Pass out . . .	0	0	0	2	2
34	Enter on next year . .	18	11	2	18	49
	Pass out . . .	0	1	0	1	2
35	Enter on next year . .	18	10	2	17	47
	Pass out . . .	1	4	0	2	7
36	Enter on next year . .	17	6	2	15	40
	&c.		&c.		&c.	

All Dr. Sprague's calculations have been separated in the four classes, and thereby facilitate a closer comparison with my results.

It will be observed that the above example is not the one given in Dr. Sprague's paper, which is for age 30. That age, however, contains the entry of a life passing out of observation by discontinuance in the first six months of assurance. He discards cases of this kind altogether, and thereby gets quit of what has been termed the year "0." This fractional part of a year can only arise when the tabulation refers to entrants at each age, and which evidently forms a prominent feature of Dr. Sprague's system.

Having accordingly re-cast all Dr. Sprague's calculations in the foregoing manner, I am able to show the years of life contributed by those who matured, discontinued, survived, died, and to compare them with the corresponding calculations according to my method.

The total years of life under observation, as given in his paper, being the portions thereof contributed by the — 30,521

	Years	
Matured	580	
Discontinued	4,005	
Survived the observations	13,007	
Died	12,929	
		30,521

According to my calculations, the corresponding figures are—

	Years	Months
Total years of life	30,579	3

of which there were contributed by the

	Years	Months		
Matured	581	6		
Discontinued	3,998	5		
Survived	13,091	3		
Died	12,907	8		
			30,579	3

These results, by two different systems, are remarkably similar. When separated in this way, the years of life afford a closer test of the weight of the observations than the mere number of lives. Thus—

While the mere numbers are		And their proportion	The proportion of the years of life are
Of the Matured	71	3·2	1·9
„ Discontinued	659	29·8	13·1
„ Survived	807	36·6	42·8
„ Died	668	30·4	42·2
	2,205	100	100

It will also be observed that the average duration of the

	Duration of each class, Years	Months		Number	Average Duration of each class	H^{M}		Scottish Experience to 1863
Matured is	= 581	6	÷	71	= 8·2 years	6·3	4·6	4·4
Discontinued	= 3,998	5	÷	659	= 6· „			
Survived	=13,091	8	÷	807	=16·2 „		5·8	9·3
Died	=12,907	8	÷	668	=19·3 „		13·5	11·4
Total	30,579	3		2,205	=13·9 „		9·2	8·6

These interesting particulars are obtained by thus separating the lives into classes. They have been derived from the total results, but their interest is greatly enhanced when we observe the portions thereof in quinquennial groups of life, or at each year of life.

Simple Illustration of the Method.

The process followed by me has already been briefly stated. Perhaps a short and simple example may be acceptable. Let there be five cases,

Entering at ages	And exiting at ages	The duration being
30 years 1 month	32 years 1 month	2 years
31 „ 1 „	33 „ 1 „	2 „
32 „ 2 „	34 „ 2 „	2 „
33 „ 3 „	35 „ 3 „	2 „
34 „ 4 „	36 „ 4 „	2 „
	In all,	10 years

The first case is aged 30 years and one month. There were thus only 11 months of a year of life, 30–31, at risk. The one life entering is placed in the column; but −1 is also entered in small type, to be deducted from that year: and it is also entered in the following year of life, 31–32, to be added, the nett result at each age being placed in the following column; and similarly for other ages. The nett numbers are summed progressively. The same process is repeated for those exiting from the observations, and the difference between the two columns of progressive summations affords the number of years of life under observation at each age.

These cases are enrolled in my formulating table thus:

Age	Entrants						Exitants						Difference	
	Number Entering		Nett Number		Progressive Summation		Number Exiting		Nett Number		Progressive Summation		Years of Life exposed to Mortality	
	Yrs.	Mos.	Yrs.	Mos.	Yrs.	Mos.	Yrs.	Mos.	Yrs.	Mos.	Yrs.	Mos.	Yrs.	Mos.
30–31	1	−1		11		11	...		...		...		0	11
31–32	1	1−1	1		1	11	...		...		...		1	11
32–33	1	1−2		11	2	10	1	−1		11		11	1	11
33–34	1	2−3		11	3	9	1	1−1	1		1	11	1	10
34–35	1	3−4		11	4	8	1	1−2		11	2	10	1	10
35–36	1	4		4	5		1	2−3		11	3	9	1	3
36–37	...		...		5		1	3−4		11	4	8		4
					5			4		4	5			
Total Number, 5							=		5					
Sum of years of life of the Entrants to the close of the observations = . .					29	1	The same for the Exitants . .				19	1	10	

The checks by summation will be observed.

Now, assume the whole number to die. They will be held to die at the end of the year of life in which they died, though actually dying in the middle of the year:—

								The Duration being Years.	Months.
That is, the entrant of	30 yrs.	1 mo.	will die at age	33,				2	11
"	"	31	"	1	"	"	34,	2	11
"	"	32	"	2	"	"	35,	2	10
"	"	33	"	3	"	"	36,	2	9
"	"	34	"	4	"	"	37,	2	8
Total durations up to end of year of death,								14	1

The tabulation being:

Age	Entrants: Number Entering Yrs.	Mos.	Nett Number Yrs.	Mos.	Progressive Summation Yrs.	Mos.	Exitants: Number Exiting Years	Progressive Summation Years	Difference: Years under observation Yrs.	Mos.
30	1	–1		11		11	...	...		11
31	1	1–1	1		1	11	...	...	1	11
32	1	1–2		11	2	10	...	...	2	10
33	1	2–3		11	3	9	1	1	2	9
34	1	3–4		11	4	8	1	2	2	8
35		4		4	5		1	3	2	
36	...		...		5		1	4	1	
37	...		...		5		1	5	...	
Sum of the years from date of entry to the close of the observations . . .					29	1	Sum of the years from the end of the year of death to the close of the observations	15	Difference, being yrs. under observations as above 14	1

The same figures are illustrated in the annexed chart.

CHART SHOWING THE MODE OF RECKONING YEARS OF LIFE.

Age at Entry	Age 30, Months 1 2 3 4 5 6 7 8 9 10 11	Age 31, Months 1 2 3 4 5 6 7 8 9 10 11	Age 32, Months 1 2 3 4 5 6 7 8 9 10 11	Age 33, Months 1 2 3 4 5 6 7 8 9 10 11	Age 34, Months 1 2 3 4 5 6 7 8 9 10 11	Age 35, Months 1 2 3 4 5 6 7 8 9 10 11	Age 36, Months 1 2 3 4 5 6 7 8 9 10 11	Age 37
$30\frac{1}{12}$	* * * * * * * * * * *	* * * * * * * * * * * *	* † † † † † † † † † † †					
$31\frac{1}{12}$		* * * * * * * * * * *	* * * * * * * * * * * *	* † † † † † † † † † † †				
$32\frac{2}{12}$			* * * * * * * * * *	* * * * * * * * * * * *	* * † † † † † † † † † †			
$33\frac{3}{12}$				* * * * * * * * *	* * * * * * * * * * * *	* * * † † † † † † † † †		
$34\frac{4}{12}$					* * * * * * * *	* * * * * * * * * * * *	* * * * † † † † † † † †	
Sum of years exposed while living	$\frac{11}{12}$	$1\frac{11}{12}$	$1\frac{11}{12}$	$1\frac{10}{12}$	$1\frac{10}{12}$	$1\frac{3}{12}$	$\frac{4}{12}$	As in narrative, p. 235
Do. up to end of year of death	$\frac{11}{12}$	$1\frac{11}{12}$	$2\frac{10}{12}$	$2\frac{9}{12}$	$2\frac{8}{12}$	2	1	

The "asterisks" indicate the term exposed to mortality while living, and the "daggers" extend the exposure to the end of the year.

A precisely similar process has been carried out with the 2,205 cards of the lives selected by Dr. Sprague for illustrating his method. I have re-computed the ages at entry and exit; and while I may have made a slip in my haste, I have been as careful as a due regard to other engagements permitted me.

To illustrate my process further I give an extract from the tabulation of the facts recorded on the cards. In the first place there are given the results when dealing with a small or a medium number of cards. The following is an extract from the manuscript of my tabulation of the facts in regard to the discontinued. Looking to the first entry. There were 25 persons entered at age 30–31. These 25 were collectively 96 months older than 30, the number 96 being obtained by mentally adding the months together while the cards are passing through the hand. These 96 months are equivalent to 8 years, so that there were only 17 persons passed through a full year of life. Inasmuch, however, as 129 months were passed on from the entrants at age 29, which are equal to 10 years and $\frac{9}{12}$, the total years of life exposed to mortality in the interval between age 30 and age 31 become 27 integral lives and nine-twelfths of a life. The same explanation applies to other ages.

Age	Entrants: Number of Lives Entering	Entrants: Adjustment of the Months	Entrants: Nett Number of Lives		Entrants: Progressive Summation of Entrants		Exitants: Number of Lives Exiting	Exitants: Adjustment of the Months	Exitants: Nett Number of Lives		Exitants: Progressive Summation of Exitants		Difference: Years of life exposed at each age	
			Yrs.	Mos.	Yrs.	Mos.			Yrs.	Mos.	Yrs.	Mos.	Yrs.	Mos.
	...		...		...				...		...		...	
30–31	25	129—96	27	9	250		18	89—104	16	9	152	4	97	8
31–32	27	96—147	22	9	272	9	17	104—74	19	6	171	10	100	11
32–33	26	147—137	26	10	299	7	25	74—129	20	5	192	3	107	4
33–34	10	137—60	16	5	316		16	129—85	19	8	211	11	104	1
34–35	15	60—97	11	11	327	11	11	85—71	12	2	224	1	103	10
35–36	10	97—39	14	10	342	9	12	71—55	13	4	237	5	105	4
36–37	25	39—128	17	7	360	4	16	55—77	14	2	251	7	108	9
37–38	31	128—127	31	1	391	5	24	77—119	20	6	272	1	119	4
38–39	18	127—86	21	5	412	10	16	119—78	19	5	291	6	121	4
39–40	30	86—144	25	2	438		26	78—145	20	5	311	11	126	1
	...		...		...		...		...		...		...	
					659						657	1	1	11
					659						659		0	
Total No. of Entrants	659	=	659				Total No. of Exitants	659 =	659					
	Sum of the years from date of Entry to close of observations				25,988.5		Sum of the years from date of Exit to close of observations				21,990			
Difference, being the sum of the years of life under observation between date of Entry and date of Exit													3,998	5

And when dealing with large numbers—the same figures being adopted.

Age at Entry	Number assuring in each month of each year of life, and total number of months older than Age at Entry (in dark type)													Totals	
	0	1	2	3	4	5	6	7	8	9	10	11		Number Assuring	Total No. of Months older than integral years of Age at Entry
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
30–31	5	3	2	2	4	4	1	...	...	1	1	2	=	25	...
	...	**3**	**4**	**6**	**16**	**20**	**6**	...	...	**9**	**10**	**22**	...	...	**96**
31–32	4	3	...	2	2	2	1	3	1	6	2	1	=	27	...
	...	**3**	...	**6**	**8**	**10**	**6**	**21**	**8**	**54**	**20**	**11**	...	...	**147**
32–33	6	...	1	3	3	1	1	2	1	2	3	3	=	26	...
	...	...	**2**	**9**	**12**	**5**	**6**	**14**	**8**	**18**	**30**	**33**	...	...	**137**
33–34	1	...	2	1	...	...	1	...	2	...	2	1	=	10	...
	...	...	**4**	**3**	...	...	**6**	...	**16**	...	**20**	**11**	...	...	**60**
34–35	2	...	2	...	2	...	...	1	2	1	2	3	=	15	...
	...	...	**4**	...	**8**	...	...	**7**	**16**	**9**	**20**	**33**	...	...	**97**
35–36	2	...	2	1	...	1	2	1	1	...	...	...	=	10	...
	...	...	**4**	**3**	...	**5**	**12**	**7**	**8**	...	...	...	...	...	**39**
36–37	5	2	1	2	1	2	...	2	5	1	3	1	=	25	...
	...	**2**	**2**	**6**	**4**	**10**	...	**14**	**40**	**9**	**30**	**11**	...	...	**128**
37–38	9	1	3	3	...	3	3	...	5	2	2	...	=	31	...
	...	**1**	**6**	**9**	...	**15**	**18**	...	**40**	**18**	**20**	...	...	...	**127**
38–39	6	...	2	...	1	1	...	2	1	...	4	1	=	18	...
	...	...	**4**	...	**4**	**5**	...	**14**	**8**	...	**40**	**11**	...	...	**86**
39–40	7	...	3	1	3	3	3	1	2	4	2	1	=	30	...
	...	...	**6**	**3**	**12**	**15**	**18**	**7**	**16**	**36**	**20**	**11**	...	...	**144**
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total No. of Entrants	107	37	43	46	57	58	54	49	55	49	56	48	=	659	...
Total No. of Months	0	37	86	138	228	290	324	343	440	441	560	528	=	...	3,415

These figures correspond with those in the previous illustration when dealing with a medium number of facts, but they show the process that would be adopted with larger numbers.

I have now to point out the immaterial differences in the numbers exposed when Dr. Sprague's results are separated into the four classes of matured, discontinued, existing, and died, and the numbers exposed in the same classes as calculated by my method. If there had been any material difference in the processes, a corresponding variation would have been exhibited in the numbers exposed of one or other of these sections.

	Dr. Sprague.	J. M.	Difference.
The total years of life are	30,521	30,579·3	58·3

The years of life for the

	Dr. S.	J. M.	Difference. Dr. S. Greater.	Less.	
Matured being .	580·	581·6	...	1·6	
Discontinued .	4,005·	3,998·5	6·7	...	
Existing .	13,007·	13,091·8	...	84·8	
Died .	12,929·	12,907·8	21·4	...	
Totals .	30,521·	30,579·3	27·11	86·2	Nett Difference, 58·3

Even if the results at the individual ages be examined, the differences are almost infinitesimal. The figures of the numbers exposed waver to the one side and to the other, and furnish the impression that they are identical.

Note.—On revising the print of the foregoing I observe that I have omitted to refer to the case of a person dying in the same year of life as that in which he entered on the risk. Thus, if a life enter at age 30$\frac{10}{12}$, and die at 30$\frac{11}{12}$, and if it be tabulated as a death at the end of that year of age, the life would only be exposed to mortality for $\frac{2}{12}$ of the year 30–31. Inasmuch, however, as we are computing the rate of mortality for entire years, the life would require to be on the risk for twelve months—that is, from 30$\frac{10}{12}$ to 31$\frac{10}{12}$, though the tabulated death be enrolled at age 31.

NUMBERS EXPOSED IN THE FOUR SECTIONS—COMPARED.

Age	Matured			Discontinued			Existing			Died			Total			Comparison of the Deaths		Comparison of the Differences				Nett Difference S. Greatest
	S.	M.		S.	M.		S.	M.		S.	M.		S.	M.		S.	M.	S. Greatest		M. Greatest		
	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	No.	No.					
2–3	...	...		...	...		...	...		...	...		...	...		...	...	...		...		...
3–4	...	...		1	1		...	...		...	...		1	1		...	...	...		...		...
4–5	...	...		1	1		...	...		1		7	2	1	7	...	...	...		...		...
5–6	...	...		1	1		...	...		1	1	8	2	2	8	...	..	...		...		...
6–7	...	...		1		10	2	1	11	3	3	4	6	6	1	...	...	...		...		...
7–8	...	...		2	1	8	5	4	8	4	4	4	11	10	8	...	...	...		...		...
8–9	...	...		3	3		9	8	4	5	5	4	17	16	8	...	...	...		...		...
9–10	...	...		7	6	4	13	12	8	6	6		26	25		...	...	...		...		...
10–11	...	..		10	9		17	17	2	8	8		35	34	2	...	...	...		...		...
11–12	2	1	10	15	13	11	18	18	6	10	9	10	45	44	1	...	...	...		...		...
12–13	2	2		16	15	10	25	24		10	10		53	51	10	...	...	...		...		...
13–14	2	2		19	19	9	33	32	9	13	12	4	67	66	10	...	...	...		...		...
14–15	1	1	9	24	22	9	44	44		15	15		84	83	6	...	...	...		...		...
15–16	3	2	10	25	25		54	53	6	18	18	5	100	99	9	...	...	...		...		...
16–17	3	3		26	27	8	58	58	7	19	19		106	108	3	...	...		Sum.		Sum.	...
17–18	3	3		34	33	5	63	62	2	20	20		120	118	7	1	...	1	1	...		1
18–19	4	3	11	38	37	10	69	67	3	21	21	4	132	130	4	...	1		1	1	1	...
19–20	6	6	3	43	43	8	75	74	6	24	24		148	148	5	1	...	1	2		1	1
20–21	8	8	3	52	51	2	83	82	7	25	24	10	168	166	10	1	1		2		1	1
21–22	6	5	7	52	51	4	93	95	1	27	28	3	178	180	3	1	...	1	3		1	2
22–23	4	3	6	57	56	8	107	109		32	31	7	200	200	9	...	2		3	2	3	...
23–24	5	4	1	68	66	6	115	115	10	40	39	6	228	225	11	1	...	1	4		3	1
24–25	6	7		71	72	5	128	130	1	45	43	2	250	252	8	3	3		4		3	1
25–26	5	5		81	80	7	142	142	7	49	49	3	277	277	5	1	1		4		3	1
26–27	5	5	2	89	87	8	154	152	8	61	59	9	309	305	3	3	1	2	6		3	3
27–28	5	5	3	84	86	9	161	163	4	68	69	1	318	324	5	1	2		6	1	4	2
28–29	3	4	1	89	87	8	173	176	8	79	77	9	344	346	2	4	2	2	8		4	4
29–30	4	4	7	88	86	8	190	189	10	90	90	9	372	371	10	1	3		8	2	6	2
Cd. fwd.																						

Numbers exposed in the Four Sections—compared—(*continued*).

Age	Matured			Discontinued			Existing			Died			Total			Comparison of the						Nett Difference S. Greatest
	S.	M.		S.	M.		S.	M.		S.	M.		S.	M.		Deaths		Differences				
																S.	M.	S. Greatest		M. Greatest		
	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	Yrs.	Yrs.	Mos.	No.	No.					
Bt. fwd.																						
30–31	8	7	7	98	97	8	199	197	1	104	105	3	409	407	7	4	2	2	10		6	4
31–32	10	10	4	98	100	11	212	212	7	119	119		439	442	10	6	4	2	12		6	6
32–33	15	13	11	107	107	4	235	238	11	128	126		485	486	2	9	7	2	14		6	8
33–34	13	14		103	104	1	264	265	8	141	139	1	521	522	15	5	9		14	4	10	4
34–35	17	16	10	102	103	10	271	272	8	156	156	6	546	549	10	5	4	1	15		10	5
35–36	15	15	5	104	105	4	279	281	10	169	169	11	567	572	6	8	5	3	18		10	8
36–37	20	18	1	106	108	9	300	297	7	179	178	11	605	603	4	8	8		18		10	8
37–38	18	18	5	122	119	4	306	306	8	186	187	6	632	631	11	8	7	1	19		10	9
38–39	16	16	1	120	121	4	320	321	11	193	190	11	649	650	3	3	10		19	7	17	2
39–40	14	15	1	127	126	1	334	331	5	203	204		678	676	7	8	4	4	23		17	6
40–41	16	15	3	127	125	9	331	335	1	214	214	8	688	690	9	5	7		23	2	19	4
41–42	15	15	3	116	115	2	339	343	2	227	225	5	697	699		5	6		23	1	20	3
42–43	16	16	6	120	119	4	330	332	1	240	237	8	706	705	7	11	5	6	29		20	9
43–44	18	17		123	123	1	332	332	5	243	242	4	716	714	10	9	11		29	2	22	7
44–45	17	17		120	122	8	325	328	4	257	259	5	719	727	5	7	7		29		22	7
45–46	17	17	11	123	120	1	316	322	9	276	271	11	732	732	8	12	11	1	30		22	8
46–47	16	16	9	110	110	3	320	320	2	280	281	10	726	729		11	9	2	32		22	10
47–48	16	16	3	99	101	2	312	313	10	291	291	8	718	722	11	9	10		32	1	23	9
48–49	16	16	4	99	97	11	309	309	11	296	294	11	720	719	1	10	12		32	2	25	8
49–50	20	18	10	87	87	3	310	311	6	305	304	6	722	722	1	13	8	5	37		25	12
50–51	19	18	8	86	83	8	309	312	5	310	310	2	724	724	11	7	13		37	6	31	6
51–52	19	17	11	78	76	9	306	309	6	320	320	5	723	724	7	10	6	4	41		31	10
52–53	16	15	5	69	70	4	302	300	11	335	333	4	722	720		15	12	3	44		31	13
53–54	14	13	10	67	67	5	289	293		339	340	5	709	714	8	9	12		44	3	34	10
54–55	14	14		68	66	1	279	283		343	344	6	704	707	7	17	11	6	50		34	16
55–56	13	12	8	59	59	8	275	277	6	346	347	1	693	696	11	13	17		50	4	38	12
56–57	11	11		58	55	8	269	270	9	355	355	1	693	692	6	14	12	2	52		38	14
57–58	11	11	5	50	49	7	262	261	2	354	355	4	677	677	6	18	16	2	54		38	16
58–59	10	11	8	41	41	9	249	252	2	352	350	1	652	655	8	13	20		54	7	45	9

59–60	8	8	6	34	35	1	247	247	11	355	354	7	644	646	1	11	11		...		45	4
60–61	6	6	2	36	34	6	239	239	7	353	352	3	634	632	6	15	10	5	59		45	19
61–62	6	6		28	27	8	235	234	6	340	343	11	609	612	1	14	11	3	62		45	17
62–63	6	5	9	22	22	4	219	220	5	327	327		574	575	6	12	18		62	6	51	11
63–64	5	4	6	19	19	5	203	204	3	316	315		543	543	2	14	13	1	63		51	12
64–65	3	3		18	17	10	194	193	7	304	301	1	519	515	6	18	16	2	65		51	14
65–66	3	3		15	14	9	172	171	10	287	286		477	475	7	23	16	7	72		51	21
66–67	3	3		11	11	3	158	160	4	264	264	3	436	438	10	14	22		72	8	59	23
67–68	3	3		10	9	11	148	148	4	251	251		412	412	3	11	14		72	3	62	10
68–69	3	3		9	8	11	131	131	11	240	241	2	383	385		19	10	9	81		62	19
69–70	3	3		8	7	9	118	117	2	224	223	8	353	351	7	10	20		81	10	72	9
70–71	3	3		5	4	6	103	105	1	215	215		326	327	7	15	10	5	86		72	14
71–72	3	3		4	3	3	93	93	10	200	201		300	301	1	11	14		86	3	75	11
72–73	3	2	10	2	1	11	77	78	2	190	188	10	272	271	9	22	13	9	95		75	20
73–74	1	1	3	...	...		66	65	1	169	165		236	231	4	22	24		95	2	77	18
74–75	1	1		...	...		56	57	3	147	146		204	204	3	17	19		95	2	79	16
75–76	1	1		...	...		50	50	5	130	131		181	182	5	19	15	4	99		79	20
76–77	1	1		...	...		39	41	7	111	111		151	153	7	16	20		99	4	83	16
77–78	1	1		...	...		33	36	6	95	93		129	130	6	12	18		99	6	89	10
78–79	...	...		...	...		30	30	5	83	83		113	113	5	9	10		99	1	90	9
79–80	...	...		...	...		23	25	5	74	72		97	97	5	10	11		99	1	91	8
80–81	...	...		...	...		19	19	8	64	61		83	80	8	10	11		99	1	92	7
81–82	...	...		...	...		10	12	1	54	55		64	67	1	9	6	3	102		92	10
82–83	...	...		...	...		9	10		45	43		54	53		13	12	1	103		92	11
83–84	...	...		...	...		8	9	1	32	29		40	38	1	8	14		103	6	98	10
84–85	...	...		...	...		4	5	2	24	23		28	28	2	5	6		103	1	99	3
85–86	...	...		...	...		3	3		19	20		22	23		2	3		103	1	100	3
86–87	...	...		...	...		3	3		17	16		20	19		4	4		103		100	4
87–88	...	...		...	...		2	2	4	13	14		15	16	4	3	2	1	104		100	4
88–89	...	...		...	...		...		1	10	10		10	10	1	4	4		104		100	4
89–90	...	...		...	...		...	...		6	7		6	7		2	3		104	1	101	3
90–91	...	...		...	...		...	...		4	4		4	4		1	3		104	2	103	1
91–92	...	...		...	...		...	...		3	3		3	3		1	1		104		103	1
92–93	...	...		...	...		...	...		2	2		2	2		...	1		104	1	104	0
93–94	...	...		...	...		...	...		2	2		2	2		...	...		...		...	...
94–95	...	...		...	...		...	...		2	1		2	1		1	1		...		...	...
95–96	...	...		...	...		...	...		1	1		1	1		...	...		...		...	...
96–97	...	...		...	...		...	...		1	...		1	...		1	1		...		...	...
Total	580	581	6	4,005	3,998	5	13,007	13,091	8	12,929	12,907	8	30,521	30,579	3	668	668		104		104	...

It has been stated that my deaths have been assumed to be on the risk up to the end of the year in which they die, and assurance claims would thus be payable six months after death, though their deaths actually took place in the course of the 12 months during the year of death. I have formed the impression, though I have not noticed that the point is clearly brought out in his paper, that Dr. Sprague has assumed his deaths to die at their nearest integral age,* which is an approximation to the actual average age. My deaths are thus six months older than his. If he has tabulated his deaths as at the beginning of the year of death, my deaths will be 12 months older than his. This difference is fully borne out and illustrated on the last column of the comparison of the deaths, where the progressive sum of the differences in the number of deaths—though the totals are equal—shows Dr. Sprague's figures always to be greater.

It may accordingly be conceded that Dr. Sprague's process of dealing with nearest ages and nearest durations is a fair approximation to the correct mode of dealing with exact ages at entry and exact ages at exit.

But I claim for my method superiority in respect—

1st. That I do not require to approximate to anything. I handle the facts as they present themselves. I make use of all the facts, and I use these facts alone. I discard nothing. Dr. Sprague deduces my results to obtain his nearest duration, and he combines that nearest duration with his nearest age. He thus uses two inner datums for his calculations which I do without. He forsakes correctness and exactness, and adopts approximations. If in some cases exactness were unattainable some such approximation would be appropriate.

2nd. In the case of the "existing", I do not require to calculate the integral years of the age in years and months at exit. The month only is necessary, so that the card is preserved for future use until the life either discontinue or die, and the integral years of the age at exit be then unalterably fixed. When the tabulation proceeds upon each age at entry the calendar year of entry furnishes the integral year of the age at exit.

* I have since ascertained that this is not the case; but, as stated on page 210 (under the heading *Duration*), the year of death has been counted as a complete year.

3rd. I claim for my method the very great benefit of being able to state that the ungraduated rate of mortality deduced from the facts, or its graduated equivalent, is the rate which has been found to prevail in each year of life between any two ages. It would be difficult to give a corresponding definition to Dr. Sprague's calculated rate of mortality. It is apparently the rate that has been found to prevail among persons who may be six months younger or six months older than the age in the tables. Even this seems too accurate a definition—because the deaths may belong to a set of lives older or younger than the lives exposed.

4th. I claim for my method the advantages flowing from separate tabulations of the several sections of the lives. We thereby obtain a knowledge of the degree of weight to be attached to the observations. We may omit the discontinued entirely, or we may omit any section of them, or we may substitute a calculated number of deaths according to some other standard, or continue them as healthy lives and observe the effect on the general rate. The effect on the general rate of mortality by lives voluntarily leaving the observations may thus be tested and traced. No doubt the separate calculation adds to the bulk of the labour, but at the same time it lightens the labour by performing it in parts and distributing it.

5th. I claim for my method the advantages of the checks in the correctness of the working as shown in the paper.

6th. I claim for my method the advantages of showing the numbers who insure in the same month as their birthday, the number who insure one month previous, two months previous, &c. This information adds to the interest of the work.

7th. I claim for my method the advantage that it is complete.* It may be abridged by adopting Dr. Sprague's approximation, or by adopting any other approximation, such as combining the exact duration with the office age at entry, which is equivalent to assuming that the date of entry is the anniversary of the date of birth.

8th. No doubt my method entails greater detail of calculation, but I draw attention to the nature of the adjustment upon the integral numbers entering or exiting at each age when effect is given to the total number of months that the lives are older than these integral years. What seemed rough and irregular

* I have omitted to state that the minimum duration of the "Died" must be one complete year. (See Note, page 240.)

becomes slightly smoothed. A wave of correction seems to pass along the column. This adjustment by exact months is greatly preferable to the adjustment of assuming every life to be six months younger than the office age at entry, as was adopted in 1837 and 1863. If the object to be attained had been to find the rate of mortality *at age x* precisely, then Dr. Sprague's method, provided his deaths were prepared on the same basis as his years of life, would be more suitable than mine. My method, however, seems to me to be more fitted for obtaining the level rate of mortality during the twelve months of each year of life, which is the practical assumption of all our office calculations.

I humbly maintain that the process adopted by me, and which has here rather egotistically perhaps been termed "my process" is rigorously accurate. Other methods may be sufficiently accurate or approximately accurate, but in a great scientific labour affecting the condition of so great an interest as the Life Assurance Companies all over the world, I would respectfully ask those who have the guidance of this work to adopt the method which can be upheld against all critics, irrespective of any considerations whatever.

Mortality during Years of Assurance.

The mode of tabulation thus formulated is the one I would have adopted if the mortality experience of the Scottish Assurance Offices had been placed in my hands. There are, however, subsidiary processes of practically handling the subject which I have not touched upon. The foregoing refers to its compilation as a *whole,* and to the development of *one* aggregate table of mortality. With so large a number of facts it will, however, be found expedient to divide the work into sections, for the purpose of facilitating the compilation, and of meeting those minor agreements and reconciliations of the numbers entering and exiting the observations that are essential to correctness—thus, after all the duplicate experiences on the same life have been withdrawn, and each life has been ascertained to render a single and undivided experience only once, it will be found desirable to deal with the facts of the entrants of each age separately, and thus *apparently* to frame a mortality experience of assurants emanating from each age. Such subsidiary tabulation, however, is merely an expedient for lessening the labour of constructing a table of the aggregate experience where each life enters the lists only once. When we really come to extract

the mortality experience of assurants at each age, the duplicate experiences of the same life require to be returned to the lists, and only those of them withheld which refer to the same life entering more than once *at the same age*. It is with these re-adjusted statistics that we really and truly proceed to prepare the mortality experiences of assurants at each age, and not with the partial tables before referred to and solely prepared for facilitating the work of the aggregate experience. The two results are quite different. The total addition of the one series of partial tables forms the one aggregate table. The others are independent and cannot be added together, but these form the most trustworthy basis for calculating those various issues depending upon and setting forth the extent and value of *selection*. I have not noticed that Dr. Sprague has referred to two sets of tables. He seems to rely upon one set—the set that properly forms the basis of the aggregate experience—as the basis of both the selected experiences for each age, and the aggregate experience.

I will now briefly refer to the mode I would pursue if I were required to tabulate the facts referring to entrants of each age for the purpose of tracing the effect of selection. I would in the first place see that the statistics had been prepared in the form that had been stated to be specially applicable to that object. So far as I can at present forecast that mode, it appears to me that there are two processes open to us, the one referring to years of life, and the other to years of assurance.

The one essential element in both methods is the actual *duration of the risk*. If that duration—obtained, in the case of the discontinued and the surviving, by taking the difference between the date of entry and the date of exit, which exit may be the date of the close of the observations; and in the case of the deceased, by taking the difference between date of entry and the next ensuing anniversary of the date of entry after death—be added to the age at entry, we shall have the age at exit; and if this age at entry be the usual office age next birthday, and the tabulation proceed upon these data, in the manner already indicated, the result would furnish the number exposed to mortality in each year of assurance of persons who when they entered were then of their usual office ages next birthday. The rates of mortality thence resulting, contrasted with the rates deduced from the aggregate statistics, dealt with in the earliest part of this paper, and exhibited in the *one* aggregate table, would show the effect of selection. The one rate of mortality,

however, would refer to years of life, reckoned by the birthday, and the other to years of assurance of persons who were, on an average, four or five months younger in their years of life,—a difference which may be overlooked.

If we attempt to deduce the rate of mortality during each year of assurance when the ages are reckoned by the birthday, we are met by the initial difficulty that the first portion of the first year of the assurance is for the number of months awanting of the year of life entered on at date of assurance, the duration being so many months and so many years. In this case, the broken period is at the *beginning* of the risk, and constitutes what has received the name of the "year 0", and the results are thus not so practically applicable to show the effects of selection. Probably the best way, under the second method, of dealing with this broken period would be to tabulate it separately and to show the number of years of life exposed in the four or five months up to the completion of the initial age in the first year of assurance, and the number of deaths arising in that period, and to set aside the resulting rate of mortality for special reference. The residue of the durations of the rates would mostly be for *complete* years. I would however give my preference to the first mode.

The experience of "years of life" and of "years of assurance" should certainly be separately tabulated, the cards being adjusted to leave room for both sets of ages; but I am clearly of opinion that the sectional experience of "policy years"—whether derived by the "Nearest Duration Method", or by the method of actual duration of the risk in combination with the office age next birthday at entry—cannot be added together to form the aggregate experience of lives passing through the same year of age, and that the sectional parts of the "years of life" investigation cannot be viewed as representing the experience of "years of assurance." The two are independent compilations.

Dr. Sprague proposes to close his observations at that anniversary of the date of entry which arises in the year 1893, so that in the case of the lives surviving the observations, the durations would be for complete years, which would also form his *integral* years. I would close my observations at 31 December 1893, being one common epoch for all offices, and exactly 30 years from the previous examination. In this case the durations of the surviving would be mostly for broken periods, but I have pointed out that broken periods do not introduce any obstacle to a correct handling of the data. A Table of the durations from

all dates of Entry to the common epoch of 31 December 1893, would facilitate and check the calculations made on the cards.

In conclusion, I would suggest that while it is most important to ascertain the general rate of mortality prevailing in each year of life, it is becoming more and more important to ascertain some knowledge of those inner "wheels of nature's mazy plan" which, in their aggregate, form the total rate of mortality referred to. The effect of medical selection safeguarding the offices is the most obvious of these. But it is sometimes thought that there are, on the part of the public, attempts to outwit the offices by adopting some one or other of the many systems of assurance now competingly offered to intending assurers, whereby a bias is thought to exist in favour of the assured and against the office. Endowment assurances payable on attaining a certain specified age, and ordinary assurances payable at death but contributed by premiums limited to a period of years, will no doubt be preferred by good lives, though not always so, because a not altogether unexceptionable life may be accepted on these scales that would not be accepted on the ordinary scales. Temporay or reduced-premium assurances may be adopted by lives who have not a similar confidence in their own prospects of longevity. Assurances without medical selection is another phase of a similar kind. We would like to know the mortality of persons who have assured their lives after having suffered some severe illness, even though at date of assurance they may have passed as good lives; of persons who have resided in tropical climates; and of persons who have lost one or other or both parents from consumption or from some hereditary ailment. Is there a material difference between the mortalities of the lives in the case of survivorship assurances or of survivorship annuities? Is there in these cases any selection *against* the office? Is the life on whose death a benefit is payable worse or better than the life that receives the benefit? How far does *occupation* affect longevity? The mass of information on these points in the possession of the offices is frequently referred to in the medical journals. At present we can only surmise. Do we not wish our surmises removed or confirmed? Probably some of these questions can best be approached by each office investigating its own experience,—the separate and independent results thereby receiving corroborative support,—and that only the more extensive and more important questions be taken up in a collective form.

Discussion on the Papers of Dr. Sprague and Mr. Meikle.

The President (Mr. A. Hendriks) said that before calling on Members to speak on the papers, he would put one question to Dr. Sprague. In his paper he said, "The total years of life at all ages according to my method were 30,538, against 30,521 in another, and $30{,}531\frac{3}{12}$ in a third", and then, in proceeding to compare it with the Institute method, it was found that the number was 31,141·5, and Dr. Sprague's surprise, which would be shared by most of them, was allayed when he found it was owing to an accidental cause consisting in a clerical mistake. He should like to ask whether that clerical mistake was rectified by a further calculation, or, if not, whether Dr. Sprague had arrived by any shorter process at the actual number? Although the Institute method would be improved upon, the question was in what way it would be done, and, with great veneration for that method, he would be glad to see that it did not differ very materially on any point from any of the methods proposed to them.

Mr. George King said Mr. Meikle had had before him more particularly the idea of a table based on the aggregate experience of all ages at entry, taking the rate of mortality for each year of age derived from all policies, no matter how long or short a period they might have been in existence; whereas Dr. Sprague had had before him more particularly the idea of select tables. Bearing that in mind, they had an explanation of the difference between the two papers. They now practically had promised for this investigation a vast body of mortality experience. About 60 offices would supply their experience; indeed, there had been practically a unanimous response to the invitation of the Institute and the Faculty in this matter. Bearing in mind, then, the large experience they had to deal with, he thought they could separate it into various classes, and could investigate separately the ordinary whole-life policies, the endowment assurance policies, and other minor groups. Dr. Sprague spoke of voluntary withdrawals and compulsory withdrawals, but if they were investigating whole-life cases, there was no such thing as a compulsory withdrawal, they were all voluntary; and if they were investigating the endowment assurances, there was very little compulsory withdrawal, and what there was took place at the later ages and was not of much consequence. He therefore thought it unnecessary to distinguish between voluntary and compulsory withdrawals. There would be minor classes, no doubt, where compulsory withdrawals would come in in some degree, but they would be of no great importance. The main difference between the papers lay in the difference between the aggregate mortality table and the select, and, with all deference to Mr. Meikle, he would throw in his vote for the select table. As Dr. Sprague had rightly pointed out, the difference in the rate of mortality due to a year more or less, especially at the beginning, in the duration of the policy, was much greater than the difference due to a year more or less in the age. That really seemed to him what

they wanted to investigate. Further, aggregate mortality tables, unless perhaps for valuation purposes, were not tables that they could rely on. The rate of mortality varied with the amount of new business and the description of new business; it varied between one company and another through causes that had nothing to do with human mortality. He was not now speaking of a question which had caused a good deal of discussion and difference of opinion—namely, the effect of withdrawals upon mortality—but he more particularly referred to the question of the admission of fresh lives, which had an immense effect on the rate of mortality in the aggregate table. Where the actual rates of mortality at the various ages remained unchanged, they would very much alter the rates appearing in the aggregate table by simply altering the proportionate numbers of new lives admitted at different periods of life. That alone showed that for calculating premiums and for other purposes the aggregate table was unsuitable. By the Institute Table, which was an aggregate table, the rates of premium for whole-life policies, and still more for term policies, were too low for the young ages and too high for the old ages, simply on account of the incidence of new business, and, therefore, for the purposes of premium calculation, the aggregate table should be discarded. He further maintained that for valuation purposes it was of use only in so far as it gave them a good approximation to the reserves that would be made by the select tables. It was only because he found by practical examination that the select tables prepared by Dr. Sprague did very closely approximate to the result of the H^M and $H^{M(5)}$ Tables that he used the latter combination. If they had differed very much, he should unhesitatingly have thrown aside the combined tables and adopted the select. Mr. Meikle had maintained that whilst for select tables they should bring in duplicate policies, if they were not simultaneous, yet for the aggregate table they should take care that only one policy on each life was brought into account. He would respectfully say that Mr. Meikle was mistaken there. He would throw out only simultaneous policies and not duplicate policies where they had not run from the beginning concurrently. Looking at the various methods suggested by the two authors for forming the mortality table, Mr. Meikle showed that his method, which was a rigorously exact one of forming aggregate tables, differed only infinitesimally from the various approximations put forward by Dr. Sprague. As to the methods which Dr. Sprague had suggested, as they differed so little from each other, he thought they should take that which was the easiest, and his view would therefore be to adopt what Dr. Sprague called the "Nearest Duration." On this matter he did not think he could do better than refer to the two tables that appeared on pages 212 and 214 of Dr. Sprague's paper. In the second table, which dealt with the exact duration method, there were 11 main columns, whereas, in the first, which dealt with the nearest duration method, there were only seven columns, and, moreover, in the table with eleven columns, in a good many instances the columns were double, so that in fact there were fourteen altogether—that was to say, that the amount of space occupied by the exact duration method table was double that of the nearest duration method. He

need not point out what extra cost and labour in printing and compiling that would entail; and not only so, but the more elaborate they were the greater the risk of error. On that ground alone the nearest duration method was the one to adopt. It had been urged that it would not enable them to deal accurately with the question of discontinuance, and that if they had the exact duration method they would be able to get at the rate of discontinuance with certainty. He did not agree with that view. He attached great importance to an investigation of the rate of discontinuance, but they could get, without very great labour, all they wanted, and as much as the facts were capable of yielding. The rate of discontinuance differed in different companies at different times. It differed according to the nature of the policy, even according to the method of the division of profits. It was not like the law of mortality, which depended much more on immutable conditions. It varied so much under varying circumstances that he did not think they could get anything very exact regarding it. It seemed to him that approximation was quite sufficient for their purpose, and that it would be a mistake to go to an enormous amount of trouble in order to obtain what they could, for all practical purposes, reach with very little trouble. With regard to the rate of mortality, the two methods would have produced practically the same result. In view of the saving of time and trouble and risk of error in dealing with a great volume of facts, he would therefore urge that the nearest duration method was the one which should be adopted.

Mr. Ralph P. Hardy said that, taking Dr. Sprague's contribution first, the conditions of the problem were stated there with outlined accuracy and with a breadth of view that left little to be desired. Some persons approached this question as one of pure mathematical deduction, but they made the great and unwarrantable assumption that the facts before them were all correctly stated and were in *pari materia*. Dr. Sprague was not engaged in searching after that phantom, the physical law of mortality. He well knew that that law was the resultant of innumerable and subtle and scarcely measurable forces that varied both in their intensity and direction in every age—changes in the earth—and that that secret was the last that nature would disclose to man's obstinate questionings. Dr. Sprague declared that "our principal object in making a mortality investigation, is to ascertain for our future guidance what has been the mortality among the lives which we classed as 'average' and accepted at the tabular rate of premium." No gentleman with the experience of Dr. Sprague could have used any other terms; he sought no phantoms whatever. He then reviewed the three methods, and finally he formulated his objections to the Institute method. But he (Mr. Hardy) failed to see that he showed any real difference, according to the conditions that he had laid down between the consequent ratios, adjusted or unadjusted, so far as they would affect either a table of net premiums or the results of a valuation. He did not think that it had been suggested that the so-called years of assurance should not be shown. That was certainly done at the last investigation and should be repeated. Let them now look impartially at the groupings of this proposed 30 years

span. The exposed to risk constituted a happy family. Taking the group of lives between, say 50 and 55, examining them, they would find aproned ecclesiastics bedding with Whitechapel butchers; the old orthodox squire sitting, perhaps to his discomfort, with the very flower of the Jockey Club; even judges, consorting for the first time with the criminals they had tried. Then there would be a sprinkling of the nobility—some pecunious but chiefly impecunious. This was the seething, discordant, inharmonious, heterogeneous mass of discrete atoms, without common life, that the purists take as lying in *pari materia.* This collection of facts came together under every conceivable variety of medical test. Was it not an abuse of the mathematical method to pretend to apply delicate corrections to the determination of such a composite denominator; and of what possible value were the deduced rates of risk of discontinuance for the first two or three years of assurance? Then as to the period when the effect of selection was supposed to be exhausted. No doubt, there came a time when such effect was no longer discernible, but whether that should be taken at five years, or at some point circling round that, was in his humble judgment of very little real importance. He did not imply that the results they brought out were not of some value. He thought they were—that they were valuable acquisitions to their knowledge, and that in proper hands they might render great service to society. Any attempt at what he called affectation of precision would add considerably to the time and cost of the proposed investigation, and they should consider whether that and the probable loss of some data would not be a heavy price to pay for an article which, after all, for the practical purpose Dr. Sprague had laid down, was no better than the old one. In speaking of Dr. Sprague's paper, he should like, after considering the matter for over 20 years, that night to venture upon a criticism. Some years ago, Dr. Sprague wrote a paper in the *Journal* in which he showed the ratio between the actual and expected deaths. He (Mr. Hardy) found he had repeated that contrast that evening, and, he said with all respect, that it was an entire fallacy. The ratio which Dr. Sprague had brought out was not truly that of the expected deaths by the H^{M} Table, but it was that of the number of deaths that were expected to happen, taking the number living brought forward according to the other table. The ratio shown by Dr. Sprague in his judgment was worthless.

Mr. Archibald Hewat said the Scottish actuaries were quite as much interested in the approaching mortality investigation as the English actuaries could possibly be. He was himself specially interested in it, for some months ago he commenced a somewhat unusual and what promised to be an interesting investigation. That was an investigation into the experience of the Widows' Funds of the five oldest and largest of the banks in Scotland, which had long had such funds. He had suggested to the managers of those banks that the time had come when the accumulated statistical information was sufficient to warrant an investigation into the mortality, marriage, and other experience of the employés of these banks and of their widows, with a view to framing tables on more reliable bases than had hitherto

been available, for valuation and other purposes in connection with such funds. He was engaged in that labour just now, at an elementary stage. When he got to the close of his labours, he would be disposed, if he thought the Council would accept it, to offer to contribute a paper on the subject, to be read before the Institute.

Mr. G. F. HARDY said that there was an unanimous agreement that for the first few years of assurance the two factors, age and duration of assurance, both affected the rate of mortality; in fact, the rate of mortality was a function of these two variables, which must both be taken into account. Mr. Meikle's came before them as an ideal method, rigorously exact, for obtaining an aggregate table. It involved a great deal of extra labour, but the idea was that by means of this additional labour, they got a table which (leaving out the effect of selection) was rigorously accurate. He (Mr. Hardy) would, however, ask if that was really the case, because, if they got no nearer exactitude than before, it was so much labour thrown away. The object being to determine the rate of mortality at each age, this could only be exactly determined from the ratio of deaths to exposures if the latter were distributed through the year of age, according to the same law as the "living" at the same age in the life table. At all insuring ages, however, they had large numbers entering towards the close of the year of age, and therefore the distribution of lives exposed to risk in each year at the insuring ages would be different from that of the lives in the mortality table, the true average age of those lives being greater than the assumed age. Again, when they reached the older ages, they would have a large number of lives withdrawn from observation during each year of age as a result of the element "existing", and the average age of the exposed to risk at the older ages of the table would be less than the assumed age. The effect of these two forces would be slightly to over-estimate the rates of mortality at the younger ages and to somewhat under-estimate them at the older, so that, notwithstanding the additional labour they were asked to take in estimating ages by months so as to obtain exact ages at entry and exit, they would not get a rigorously exact result. If they were to adopt a method similar to that proposed by Dr. Sprague, which he called the "Nearest Duration Method", it would be well first to test the method by a rather more representative body of facts than they had yet had before them. In the present instance the lives were assured, he believed, under a scale of premiums proceeding by half-ages, and this would materially affect the distribution of the lives through the age at entry. Then they had the fact that these were female lives, and they did not know whether exactly the same law would regulate the ages at entry as in the case of male lives. What they required to know was, given a general representative body of facts, including ordinary whole-life and endowment assurances, how near did they get to the true average ages at entry and exit by adopting the nearest entry age and the nearest duration method. As far as one could see, there would be a fair balance of error, but that was a question which could only be decided by an appeal to the actual facts. As regards the correction introduced in a note by Mr. Meikle, by which the lives

dying in the age at entry were assumed to be at risk for 12 months after entry, this seemed to involve an error. They must settle first of all whether they were going to deal with policy years or years of age, and whichever they selected with, they must take each death as a life exposed to risk up to the close of the policy year or the year of age, as the case might be, but not shift from one to the other. In other words, lives dying between ages x and $x+1$ could not possibly be brought into account in determining the rate of mortality between $x+1$ and $x+2$, as was done if the proposed corrrection was employed.

Mr. W. J. H. Whittall agreed with Mr. G. F. Hardy as to the danger of small inaccuracies in the construction of a table which proceeded by years of age, as Mr. Meikle's did, through the necessity for introducing fractional periods in respect to the years of entry and exit. He (Mr. Whittall) had pointed out that this was inseparable from any system of table following life years. It was necessary to bear in mind the difference between the principles upon which Mr. Meikle and Dr. Sprague had proceeded in order to understand thoroughly the remarkably different conclusions at which they had arrived. Mr. Meikle said of his own plan, "I make use of all the facts, I discard nothing." But of Dr. Sprague's method he said, "He forsakes correctness and exactitude and adopts approximations." That was perfectly correct if they approached Mr. Meikle's point of view, which was a point of view of exact ages or exact years of life; but if they approached the problem from Dr. Sprague's point of view exactly the reverse would hold good, and Dr. Sprague might say the same thing about Mr. Meikle's method. Another point in connection with Mr. Meikle's paper was his device of taking a minimum duration of one year for such of the deaths as occurred in the first year. He (Mr. Whittall) confessed that on briefly considering the matter he could see no reason for adopting that plan. Regarding Dr. Sprague's paper he ventured to enter a protest on his use of the words "Nearest Duration Method" and "Exact Duration Method." He was aware that the subject of his table was "A method of tabulating the facts for the purpose of ascertaining the experience of Companies." He feared, however, that in the result his method would be spoken of as "Dr. Sprague's nearest duration method of ascertaining the rate of mortality." He would therefore point out that the great distinction they had to bear in mind was that his method followed policy years. If they asked themselves what it was that gave the name of Dr. Sprague's nearest duration method, they found that it was only to the discontinuances that it really did apply. It seemed to him a little unfortunate to call a method by a name which was indicative of the mode by which only one of several elements was treated. Coming to the real characteristics of Dr. Sprague's method, he was pleased to see that Dr. Sprague had independently arrived at a formula identical with that which he had advocated. As Mr. King had said, Dr. Sprague had not that evening indulged in formulas. He (Mr. Whittall) for one reason wished he had. It would be an advantage if the nearest duration method of tabulating facts provided not only for registering them under the duration of the policies, but also under the age at exit, and that if

Dr. Sprague had proceeded to illustrate his method by a formula he would have found the necessity for using the age at exit. In other ways it would be useful if the age at exit could be used in preference to the duration of the policy. The first thing they had to decide was whether or not policy years should be adopted. In that, he (Mr. Whittall) thoroughly concurred, but at the last meeting and since, strong views had been expressed, notably by Mr. Ryan, as to the advisability of investigating the rate of discontinuance. He agreed that there would be a great deal to be learned from the rate of discontinuance, but he would not advocate it if there was going to be an enormous increase in the work or printing to be done. He believed, however, the rate could be observed more accurately than by the nearest duration method, and without any equivalent increase of work or expense. The discontinuants could be sampled by a haphazard process, and thus could be ascertained the mean period of the policy year at which they occurred. They would then merely need to be tabulated just like the deaths—namely, in the exact policy years in which they occurred. The ascertained average correction could be applied in respect of the discontinuants when calculating the exposed to risk of mortality; and when calculating the exposed to risk of discontinuance, a correction could be applied in respect of the deaths, based upon the assumption of equal distribution throughout the year. He thought this was one point which ought to be finally decided upon.

Mr. T. J. Searle asked Dr. Sprague whether it was not possible for him to give the results of a comparison with the H^M Table, as they would doubtless be very useful. Dr. Sprague had said that the reason why he could not give these results was that it was necessary to close his year on 31 December, but he (Mr. Searle) could not bring himself to think that that was necessary. It might, to his mind, just as well end on 1 March.

Mr. H. C. Thiselton said they had at least three methods before them, as there were two new methods stated, and it was well known that there were influential parties in the Institute who were in favour of having the experience treated on the lines of the old experience. It seemed to be thought that one of these systems must of necessity be chosen, but if instead of three methods there were six, if these matters really were proper subjects of investigation, and if no definite decision could be arrived at, why not try two or three of those systems. There were plenty of members of the Institute who would be willing to work upon the investigation. The nearness of the results brought out by Dr. Sprague and Mr. Meikle was no guide to the nearness of the results to be obtained by treating the larger experience by two or three different methods.

The President (Mr. A. Hendriks), in moving the usual vote of thanks, said the papers had given rise to a very animated and excellent debate. It brought them another step in advance in the matter which they had before them and in which they all took so much interest. The subject would be approached on all sides, by those who held different views, from the point of view of each, and every one of them expressing sincere desire that ultimately the best

method would be adopted. The greatness of the subject was self-evident. Mr. Hardy had referred to the composition of that mass of lives which had to be examined. They found the peer, the peasant, and all ranks of life mixed up in the investigation, and they had not to determine the mortality amongst any particular class, but amongst all classes collectively who insured with them, and he felt sure that when they approached the subject they would more than ever remember that it was large and collective results that they wanted. He did not deprecate further enquiry into the component parts of the collective experience, and he felt sure that the younger members would not be wanting in their help to carry out the work. It would be approached by all from the point of view of doing what they could in their time to hand down to their successors the great advantages which they had derived from the Institute Table. The Institute Table at the time of that investigation did not have the advantage of the large number of minds which were now giving themselves to the consideration of the subject, yet, taking it all in all, at that time the best mode was adopted according to the information then available, and what they had to consider now was the best mode for the present moment. They should do the best they could to bring the investigation up to the needs of the time being, so that 30 years hence, or during the period of those 30 years, their successors in office would be able to say that the greatest care and deliberation had been evinced by every member of the Institute in endeavouring to arrive at true results.

Dr. Sprague, in reply, said he heartily sympathized with what the President had said, that it should be the aim of each of them to do what he could in his time to promote the exact knowledge of a subject in which they were all interested, and thus to advance the theory of life assurance. With regard to Mr. Meikle's paper, it might be interesting to explain that after he (Dr. Sprague) had drawn up his memorandum and shown it to Mr. Meikle, that gentleman expressed a wish to have the cards relating to the 2,205 lives placed at his disposal, so that he might calculate them in his own way. He was pleased to accede to that request, and the cards were handed to Mr. Meikle with the result stated in his paper. The comparison of having the same material treated by two different methods had been eminently satisfactory; it was a thorough test of the two methods. The President had asked him to explain the difference between the numbers exposed to risk by the Institute method and by his own, but he thought the answer to that question was given in the paper. He had there stated that in order to make a direct comparison between the results that could be brought out, by the Institute method and by his own, the whole work would have to be done over again, because, for the convenient application of the Institute method, it was essential that the observations should close on 31 December, whereas in his own place they closed on 1 March. No doubt by proper adjustment they might define their calendar year as one terminating on 1 March, but in doing this there would be a great many things to attend to, the whole of the cards would have to be gone over again, and it would cost

a great deal of trouble, and there would be a great risk of error. For these reasons he had not thought it worth while to pursue the matter further. The President asked whether he (Dr. Sprague) thought it would have reduced the difference between the two methods considerably if it had been gone into? No doubt it would have done so. He expected that the results would have been made to agree closely. If his observations had been made up to 31 December there would have been very little difference in the number of lives. Mr. King had said that with whole-life policies there were no compulsory withdrawals, but according to the plan he went upon he had the compulsory withdrawal of a life which went abroad and was charged an extra premium. As to Mr. Hardy's remarks on the calculation of the expected deaths, he would be glad to see them in writing before saying anything about them. Mr. Hardy said that the usual method of calculating expected deaths was not strictly correct. Very likely he was right, but when he went on to say that the results were entirely worthless he (Dr. Sprague) thought he was going much too far. The results might not be strictly and theoretically correct, but they were very useful for purposes of comparison. He had used them for a great many years and saw no reason to consider them worthless. He hoped Mr. Hardy would elaborate that point. Then with regard to Mr. Whittall's criticism of his term "Nearest Duration", there was a great deal in his remarks, but as all his (Dr. Sprague's) three methods proceeded by policy years and not by calendar years, he required three names to distinguish them from each other, and he thought that for the purpose of his paper the names he adopted were very suitable, though for other purposes some other name might probably be better. He was glad that there seemed to be such a general agreement in favour of the use of the policy year in preference to the calendar year. Without this it was not possible to get at the effect of the duration of the policy on the rate of mortality, which was certainly a matter of practical importance. He understood Mr. Hardy to say that the results attained in that way did not influence their calculations. He (Dr. Sprague) thought they did. He thought that if Mr. Hardy would pay him the compliment of examining his short term rates—[Mr. Hardy—I do not agree with them]—he would find that they were based upon the results got out by a study of selection. [Mr. Hardy—That is where I should venture to differ from you.]—He (Dr. Sprague) was quite satisfied with those rates, but whether they were right or wrong they were a practical illustration of the importance of studying selection.

ACTUARIAL NOTE.

Deferred Assurances with Returnable Premiums. By JAMES STIRLING, F.F.A., *Edinburgh.*

RECENTLY I had occasion to construct tables of rates for deferred assurances on the lives of children, with return of premiums in the event of death before the commencement of risk; and it occurred to me that it might be possible in this operation to employ advantageously the rates for children's endowments already calculated. I accordingly looked into the matter with this object in view, and succeeded in obtaining the following formula, which, so far as I know, has not before been published, namely:

$$\mathrm{P}'_x = \frac{\mathrm{P}_{x+n} \times \mathbf{a}_{x+n}}{\dfrac{1}{\mathrm{R}.\mathrm{P}_{x.\overset{1}{\overline{n|}}}} + \mathbf{a}_{x+n}},$$

where x = present age,

$x+n$ = age when assurance begins,

P'_x = premium for deferred assurance (to be found),

P_{x+n} = premium at age $(x+n)$ for immediate assurance, and

$\mathrm{R}.\mathrm{P}_{x.\overset{1}{\overline{n|}}}$ = premium for pure endowment on (x) attaining age $(x+n)$, with return.

The formula may be deduced as follows, namely:

The annual premium required for a whole-life assurance at age $(x+n)$ being P_{x+n}, and P'_x being the premium receivable in the case we are considering, $(\mathrm{P}_{x+n}-\mathrm{P}'_x)\mathbf{a}_{x+n}$ is the sum the office must have in hand when (x) attains age $(x+n)$. Making use of the endowment table, we find that the annual premium (returnable in the event of death before age $(x+n)$) required to provide this sum is $(\mathrm{P}_{x+n}-\mathrm{P}'_x)\mathbf{a}_{x+n} \times \mathrm{R}.\mathrm{P}_{x.\overset{1}{\overline{n|}}}$, which, of course, must equal P'_x. From this equation we at once obtain the formula given above.

Most offices have the rates for children's endowments with return already calculated, and in constructing rates for deferred

benefits, if the above formula be used, the only point to be considered is the loading of P_{x+n}. Probably some actuaries might be disposed to take their published rate for P_{x+n} also, seeing that when the risk begins there will be in hand a considerable sum to balance the want of a medical examination. But whatever premium be taken for P_{x+n}, and however it may be loaded, if the assurances are all to commence at the same age, $x+n$, the numerator is the same for each age at entry, and to construct a table it is only necessary to take out the reciprocals of $R.P_{x.\overline{n}|}^{1}$, add $\mathbf{a}_{x+n}$ to each, and divide into $P_{x+n}\times\mathbf{a}_{x+n}$.

The formula is easily altered to apply to endowment assurances and limited-payment policies.

[It may be easily shown that, if in the above formula *net* premiums are inserted throughout, the result is the ordinary formula for the net premium for the benefit in question. Thus, we have

$$R.P_{x.\overline{n}|}^{1}=\frac{D_{x+n}}{N_{x-1}-N_{x+n-1}-(R_x-R_{x+n}-nM_{x+n})}$$

$$\therefore \text{ as } P'_x=\frac{P_{x+n}\times\mathbf{a}_{x+n}}{\dfrac{1}{R.P_{x.\overline{n}|}^{1}}+\mathbf{a}_{x+n}} \quad \text{(see above)}$$

$$P'_x=\frac{\dfrac{M_{x+n}}{D_{x+n}}}{\dfrac{N_{x-1}-(R_x-R_{x+n}-nM_{x+n})}{D_{x+n}}}$$

$$=\frac{M_{x+n}}{N_{x-1}-(R_x-R_{x+n}-nM_{x+n})}.$$

This will be recognized as being the formula for the net annual premium to provide a deferred assurance with return of the premiums in the event of death before the expiry of the term of deferment.

Ed. *J.I.A.*]

JOURNAL

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INSTITUTE OF ACTUARIES.

Notes on the Use of Scales of Premium reduced in anticipation of future Bonuses. By G. F. Hardy, F.I.A., *Actuary to the English & Scottish Law Life Assurance Association.*

[Read before the Institute, 26 February 1894.]

The idea of reducing the ordinary with-profit premiums in anticipation of such future bonuses as past experience shows may be safely reckoned upon, is not a modern one. The adoption of such tables has, however, become more common in recent years; and since their use involves the consideration of various practical points of importance—such as the basis upon which the reductions should be computed, the reserves and surrender-values attaching to the policies, the course to be pursued when the profits exceed or fall short of their anticipated amount, and the manner in which the calculations are affected by changes in the rate of interest and mortality—the subject appears to be one well suitable for discussion by this Institute.

Each method of division of surplus in conjunction with which the reduced-premium system may be employed presents us with a distinct problem; but, beyond general principles, I only propose to discuss the three common methods of surplus distribution:

1°. The cash bonus, of fixed percentage of the premiums paid during the quinquennium.

2°. The simple reversionary bonus, of fixed percentage on the sums assured.

3°. The compound reversionary bonus, of fixed percentage on the sum assured and existing bonuses.

1°. *Cash Bonuses.*—The first case is perfectly simple if interim bonuses are given, as the reductions of premium become merely loans at interest, repayable out of the bonus when declared, while if death takes place during the quinquennium the interim bonus is sufficient to repay the advance. The reserves at a valuation will be the same as for ordinary policies.

If no interim bonuses are given—an arrangement not very probable—the element of mortality must be taken into account, and the amount of reduction will then vary with the age. If we suppose the cash bonuses to be payable at the end of the fifth, tenth, &c., policy years, and the proportion to be anticipated to be KP'_x, the value of these bonuses will be represented by the series

$$KP'_x\left\{\frac{5D_{x+5}+5D_{x+10}+\&c.}{D_x}\right\}.$$

By analogy with the symbol $a_x^{(m)}$ representing an annuity of 1 payable m times a year, we may represent the above expression by the symbol $KP'_x a_x^{(\frac{1}{5})}$, that is an annuity of KP'_x payable quinquennially. The annual reduction of premium will therefore be $KP'_x\frac{a_x^{(\frac{1}{5})}}{\mathbf{a}_x}$, which, by writing $m=\frac{1}{5}$ in the usual approximate formula $a_x^{(m)}$

$$=a_x+\frac{m-1}{2m}\text{, becomes } KP'_x\frac{a-2}{a+1}$$

$$=KP'_x\{1-3(P_x+d)\},$$

Again, if we assume that, generally,

$$(D_{x+3}+D_{x+4}+\ldots D_{x+7})=5D_{x+5}$$

nearly, then we have as another approximate value

$\dfrac{KP'_x(D_{x+3}+D_{x+4}+\&c.)}{D_x}=KP'_x\dfrac{N_{x+2}}{D_x}$; and for the annual reduction $KP'_x\dfrac{N_{x+2}}{N_{x-1}}$.

The rate of interest in these calculations should not be less than that actually earned by the office on its funds, and may very fairly, I think, be taken at the same rate as charged for

loans upon policies. I have throughout, in the tables that follow, assumed a rate of $4\frac{1}{2}$ per-cent per annum, which appears a fair average rate to employ, the 4 per-cent tables being merely added for comparison.

With respect to the rates of mortality to be employed, select tables, such as those constructed by Mr. Sprague, might be used, but as the use of the H^M Table requires less labour and is sufficient for purposes of illustration, the tables that follow are based upon that standard of mortality. It is evident that, in practice, where reversionary bonuses are converted into reductions of premium, a standard of mortality should be employed somewhat more favourable than the average experience of the society, to allow for the option exercised by the policyholders. It is assumed throughout that sums assured are payable at the moment of death, although, for the sake of clearness, the ordinary commutation symbols are used in the formulas.

The following table represents the value of the ratio $\frac{a^{(\cdot 2)}}{\mathbf{a}}$ for quinquennial ages, and the percentage reduction of the premiums corresponding to cash bonuses of 25 per-cent on the premiums paid during the quinquennium.

TABLE I.

Age x	H^M 4 PER-CENT		H^M $4\frac{1}{2}$ PER-CENT	
	$\frac{a_x^{(\cdot 2)}}{\mathbf{a}_x}$	Reduction per-cent of Annual Premium	$\frac{a_x^{(\cdot 2)}}{\mathbf{a}_x}$	Reduction per-cent of Annual Premium
20	·8519	21·3	·8412	21·0
25	·8466	21·2	·8361	20·9
30	·8397	21·0	·8314	20·8
35	·8310	20·8	·8211	20·5
40	·8202	20·5	·8105	20·2
45	·8056	20·1	·7961	19·9
50	·7865	19·7	·7821	19·5
55	·7608	19·0	·7519	18·8
60	·7262	18·2	·7176	17·9
65	·6802	17·0	·6720	16·8
70	·6160	15·4	·6083	15·2
75	·5334	13·3	·5263	13·2
80	·4301	10·8	·4241	10·6

This table shows that the initial reduction of premium becomes more than an equivalent for the future bonuses as the

policy increases in duration, and therefore that a somewhat larger reserve should be made for the policies under the reduced premiums than for ordinary policies.

The additional reserve for any given policy may be simply expressed.

Supposing K to be the percentage of bonus anticipated in the construction of the original table of premiums: the reduction of premium at age x is then $KP'_x \frac{a_x^{(\cdot 2)}}{d_x}$ or $KP'_x\{1-3(P_x+d)\}$, as shown above, while after n years the reduction corresponding to the value of future bonuses of KP'_x is $KP'_x\{1-3(P_{x+n}+d)\}$, the value of the difference of their reductions is the additional reserve required, namely,

$$3KP'_x(1+a_{x+n})(P_{x+n}-P_x)$$
$$=3K.P'_x\,{}_nV_x,$$

(in which ${}_nV_x$ should strictly be taken at $4\frac{1}{2}$ per-cent, or whatever rate may be used in discounting the bonuses). If KP'_x have an average value of about 1 per-cent, these policies should require an addition roughly of about 3 per-cent to their reserve values as obtained in the ordinary manner.

As the main bulk of the surplus, however, existing at any moment (sufficient, that is, to provide the minimum bonus) belongs to the ordinary policies, the surrender-values granted to the minimum-premium policies should clearly be reduced by the extent to which the expected bonus has been forestalled by the reduction of premium during the current quinquennium. It would be sufficient, in general, to deduct from the ordinary surrender-values the total of such premium reductions for the quinquennium without interest.

2°. *Simple Reversionary Bonuses.*—Dealing now with the case of bonuses distributed as a fixed percentage addition to the sum assured, as the benefit is not here affected by the method of computing the office premiums, we are able to calculate tables of the reductions to be made from the ordinary rates corresponding to given rates of bonus.

If no interim bonuses are given and b = the rate per annum of the minimum reversionary bonus, the reduction of premium will be given by the formula

$$\frac{5b(M_{x+5}+M_{x+10}+\&c.)}{N_{x-1}},$$

while the additional reduction corresponding to an interim bonus of b' per annum will be

$$b'\left[\frac{(C_x+2C_{x+1}+\ldots 5C_{x+4})+(C_{x+5}+2C_{x+6}+\ldots 5C_{x+9})+\&c.}{N_{x-1}}\right]$$

$$=\frac{b'R_x-5b'(M_{x+5}+M_{x+10}+\&c.)}{N_{x-1}}.$$

It will be sufficient to compute an illustrative table of the reductions corresponding to interim and quinquennial bonuses of 30*s.* per-cent per annum.

The following table shows the value of the future bonuses at quinquennial ages and the corresponding whole-life reductions of premium at those ages:

TABLE II.

Age	H^M 4 per-cent		H^M 4½ per-cent	
	Value of future Reversionary Bonus of 30/- per-cent on £100 Policy	Equivalent Whole-Life Reduction of Premium	Value of future Reversionary Bonus of 30/- per-cent on £100 Policy	Equivalent Whole-Life Reduction of Premium
20	10·62	·541	8·75	·479
25	11·17	·589	9·32	·527
30	11·64	·642	9·85	·580
35	12·01	·698	10·31	·637
40	12·24	·758	10·66	·699
45	12·25	·822	10·84	·765
50	12·03	·889	10·80	·835
55	11·52	·957	10·49	·907
60	10·72	1·025	9·90	·980
65	9·69	1·092	9·07	1·052
70	8·44	1·167	8·00	1·122
75	7·10	1·216	6·79	1·187
80	5·84	1·268	5·64	1·243

Since the amount of reduction increases with the age, it is clear that the reserves required for the reduced-premium policies will be less than for the ordinary policies, and a reduction may safely be made (supposing always an adequate reserve is made for the latter) equivalent to the difference, at say 4½ per-cent, between the future value of the minimum bonuses and of the reduction of premium. The following table will then show the difference between the reserve values for the two classes of policies.

Table III.

Abatements from Whole-Life Policy-Values to obtain Reduced-Premium Policy-Values (simple Reversionary Bonuses as above).

Age at Entry	Duration of Policy					
	10 Years	20 Years	30 Years	40 Years	50 Years	60 Years
20	1·71	3·35	4·60	5·16	4·59	3·47
25	1·79	3·38	4·39	4·53	3·77	...
30	1·81	3·30	4·04	3·87	3·40	...
35	1·82	3·12	3·58	3·14	...	...
40	1·76	2·84	3·02	2·47	...	...
45	1·63	2·47	2·41	...	...	...
50	1·46	2·05	1·85	...	...	...
55	1·25	1·60	...	...	...	...
60	1·02	1·19	...	...	...	...

These differences are seen to increase to a maximum and then diminish (as would be expected), and this feature suggests that they may possibly be conveniently represented by a variation in the rate of interest in the reserve values.

A comparison of the values in the last table with the differences between the 3 per-cent and 4 per-cent reserves (given below) will show that a difference of 1 per-cent in the valuation rate of interest represents fairly well the difference in the required reserves.

Table IV.

Differences between the 3 *per-cent and* 4 *per-cent* H^{M} *Policy-Values (continuous).*

Age at Entry	Duration of Policy					
	10 Years	20 Years	30 Years	40 Years	50 Years	60 Years
20	1·73	3·22	4·17	4·32	3·64	2·52
25	1·80	3·23	3·98	3·74	2·96	...
30	1·86	3·19	3·69	3·29	2·34	...
35	1·89	3·08	3·31	2·69	...	...
40	1·89	2·87	2·85	2·13	...	...
45	1·84	2·61	2·35	...	...	...
50	1·75	2·29	1·88	...	...	...
55	1·63	1·92	...	...	...	...
60	1·46	1·56	...	...	...	...

There is no difficulty, however, in obtaining a more exact estimate of the abatement from the total reserves that may be safely made at a valuation in respect of these policies. All that is necessary is to value the total reductions of premium for each present age, by say $4\frac{1}{2}$ per-cent annuities, and set against them the value of the anticipated bonuses; the latter being found by multiplying the sums assured under the reduced-premium policies by the suitable constant at each age, specimens of which are given in Table II, and for compound bonuses in Table V.

3°. *Compound Reversionary Bonuses.*—Making the same assumptions as before, if b be the minimum compound reversionary bonus per annum, and b_1 the interim bonus, the value of the sum assured and future bonuses will be represented by the series (omitting for clearness the denominator D_x)

$$
\begin{aligned}
&[(1+b_1)C_x+(1+2b_1)C_{x+1}+\ .\ .\ .\ (1+5b_1)C_{x+4}] \\
&+(1+5b)[(1+b_1)C_{x+5}+(1+2b_1)C_{x+6}+\ .\ .\ .\ (1+5b_1)C_{x+9}] \\
&+(1+5b)^2[\qquad \&c. \qquad] \qquad +\&c. \\
&=[(M_x-M_{x+5})+b_1(R_x-R_{x+5}-5M_{x+5})] \\
&+(1+5b)[(M_{x+5}-M_{x+10})+b_1(R_{x+5}-R_{x+10}-5M_{x+10})] \\
&+(1+5b)^2[\qquad \&c. \qquad] \qquad +\&c.
\end{aligned}
$$

The same series with the denominator N_{x-1} will give the corresponding annual premium. If now we write

$$(1+5b)^{\frac{x}{5}}[(M_x-M_{x+5})+b_1(R_x-R_{x+5}-5M_{x+5})]=Q_x$$

then the reduction in the annual premium equivalent to the bonuses will be

$$\frac{Q_x+Q_{x+5}+\&c.}{(1+5b)^{\frac{x}{5}}N_{x-1}}-P_x$$

where P_x is the ordinary net premium at the rate employed in the formula, say $4\frac{1}{2}$ per-cent.

The function K_x is, of course, formed with great facility for quinquennial ages, and these are all the values that would be computed in practice, the reductions of premium for the remaining ages being interpolated.

Taking again the example of a 30*s*. bonus (both interim and quinquennial), we arrive at the following values for the reductions of premium and their capitalized values:

Table V.

Age	H^M 4 per-cent		H^M $4\frac{1}{2}$ per-cent	
	Value of future Compound Bonus of 30/- per-cent on £100 Policy	Equivalent Whole-Life Reduction of Premium	Value of future Compound Bonus of 30/- per-cent on £100 Policy	Equivalent Whole-Life Reduction of Premium
20	14·44	·735	11·81	·646
25	14·82	·781	12·30	·699
30	15·05	·830	12·68	·747
35	15·14	·880	12·95	·801
40	15·05	·933	13·08	·857
45	14·69	·986	12·98	·924
50	14·07	1·040	12·62	·976
55	13·17	1·093	11·99	1·036
60	11·98	1·146	11·07	1·096
65	10·61	1·196	9·94	1·153
70	9·07	1·243	8·61	1·208
75	7·50	1·286	7·20	1·258
80	6·08	1·321	5·90	1·301

Here the reductions of premium are, of course, larger than with the simple reversionary bonuses, but it will be observed that they increase more slowly: hence the difference in the requisite reserves for policies under the ordinary table and those under the reduced-premium table is less than in the former case.

We do not need to consider, in comparing the reserves at successive stages in the duration of the policies, the necessity for earning the compound bonuses upon the reversionary bonuses already existing. These can obviously be provided for by making a reserve for the existing bonuses at a sufficiently low rate of interest. If, *e.g.*, the office is making a rate $1\frac{1}{2}$ per-cent in excess of the rate at which the reserve for existing bonuses is computed, they are obviously just earning the requisite 30*s.* per-cent per annum upon these bonuses—indeed, under the conditions of practice, somewhat more. We need to consider, therefore, at any given moment, only the future compound bonuses upon the sum assured, and we then obtain the following difference in the reserves under the ordinary and reduced-premium policies:

TABLE VI.

Abatement from Whole-Life Policy-Values to obtain Reduced-Premium Policy-Values (compound Reversionary Bonuses as above).

Age at Entry	POLICIES IN FORCE					
	10 Years	20 Years	30 Years	40 Years	50 Years	60 Years
20	1·69	3·21	4·25	4·53	4·00	2·97
25	1·70	3·13	3·94	3·95	3·22	...
30	1·69	2·97	3·53	3·29	2·51	...
35	1·65	2·73	3·04	2·62	...	...
40	1·53	2·41	2·50	2·01	...	...
45	1·39	2·04	1·95	...	...	...
50	1·21	1·66	1·47	...	...	...
55	1·01	1·27	...	...	...	...
60	·81	·93	...	...	...	...

These differences would, perhaps, be fairly represented in practice by a difference of about $\frac{3}{4}$ per-cent in the valuation rate.

It might appear that we are led to the somewhat anomalous result, that the reserve required to be made for the ordinary policies, when the method of distribution is by way of compound reversionary bonus, is less than when by simple reversionary bonuses. This, however, is strictly true (if we exclude the existing bonus additions) under the conditions we are dealing with, namely, where the premiums are specially loaded to provide the given rate of bonus (simple or compound, as the case may be), such profit-loading being here represented by the difference between the ordinary and reduced premiums. On the other hand, the reserve for the existing bonuses may in the first case be calculated at a true rate of interest, while in the second case, as already stated, the reserves must be taken at a rate sufficiently low to allow of the future bonuses being earned out of excess interest.

To sum up, in general terms, the question of reserves: reduced-premium policies do not require so large a reserve as ordinary policies, and the difference may be conveniently expressed by increasing somewhat the valuation rate for these policies.

A strong company will, no doubt, seek to make this reduction in the reserves as small as possible, but if the business done under the reduced premiums is large, it will find a difficulty in disregarding it. It is sufficiently obvious that, whatever course is taken at a valuation, a proper reduction must be made in the

surrender-values of such policies. The exact mode of carrying out this conclusion in practice will depend on the method used for computing the surrender-values for the ordinary policies; where possible, an increase in the rate of interest used for that purpose will be found the simplest plan that can well be adopted, or as an alternative the deduction of a diminishing percentage of the surrender-value. Where a policy has not been in force a sufficient time to entitle it to a surrender-value, it is clear that the reduced-premium policies will have an advantage over those taken out under the ordinary table. This is an advantage, however, which is of course already possessed by policies under ascending premiums, and by whole-life policies over endowment assurances. It might, perhaps, be met in practice by giving smaller commuted commissions in the case of reduced-premium assurances.

It remains to consider the question of additional or defective bonuses. In the case of compound reversionary bonus, any additional profits may, perhaps, be fairly allotted upon the same plan to both classes of policies, since the additional bonus-earning power of the ordinary policies will generally be fairly compensated by the larger sum upon which the bonus additions will be computed. In the case, however, of simple reversionary bonuses, this is not so; and, theoretically, the additional bonus should be smaller on the reduced-premium assurances than on the ordinary policies, though the difference would be too insignificant to take account of in practice. The case of defective bonuses cannot be met properly by increasing the premiums, as is obvious if we take the extreme case and suppose all future bonuses to cease. The raising of the reduced premiums to the ordinary scale will not in these circumstances be sufficient to put both classes of policy-holders on the same level for the future. It is preferable, therefore, in the unfortunate contingency of a defective bonus, to operate upon the sum assured, and reduce it by the amount by which the bonus falls short of the expectation.

As to the circumstances under which reduced scales of premiums can be adopted with advantage, it will not be sufficient to consider merely the bonuses at present earned by the company. It is evident that these may, in a given case, be in part due to large accumulated profits, and that, apart from the fact that these belong equitably to the old policyholders, any considerable influx of new business, by spreading the accumulated surplus over a larger area, must diminish the rate of bonus. In fact, it appears

to me that, in adopting any such scale of reduced premiums, an office must consider, not merely the rate of bonus it has hitherto been able to declare, but rather the cost at which it can now transact its business, and the rate of interest at which it can hope to invest its funds. The fact that the premiums are only conditionally reduced, and that they may be raised or the sum assured reduced if the bonus on the ordinary policies falls short of the anticipated amount, makes the use of such tables safe enough, but any such revision would obviously be an unpleasant alternative in practice.

With respect to the effect upon the estimated reductions of any future changes in the rates of interest or mortality, the figures in Tables II and V enable us readily to give a numerical answer, on the assumption that such changes as may occur in the mortality can be represented by varying the age. We may, however, look at the matter somewhat more broadly, considering the reductions of premium to be advanced by the office in return for certain varying reversionary sums, computed at a fixed rate of interest, say $4\frac{1}{2}$ per-cent. This rate is not affected by any fall in the rate earned by the assurance fund; on the other hand, if a fall should occur in the rate of mortality (which, as already stated, should be taken in the calculations, at least as low as that experienced by the society), it is evident that upon this particular part of the transaction there will be a loss. This result of course means merely that the profit from any fall in the rate of mortality will be less in the case of the reduced-premium policies than in the ordinary business; while on the other hand, the loss from any fall in the rate of interest will also be less.

The most obvious disadvantage attaching to the use of these reduced premiums is the probable effect upon the non-profit business of the office employing them. If such business is considerable, this alone may be a sufficient reason for not embarking in the new tables. On the other hand, there are some possible advantages to be gained by a strong office, economically managed, in the smaller reserves required to be held against the reduced-premium policies, the smaller commission payable, and possibly in the accession of business which may result in meeting a want of the insuring public.

Discussion.

The Chairman (Mr. T. G. C. Browne) said that when reduced tables of premiums took the form of anticipating a certain rate of bonus he had great doubts as to their expediency. However these schemes might be guarded with saving clauses, the public undoubtedly looked upon the reductions as permanent; and in the event of the sum assured having to be reduced or the reduction having to be reduced, the confidence of the public would be shaken in the office, however unreasonable that feeling might be. The actuary who assumed that throughout the next 40 or 50 years a reversionary bonus of 30*s.* could be safely depended upon took a somewhat sanguine view of the position, looking to the falling rate of interest and the tendency of expenses to increase. There was another point which might be said to be purely a trading one, namely, the effect that these tables would have in reducing the volume of non-participating business, and the secondary effect which a reduction of such business would have upon the bonuses of the participating policyholders. As an alternative he much preferred to give a reduction of premium in the form of a credit of a certain percentage, say 20 or 25, which was to form a debt secured not only on the bonus but on the policy itself. The only advantage which this method gave to the credit policyholder as distinguished from the ordinary policyholder was that it allowed him a surrender-value of moderate amount from the beginning of the policy in place of his having to wait one, two, or three years, as the case might be. This system of giving a reduction of a percentage of the premium by way of a debt fitted in most conveniently with those systems of division which gave a large cash bonus to start with, such as dividing the profits in proportion to the premiums paid during the quinquennium, assuming that that cash bonus was maintained fairly throughout life. The reduction in that case might be so adjusted as, almost certainly, to wipe off the debt of each five years at the end of each quinquennium, while, if they turned to an office whose system yielded a moderate cash bonus to start with, but having a tendency to increase, they probably found that even with a smaller reduction they ran a chance of having to carry forward a balance of unliquidated debt for at least two or three quinquenniums after the policy was effected.

Mr. C. D. Higham said that in reference to Mr. Hardy's formula as to compound reversionary bonuses he doubted whether it was right in these cases to allow bonus on bonus at all. If it were an ordinary policy, and the assured had surrendered his bonus, he would not be allowed the compound accretion, and he (Mr. Higham) failed to understand why in the present case he should stand in a better position, seeing that the whole essence of the formula was a surrender of the future additions. He was glad to see Mr. Hardy's claim that future profits should be looked to and not past experience, as also his remarks on the injury which may be done by "watering" (to use a Stock Exchange term) the indefinite surplus often possessed by old companies by too large an introduction of new business. It was flat heresy to hint that a large new business was not always desirable, at any rate for the policyholders, and yet it was not sufficiently recollected that a new with-profit policy introduced a

new partner into the concern. As to possible defective bonus he felt that the conditional element was not made sufficiently plain in respect of these policies, for though no doubt every precaution was taken that there should be no diminution in the future, yet the public ought to be made clearly to understand that it was possible that the sum assured might be diminished. Why a strong office, as Mr. Hardy suggested, should find it advantageous to hold small reserves was not apparent; nor had he ever found any system of assurance which appeared to an agent rightly to justify a smaller commission. His own experience was that this kind of policy was very popular and pleased both parties, for the assured got his increased assurance at the younger age instead of taking out a further policy later, while the office secured the future amount from the machinations which might have led the policyholder into another fold: only caution must be exercised to see that there was no undue selection against the office by a doubtful life increasing his proposal to the company's disadvantage. To some extent these assurances were a return to more primitive methods, for the bonus system was not introduced with any view of taking care of the savings of the assured, but merely that a margin which was necessary for safety should be equitably dealt with. Perhaps with these minimum premium assurances the margin was in the sum assured, which might be reduced, though with more accurate data and more experienced methods there should be little probability of so unfortunate a result.

Mr. W. J. H. Whittall said there had been of late a great extension of the principle involved in these tables. There was undoubtedly at the present day a demand for cheap insurance. No doubt in these reduced premium scales there had been found just the sort of insurance that people desired. It was not tainted in any way with the errors of assessmentism, and, on the other hand, the actuary had a *locus penitentiæ* if he took too rosy a view of the future. But for fear of after consequences most companies would shrink from putting forward a table of premiums which there was not a fair prospect of their being able to maintain. Mr. Hardy had pointed out and Mr. Browne had strongly dwelt upon the very inconvenient effects which would ensue to an office should they have to raise the premiums. As Mr. Browne had said, make all the safeguards they would, and put them forward never so prominently, the public would persist in considering that a table of premiums printed by a company was something approaching a pledge. The success of these schemes was partly due, no doubt, to the high repute for caution which British offices had earned, and on that account it strongly behoved every company to be most cautious in putting forward schemes of that kind. He suggested that, where possible, an office contemplating one of these reduced premium schemes should take care to allow an increasing margin of bonus. For instance, supposing the office to be one which allotted compound reversionary bonuses such as had been described, then it might adopt the plan of anticipating a portion, not of the compound bonus, but merely of a simple or uniform bonus at the same rate. The author, referring to Table II, stated that, "Since the

amount of reduction increases with the age, it is clear that the reserves required for the minimum premium policies will be less than for the ordinary policies." It was not at all clear to him when he first read it. He could not make out what had become of the existing bonuses which were declared before the date of valuation, but upon looking into the matter more closely he found that if the separate bonus transaction was valued, as they might value an ordinary policy, by the retrospective method, they obtained an expression which was quite analogous to that of the ordinary policy value, namely—

$$\frac{r_x(\mathrm{N}_{x-1}-\mathrm{N}_{x+n-1})-b(\mathrm{R}_x-\mathrm{R}_{x+n})}{\mathrm{D}_{x+n}}$$

(where b = rate of bonus anticipated and
r_x = corresponding reduction at age x).

This, it was quite apparent, was equal to the formula which appeared to have been used by Mr. Hardy in obtaining the abatement in table 3, namely—

$$b\times\frac{\mathrm{R}_{x+n}}{\mathrm{D}_{x+n}}-r_x\times\frac{\mathrm{N}_{x+n-1}}{\mathrm{D}_{x+n}}.$$

The author stated, in discussing the effect of a change in the rate of mortality, that a low mortality would cause a loss on that particular part of the transaction—that is to say, on the bonus transaction. It was with hesitation that he (Mr. Whittall) expressed any disagreement from that view, but as far as he could see it seemed to be the reverse. The office could afford to make a smaller reserve at the date of valuation simply because it had a larger amount of profit to get from the transaction in after years. The longer a man lived the greater therefore would be the profit made by the office. If there were anything in this view it would be a little remarkable that the office would in the event contemplated be on the right side of the hedge in respect of the bonus transaction, just as it was in regard to the corpus of the policy.

Mr. JAMES SORLEY said he had been struck with the practical judgment with which Mr. Hardy had handled the problem, apart altogether from its mathematical development. He instanced the useful practical suggestion that these reductions of premium might be calculated at the same rate as they would charge for loans on policies, say at 4½ per-cent. Then, again, the suggestion that if they were to allow in their valuations for this system of anticipating profits they might fairly do so by making an increase in the valuation rate of interest, was very ingenious. Something analogous might be done in the case of limited payment policy valuations. Clearly it was unfair with single or limited payment policies to appropriate all the loadings as received, and to leave the future to look after itself. Instead, however, of elaborate calculations spreading the loading over future years, the simplest and most practical way appeared to be to adopt such a lower rate of interest as would adequately represent the value of the margin requiring to be reserved. The question of interest, as affecting the reduced premiums with which

they had been dealing, was a little puzzling. For instance, it had been suggested that because there had been a fall in the rate of interest that could be realized on investments, those reduced premiums must be raised. To non-experts this looked reasonable, but the obvious theoretical result of such a fall was that the reduced premiums must be still further diminished because the value of the bonus discounted increased with the lowering of the rate of interest. What a fall in the rate of interest required was a reconsideration of the reduced premium system—either the raising of the full-profit premiums or the diminishing of the rate of bonus discounted, if that rate was as high as the company could safely afford to anticipate prior to the fall in interest occurring. He disagreed with the author in his statement that defective bonus could not be met properly by increasing the premium, but only by operating on the sum assured. Theoretically it was very easy to operate on the premiums. If the bonus on the full-profit policy disappeared altogether then the rate of premium on the reduced premium policy would gradually rise and ultimately considerably exceed the rate of premium on the full-profit policy. That was as it should be. The transaction would then be very similar to an assurance on the ascending scale of premiums offered by many companies. It was a suicidal thing for any office to discount a rate of bonus of which it was not absolutely sure as a minimum: it was much better that the policyholder should face his loss at once and pay his deficiency year by year than allow it to accumulate at compound interest and mortality against him till death, so that the longer a man lived and the more premiums he paid the less would become the sum assured. If they adopted the system of simply raising the premium as required it would come to this—that if the individual policyholder was in the exceptional position of having insured with reference to the amount of premium he wanted to pay, and not to the amount of insurance he wanted to have, he could get the sum assured reduced by endorsement on the policy, so as to keep his premium uniform, while, if the deficiency in the bonus had to be deducted from the sum assured, he would have no corresponding option of keeping his sum assured uniform—at least, without medical examination and other formalities.

Mr. W. O. Nash said that reference had been made to the fact that past bonuses were not a safe guide to the profits of the future. It seemed to him that having obtained their minimum premium by deducting the value of the bonus anticipated, the most difficult part of the problem then arose—namely, to analyze the reduced premium into its constituent elements, and to realize what rate of mortality, what rate of interest, and what expense rate would enable them to conduct their business in the future and to pay the full sum assured. These minimum premiums approached the H^M 4 per-cent net values, and from that point of view they saw that if the mortality of the future was truly represented by the H^M mortality, if they earned 4 per-cent on their funds, their business must be conducted actually without expense, or else the premiums would not provide the sum assured. Admitting that, they would have some idea as to which offices were really justified in employing these tables and which were not. It

was necessary that they should have a fair prospect of making 4 per-cent interest; that the expenses should be met by that undivided fund possessed by many offices, aided by miscellaneous sources of profit; and no office, under present circumstances, was justified in adopting one of these low tables of premium unless it was convinced that the mortality would be less than the H^M. The correspondence that the author had pointed out in the difference between the reserve values of ordinary policies and of reduced premium policies, and the difference between the 3 and 4 per-cent net reserve values, arose from the fact that in charging these low premiums they were practically conducting their business on a 4 per-cent basis, and therefore had to carry that rate through all their calculations. He suggested that the policies referred to certainly met a want of the public, and if the office employing them had really a sound foundation for doing so great benefit might result. Further, if the present conditions altered as regards interest or mortality or expense, and it seemed likely that the office would be unable to maintain its bonus, then there was a way out of the difficulty by closing the present series of policies and starting a new one.

Mr. R. P. Hardy said the paper was a modern rendering of a paper read before the Institute by Mr. Robert Tucker some 32 years ago (*J.I.A.*, ix, p. 245), in which he illustrated mathematically how premiums should be estimated to provide for certain contingencies. Unless he did the author an injustice he wished to read between the lines a note of warning. He (Mr. R. P. Hardy) believed that these systems were highly dangerous. Who could say what was going to be the future rate of expense? Who could predict whether the rates of interest now obtained would hold over the long future for which the contracts extended? Nor could they assume that the *quasi* profits now made from surrenders might not be seriously interfered with by the legislature, as had been done in America. He would be a bold man who upon the records of his own past entered into serious engagements, pledging himself over these long periods. Why had this all come about? Was it not because insurance offices had not been firm in insisting that the principles of individualism or investment, and collectivity or assurance were discordant, and that a man must choose the one or the other—that it was impossible to make an agreement between those diverging two principles. Mr. Whittall had said that the office had a *locus penitentiæ*, that if it found that it had made a mistake it could recall it. Yes, and a very bad penitential quarter of an hour some offices would have to suffer when the adjustment had to be made. Supposing the sum assured under a policy had to be reduced how would a mortgagee look, or how would a creditor feel, if he held such a policy as security? Mr. G. F. Hardy had suggested the reduction of the surrender-values, but would that be a very popular proceeding? Who could think of reducing the commission to the agents? There would be an insuperable difficulty in making agents believe that the sums they now received were a great deal too high. This was a question which he had watched with

great interest, and he hoped that those who might adopt the system described by the author would not be disappointed.

Mr. H. C. Thiselton said the system discussed by Mr. G. F. Hardy was introduced by the Scottish Amicable in 1854, and had since been adopted by 12 other offices. He thought the time had come when it should be placed on record in what way this system could be safely and equitably worked. With regard to the remark that a man under the compound reversionary bonus system, if he surrendered past bonuses, forfeited thereby his title to future compound bonuses, there was no doubt that in discounting these bonuses to reduce the man's premium they did give him the full benefit of the future compound bonus; and not only did he receive the bonus in cash, but he even received it in advance. But the man who surrendered the bonus in cash ought to receive in the surrender-value the full equivalent of his right to future bonuses. The only point was to take care that the man who surrendered the bonus and the man who received the bonus in reduction of premium were placed on an equitable footing. The author had worked his premiums out at two rates—4 and $4\frac{1}{2}$ per-cent, and seemed to prefer the $4\frac{1}{2}$. He agreed that they could not afford to do it at any rate under $4\frac{1}{2}$; but, taking the two rates side by side, it was apparent that the reduction of premium increased when they lowered the rate of interest at which the future bonuses were discounted. This was of great importance, because it showed that the danger did not come from the reduced premium policy. One interesting point which had not been referred to was that Tables II and V placed side by side enabled them to determine what simple bonus corresponded to a 30*s*. compound bonus. These tables would enable them to obtain the precise equivalent for a 30*s*. compound bonus, and showed that at age 20 it was worth 40*s*. 6*d*. in a simple bonus, and so on. As to the additional and defective bonuses, he could not agree with the author's system of treating the latter. No office had yet had to deal with defective bonuses, and he would therefore refer principally to additional bonuses. The author suggested that the additional bonus should not be allotted as a compound bonus, because the profit earning power of the policy with the previous bonus attached was greater than that of the reduced premium policy, and, therefore, to make up for that it would be quite sufficient to allot to the reduced premium policy the simple bonus. He thought there was a fallacy in this. Was the profit earning power of the reduced premium policy any smaller? Adopting Mr. Hardy's suggestion that they should regard the reduction of premium as invested in the purchase of a series of reversions, were they not practically getting $4\frac{1}{2}$ per-cent on them, and might not they justly infer that the profit earning power of these reduced premium policies was really greater than that of the policy on which all previous bonus remained uncommuted? Both additional and defective bonuses must be calculated by working out a scale showing the bonuses which had been actually discounted and another showing the bonus declared on a policy in which none of the previous bonuses had been commuted. They might print the first scale on the back of the policy and leave two blank columns, one to

contain the bonus actually declared, and the other to contain the differences between the first and second columns, which would be the amounts by which the sum assured would be decreased or increased. One thing to be provided against was that when a man had side by side two tables he might exercise a very strong selection against the office, according to which table he chose. It did not follow, however, because that danger existed they were not to put into practice these systems, which really contained the true principle of insurance. The danger could be surely provided against by calculating the reduction of premium at $4\frac{1}{2}$ per-cent or even higher. He should not hesitate if they found the experience of an office showing a much heavier rate of mortality under these policies to calculate the reduction of premium even at as high a rate as 5 per-cent.

Mr. G. King said Mr. R. P. Hardy had arraigned a number of actuaries, and amongst them he felt he was included. He could not allow judgment to go by default. He agreed that in introducing a minimum premium scale caution was required, and it was quite right to raise a note of warning. It did not follow that because caution was required they must not advance, as Mr. Hardy had seemed to say. They might advance with caution, and they must advance. The introduction of these scales of premium was simply an attempt to adjust their rates to their latest knowledge. With-profit rates were too high for the mere purpose of safety—in fact, all offices having non-profit tables alongside of the with-profit tables admitted that. The purpose of introducing these minimum rates was to bring their rates as closely as possible to the real cost of insurance, and they were quite safe in doing so if they reserved the power in the event of an unfavourable contingency to raise the rates or to adjust the sum assured. He admitted there would be very considerable embarrassment were either of these courses necessary, but the actuary must be specially cautious in his general administration, The author said caution was required in these special forms of tables, because if they introduced too much new business by spreading accumulated surplus over a larger area they must diminish the rate of bonus. He (Mr. King) could not see that that argument applied more particularly to these special tables than to any other. The same remark would exactly apply to business of any kind. They must be careful of the amount of new business as compared with the old if they were to maintain their bonuses, and therefore a large new business was not what a wise actuary should indulge in. He must not get so much new business as to materially disturb the proportion between the reserves and the existing business. He did not agree with the way in which the compound reversionary bonuses had been dealt with by the author. The bonus being *ipso facto* discounted by this method of reducing premiums, it necessarily followed that there was no compound bonus, and therefore the author's formula for simple bonuses ought to be adopted even by companies granting compound bonus. There was a difference between bonuses thus discounted and the reduction of premiums given ordinarily in exchange for reversionary bonus. In the latter case the company had a reserve in hand of the bonus already declared, and there might be some reason in allowing a compound bonus in the case

of ordinary reductions of premiums, because reserves made for this reduction would no doubt earn bonus; but where they discounted future bonus, and had no reserves for it, there was no reason why they should be considered compound bonus at all. The reductions in Table II, where simple bonuses were in question, were reasonable and safe if caution was exercised, but they ought not to go beyond that, and they ought not to deal with compound bonus. One very important matter was the surrender-value. The proper course was to take the value in the ordinary way, then to debit it with the difference in value between the future discounted bonus and future reductions of premiums, and again with the reductions of premiums that had been allowed during the current quinquennium. By doing that they acted fairly between the different policyholders and safely for the office. He had not found that the introduction of tables of this kind interfered with the non-profit business. If it was made clear to the insuring public that there was a liability to a reduction of the sum assured under these special tables that did not suit various purposes, and non-profit policies were still freely taken out.

The CHAIRMAN having proposed the usual vote of thanks to the author,

Mr. G. F. HARDY, in reply, said he could not agree with Mr. King that compound bonuses ought not to be considered in estimating the reduction to be made in view of future reversionary bonuses. What they had to consider in determining these deductions was the benefit which the ordinary policyholder received, and the reduced premium policyholder was asked to forego. It was obvious that this benefit was considerably larger in the case of a compound than of a simple bonus, and the reduction of premium should surely be correspondingly greater. As to the selection exercised against the office in the table, that was a point to be kept in view, but it would naturally apply to all tables used by an office. There was no doubt a certain selection exercised against an office, when a man determined whether he would insure under a reduced premium or a full premium table, or under an ascending or descending scale, or under an endowment assurance. All these differing methods of insurance naturally gave a certain option to the assured which, if he knew all about his future prospects of longevity, he would be able to exercise against the office. Mr. Sorley thought the premiums should be increased in the case of a defective bonus, rather than the sums assured reduced; and here again he (Mr. Hardy) was not altogether inclined to agree with his critics. A picture had been drawn of the disappointment of an old policyholder on finding his sum assured gradually diminished, but it was an open question whether he would not be equally disappointed if, on the other hand, he found the premiums gradually rising. If they undertook to declare a certain bonus on an ordinary policy, and found the bonus defective, it was easy to show the policyholder that the reversionary bonus being short of the anticipated amount the sum assured must be correspondingly diminished; but. if that defect in the bonus was converted into an increase of premium it would be necessary to lay down in advance the terms upon which this was to be done.

Reversionary Interests contingent on survival of the Reversioner.
By SAMUEL GEORGE WARNER, F.I.A., *Actuary and Secretary of the Law Union and Crown Insurance Company.*

[Read before the Institute, 2 April 1894.]

IN venturing to direct the attention of the Institute of Actuaries once more to the subject of contingent reversionary interests, I am deeply sensible that the topic has been treated by some of the most brilliant and experienced members of our profession, and that it may therefore seem presumptuous of me to choose it for further comment. I can but plead in extenuation the very great practical importance of the subject—so great as to make it one upon which we can hardly confer too freely or too often. The only other prefatory remark I would make is a disclaimer of any ambitious design, or plea for originality, as regards the paper itself. Its purpose, so far as I am concerned, will be amply fulfilled should it succeed in eliciting a useful discussion.

I have tried to indicate by my title not only the class of securities to be considered, but also, as far as possible, the standpoint from which I shall consider them.

Contingent reversions and reversionary life interests, as dealt with by life assurance companies, form a class by themselves amongst our investments, inasmuch as we must call in the aid of life assurance to perfect their value as securities.

We become, in fact, in such cases our own customers as regards the business of life assurance, by virtue of transacting the business of investment. Either a policy is effected by the company itself, or one is taken out at its instance for such an amount as it shall determine. This feature seems to me to differentiate these transactions from all others; offering on one hand a special inducement to entertain them because of the life assurance they involve, but presenting on the other hand, and for the same reason, complex problems which demand careful consideration, and which have in fact furnished fruitful ground of controversy.

As the result of that controversy, there are before us two distinct and well-defined modes of dealing with such investments; each proceeds on broad and easily-understood principles; a full explanation and defence of each is on record in our *Journal*. The earlier method is associated with the name of the late Mr. Jellicoe, the later with that of Dr. Sprague. As frequent reference to

these two sets of formulas will be necessary, I have tried to find names for them, founded on the principles they involve; so as to avoid constant personal allusion to their authors. Both proceed by creating a life assurance which eliminates the contingency, valuing the benefit as if in possession, and deducting from the result so obtained the value of such an annuity as will provide premium and interest during the period of its deferment. The difference between the two methods is in the rate of interest at which the annuity-value is computed; Mr. Jellicoe using $3\frac{1}{2}$ per-cent, the rate at which an annuity may be purchased, while Dr. Sprague uses 6 per-cent, the rate at which an absolute reversion may be bought.

I shall therefore—for the purposes of this paper—ask leave to designate Mr. Jellicoe's as the "Annuity Formulas", and Dr. Sprague's as the "Investment Formulas."

I venture to think that each method represents an extreme view, and is open to some criticism on practical grounds. The Annuity Formulas are undoubtedly too stringent, commuting as they do the interest, as well as the premium, at $3\frac{1}{2}$ per-cent. In this respect they are consistent with, as they are derived from, the old formula for an absolute reversion, $1-d_{(5)}\,(1+a_x)_{(3\frac{1}{2})}$, which assumed that an annuity to provide interest on the investment was actually purchased. It is probably safe to say that this method, as regards absolute reversions, is now practically abandoned by the companies which compete for such securities. The market would present so few opportunities to an investor limiting his offers to prices so arrived at, that he might save himself the trouble of entering it. As all contingent reversionary interests are, for our purposes, made absolute by assurance, I do not see that we have any right, or that there is any real occasion, to treat them—once they have so been made absolute—otherwise than as we are prepared to treat absolute reversions generally. In so far, therefore, as they claim to commute interest at $3\frac{1}{2}$ per-cent, the Annuity Formulas are severer than the conditions of modern practice warrant.

The arguments which appear to me to militate against the Investment Formulas are of a different kind, and I foresee more difficulty in an attempt to establish their cogency. They are less obvious than those which apply to the Annuity Formulas, but, I think, equally practical, and of greater importance, as probably the Investment Formulas dominate the practice of most of us nowadays. The commutation of premiums at the investment

rate admits of a perfect theoretical defence, so long as we regard the matter from the standpoint of the investment only. Regarded in its relation to the life assurance fund, however, such a practice involves the sacrifice, for the benefit of the insured, of the profit made by the company in the excess of its investment over its valuation rate. Nor is the advantage so given that of the difference between the valuation rate and the *average* rate realized by the company on its investments. It is the highest rate that can be obtained upon any class of security which is thus employed in the capitalization of premiums. I confess to a strong dislike of the hybrid function thus brought into our calculations, in the shape of a premium commuted at possibly double the rate at which it is constructed. What such a process actually does is to give to the lives insured in connection with reversionary transactions, the benefit of the full rate of interest at which their premiums are invested by the company, that rate being an exceptionally high one. Such a concession would be impossible, and undreamed of, in connection with any other class of policy-holders. In these days of keen competition we have had some sharp discussions as to the propriety of creating privileged classes of insurers; but I do not think there has been any suggestion to form such a class for the benefit of the borrowers on and vendors of contingent reversionary interests. The profit made by a life assurance company out of surplus interest does not, any more than its profit from other sources, belong to its non-participating policyholders. By this method, however, a section of them obtain that profit, and probably more than their share, while as regards with-profit policies (which are quite possible in connection with reversionary life interests), it is easily conceivable that they might get the benefit twice over—once indirectly by the process now before us, and again directly in accordance with the company's method of distribution.

The subject of reversionary life interests stands somewhat apart from the other branches of the question. The magnitude of the figures generally involved, and the large differences of net result which different methods of valuation yield, cannot but impress upon us that these are investments peculiarly difficult to handle, demanding the utmost prudence and care. These circumstances appear to me to intensify, as regards such securities, the inadvisability of commuting premiums at investment rates, especially as it is in cases where the interval between the ages of reversioner and life tenant is comparatively short that such a

process becomes at once further-reaching in its assumptions as to the future, and more attractive in its results to a vendor or borrower. After all, at the back of the reasoning which shows that each investment provides the rate of interest assumed therein independently of other transactions, lies the primary assumption that the sums involved are, on an average, fairly equal; but I think practice teaches us that this does not, as a rule, hold good when we include reversionary life interests. They constitute almost a class by themselves in this respect, and a class which we can scarcely hope will be large enough, in the case of any one company, to form its own average.

Before closing these comments on the Investment Formulas, I should like to express my very deep indebtedness to their author for his labour in this and other branches of our science, and for its luminous results. If we criticize any of his conclusions, we are enabled to do so chiefly by his own light, and by the aid of his own work in making the subject clear and comprehensible.

The preceding discussion of existing methods will have indicated pretty clearly my own suggestion as to the way in which we should deal with contingent reversionary interests. It is a very simple and obvious one. I think the best and safest course to follow is to make the interest an absolute one by insurance, at such a price as we would charge for any other insurance, and then to purchase the absolute reversion, so formed, at such a price as we would give for any other absolute reversion.

Translated into symbols, these principles give as the value of a contingent reversion to 1:

$$\mathrm{A}_{xy_{(j)}} - \mathrm{P}^1_{xy}(1+a_{xy})_{(i)} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and for the value of a reversionary life interest of 1 per annum

$$\left\{\mathrm{A}_{xy_{(j)}} - \mathrm{P}_x(1+a_{xy})_{(i)}\right\} \times \frac{1}{\mathrm{P}_x + d_{(\bar{j})}} \quad . \quad . \quad . \quad (2)$$

In the above, which I shall venture to call the "Combined Formulas", P is of course an office premium, i is the commutation rate, j the investment rate, and $\bar{j}$ (in formula 2) the rate to be yielded by the life interest when in possession.

I have selected a few typical cases in order to show the practical effect of the Combined as compared with the Annuity and Investment Formulas. These are set out in the following table, as to which, however, a few explanatory remarks are first necessary.

As regards the contingent reversion to £1,000, I have worked out the cases by all three formulas as follows:

Annuity Formula, $1-(P^1_{xy}+d_5)(1+a_{xy})_{3\frac{1}{2}}$,

Investment Formula, $1-(P^1_{xy}+d_6)(1+a_{xy})_6$,

Combined Formula, $A_{xy(6)}-P^1_{xy}(1+a_{xy})_{3\frac{1}{2}}$.

The office survivorship premiums assumed throughout are the following:

	£	s.	d.	
30–50 . .	1	9	0	per-cent per annum
30–60 . .	1	6	0	" "
30–70 . .	1	4	0	" "
40–50 . .	2	1	0	" "
40–60 . .	1	15	6	" "
40–70 . .	1	11	0	" "
50–60 . .	2	17	0	" "
50–70 . .	2	8	0	" "

As regards the reversionary life interest of £100, I have based my work on the tables given by Dr. Sprague in his paper of 30 November 1868 (*J.I.A.*, vol. xiv, pp. 432–433) and have as far as possible brought my formula, as regards rates of interest, into line with his, so as to produce an efficient comparison. The formula I select as his, from those given in that paper, is No. 10 (*J.I.A.*, vol. xiv, p. 428) which assumes the investment to yield 6 per-cent in reversion and 5 per-cent in possession. I follow Dr. Sprague in correcting my formula, as he does his, by deducting $\frac{1}{2}A_{xy}$; and I have also ventured to make this deduction from the results as given by the Annuity Formulas before setting them down in the table. This is done so as to show more clearly the result of the comparison; the reasoning by which Dr. Sprague establishes the correction being perfectly general and applying equally to the Annuity, Investment, or Combined Formulas.

The respective formulas as used in the table are therefore—

Annuity Formula, $\dfrac{1}{P_x+d_5}-(1+a_{xy})_{3\frac{1}{2}}-\frac{1}{2}A_{xy(5)}$,

Investment Formula, $\dfrac{1}{P_x+d_5}-\frac{1}{2}-(1+a_{xy})_6\times\left\{\dfrac{P_x+d_6}{P_x+d_5}-\dfrac{d_6}{2}\right\}$,

Combined Formula, $\left\{A_{xy(6)}-P_x(1+a_{xy})_{3\frac{1}{2}}\right\}\times\dfrac{1}{P_x+d_5}-\frac{1}{2}A_{xy(6)}$.

I have used throughout the office premiums suggested by

Dr. Sprague, namely, for age 30 £2. 4*s.* 9*d.* per-cent per annum, for age 40 £2. 19*s.* 2*d.*, and for age 50 £4. 3*s.* 6*d.*

Age of Rever-sioner	Age of Tenant for Life	Value of Contingent Reversion to £1,000			Value of Reversionary Life Interest, £100 per Annum		
		Annuity Formula	Investment Formula	Combined Formula	Annuity Formula	Investment Formula	Combined Formula
30	50	195	250	215	111·8	217·7	142·6
	60	384	398	379	384·2	427·6	379·2
	70	555	544	534	648·0	647·3	621·0
40	50	156	214	168	31·1	135·9	51·2
	60	355	373	347	278·9	327·5	272·2
	70	537	529	516	528·5	534·2	503·6
50	60	283	308	271	147·2	197·6	137·4
	70	487	483	465	368·2	381·1	346·1

This table presents some features which appear anomalous, and which my critics will doubtless not fail to point out. In six of the eight values, both of contingent reversions and of reversionary life interests, the Combined Formula gives a result lower than is obtained by the Annuity Formula; whereas the objection made to the latter was its undue stringency. This is due to the fact that the Annuity Formula assumes only 5 per-cent as the rate of interest upon the investment; the effect of which is, notwithstanding the commutation of the interest at $3\frac{1}{2}$ per-cent, to bring out at certain ages larger values than are obtained by use of the pure A_x function at a rate 1 per-cent higher. This fact is familiar to anyone who has compared the Carlisle A_x 6 per-cent table with one constructed upon the basis $1-d_5(1+a_x)_{3\frac{1}{2}}$. The comparison is therefore more instructive as between the results obtained by the Investment and the Combined Formulas. Here, as is natural, the latter show a distinct reduction throughout, which is relatively greater as the interval between the ages of reversioner and life tenant decreases.

We cannot, however, be said to have as yet arrived at satisfactory results. The figures shown have some value as exhibiting the exact effect, other things being equal, of the suggested use of the Combined instead of the Investment Formulas; but we can scarcely regard a set of results, lower on the whole than those of either existing method, as practically useful.

There is however no reason, inherent in the Combined Formulas, for the rigid use of the investment rates assumed in the examples. One of the recommendations of the suggested methods is, in fact, their elasticity in this respect. It does not seem, for instance, to be an unreasonable assumption, in view of the prices now and for some time past ruling in the reversion market, to take the rate of interest realizable on such securities as $5\frac{1}{2}$ instead of 6 per-cent, and modify the formulas accordingly. With regard to the reversionary life interest formula, a further concession is possible should it be deemed expedient—in the shape of a reduction of the rate of interest assumed to be yielded by the income when in possession—say to $4\frac{1}{2}$ per-cent. The combination of these assumptions would give reversionary life interests a much higher value; and ought, I think, to mark the maximum for a prudent investor. Either alteration, however, might be made without the other; the reversion computed at $5\frac{1}{2}$ and the life interest in possession at 5 per-cent, or the latter at $4\frac{1}{2}$ and the former at 6 per-cent. There are thus, for reversionary life interests, three distinct modifications possible; and I have set out their results, together with those for the contingent reversion, in the following table.

The formulas used are:

For contingent reversions,

$$\mathrm{A}_{xy(5\frac{1}{2})} - \mathrm{P}^1_{xy}(1+a_{xy})_{3\frac{1}{2}};$$

For reversionary life interests,

$$(1)\quad \left\{\mathrm{A}_{xy(5\frac{1}{2})} - \mathrm{P}_x(1+a_{xy})_{3\frac{1}{2}}\right\} \times \frac{1}{\mathrm{P}_x + d_{4\frac{1}{2}}} - \tfrac{1}{2}\mathrm{A}_{xy(5\frac{1}{2})},$$

$$(2)\quad \left\{\mathrm{A}_{xy(5\frac{1}{2})} - \mathrm{P}_x(1+a_{xy})_{3\frac{1}{2}}\right\} \times \frac{1}{\mathrm{P}_x + d_5} - \tfrac{1}{2}\mathrm{A}_{xy(5\frac{1}{2})},$$

$$(3)\quad \left\{\mathrm{A}_{xy(6)} - \mathrm{P}_x(1+a_{xy})_{(3\frac{1}{2})}\right\} \times \frac{1}{\mathrm{P}_x + d_{4\frac{1}{2}}} - \tfrac{1}{2}\mathrm{A}_{xy(6)}.$$

The $5\frac{1}{2}$ per-cent values are found approximately, by taking the mean between $a_{xy(5)}$ and $a_{xy(6)}$ and then employing the formula $\mathrm{A} = 1 - d(1+a)$.

Results of Combined Formulas, on different Assumptions as to Rates of Interest.

Age of Reversioner	Age of Life Tenant	Value of Contingent Reversion to £1,000	Value of Reversionary Life Interest of £100 per Annum		
			(1)	(2)	(3)
30	50	240	189·5	175·7	153·9
	60	403	440·3	412·7	407·4
	70	556	698·6	651·0	666·5
40	50	204	89·0	82·2	55·7
	60	371	322·8	302·1	290·8
	70	538	565·8	530·4	537·2
50	60	296	173·0	162·7	145·9
	70	486	390·1	368·8	366·4

We are now in a position to gauge with substantial accuracy the practical effect of using the Combined Formulas. As regards contingent reversions, the results at $5\frac{1}{2}$ per-cent are practically coincident with those of the Investment Formulas at 6 per-cent. The case of reversionary life interests presents several features worthy of notice. Under method (1), which values the reversion at $5\frac{1}{2}$ per-cent and the life interest at $4\frac{1}{2}$ per-cent, the results are in four cases lower and in four cases higher than those by the Investment Formulas: the lower values occurring where the interval of ages is 20 years or less. Under method (2), valuing the reversion as before but the life interest at 5 per-cent, the results are, with one exception, between those given by the Annuity and Investment Formulas; the exception being the case in which there is so long an interval as 40 years between the ages. There the three results are so near as to be practically coincident, in accordance with the well known law that differences of method are minimized in effect by a protracted interval of this kind. Method (3), which assumes 6 per-cent for the reversion and $4\frac{1}{2}$ per-cent for the life interest, gives, except in two cases, results rather lower than those of method (2): the exceptions being the instances in which the life-tenant's age is 70, and that of the reversioner 30 and 40 respectively, and where consequently the life interest is likely to fall into possession soon, and has a high relative value as compared with the reversion.

The second method appears to me to be, under our present financial conditions, the fairest and most practicable, for the following reasons. The immediate transaction is the purchase of

a reversion, and this it is therefore equitable to value at the rate now obtaining for securities of that class. The life interest, on the other hand, being in reversion, may reasonably be so dealt with as to be more remunerative to an investor than a similar security in possession. Hence, even admitting, as we probably should, that well-secured immediate life interests are freely dealt with now at rates ranging from 4 to $4\frac{1}{2}$ per-cent, the 5 per-cent rate in such formulas as we are now discussing appears to be justifiable.

In one feature it will be noticed that all the modifications oı the Combined Formulas agree; namely, a large reduction of the values given by the Investment Formulas for reversionary life interests when the interval between the ages is only 10 years. This appears to me to be one of their most attractive characteristics. It is difficult to see the wisdom of competing at high prices for investments such as these. We are accustomed to say that the great inducement they offer is the life insurance involved; but large life insurances, "watered" by the commutation process which we have been considering, may conceivably, I think, become a bane instead of a boon; the mischief of the process being that it may take place undetected, and not reveal its true character for many years.

If it be urged that there is no logical ground for declining to regard premiums invested at 6 per-cent as capable of commutation accordingly, I reply that the argument goes either too far or not far enough. Logic involves a premium not only commuted, but constructed at 6 per-cent—and would force us relentlessly on until we made these investments upon net formulas. The question therefore is one, not of logical consistency, but of practical policy. At some point we must draw the line between theory and practice. The whole question is, how far shall we go? Rightly or wrongly, my contention is that the limit of prudence is reached when we treat the investment as an investment, and the insurance as an insurance, dealing with each precisely as we should if it came before us separately.

The fact that all proposed increases of values given for contingent reversionary interests are steps towards the employment of net formulas is well illustrated by the ingenious and interesting paper read here last year by Mr. W. B. Paterson (*J.I.A.*, xxx, pp. 536 *et seq.*). It would, in any case, be impossible for the next writer on the subject to leave unnoticed so original and forcible a treatment of it. But apart from this, the line of

argument I have adopted makes it necessary to examine what is in effect a recommendation of an entirely opposite kind.

Mr. Paterson points out that premiums need not be paid in advance, if security exist for an equivalent reversionary charge, and he suggests their postponement accordingly; proceeding, by reasoning which I need not recapitulate, to evolve the formulas: for the value of contingent reversion of 1,

$$\frac{1-d_5(1+a_{xy})_{3\frac{1}{2}}}{1+\dfrac{\mathrm{P}^1_{xy}}{\mathrm{P}_{xy}^{\ 1}}};$$

and for the value of a reversionary life interest of 1,

$$\sqrt{v_5}\times\frac{1-d_5(1+a_{xy})_{3\frac{1}{2}}}{\dfrac{\mathrm{P}_x(1+a_x)}{a_x-a_{xy}}+d_5}.$$

There are other formulas for the latter benefit, but I select the above as corresponding to the Annuity Formula, with which it is compared as regards financial result in the illustrative instances given by Mr. Paterson. On the principle already adopted of avoiding personal allusion, I shall call these the "Postponement Formulas."

The exact nature of the departure from existing methods which the Postponement Formulas involve, is a little obscured by the rates of interest employed in constructing the postponed premiums; and upon this point I should like to take the present opportunity of rectifying a certain misconception. One of the criticisms made upon the formulas, as they affect contingent reversions, was that the assumption of 4 per-cent interest in constructing the postponed premium had an important influence on the increase of value. It was replied that, as a matter of fact this did not make much difference, and that the substitution of 5 per-cent would not materially diminish the result; and Mr. Lidstone in an interesting letter (*Post Magazine*, 1893, p. 293), taking up the test case of a contingent reversion to £1,000, reversioner aged 30 and life-tenant 50, uses 5 per-cent, and brings out a result differing from Mr. Paterson's by £5 only, thus apparently confirming the above contention.

I would point out, however, that this result is obtained by working on $\dfrac{\mathrm{P}^1_{xy}}{\mathrm{P}_{xy}^{\ 1}}$, which is not the formula first given in the paper.

That is $\frac{A^1_{xy}}{A_{xy}^{\ 1}}$, from which the other is afterwards derived by dividing numerator and denominator by $(1+a_{xy})$. Such a process is not possible unless the factor be the same in each case, and therefore, when we start our investigation of the result of replacing 4 per-cent by 5 per-cent from the function $\frac{P^1_{xy}}{P_{xy}^{\ 1}}$, we assume that the latter rate is used in the commutation of the premiums as well as in the purchase of the reversionary charge. Such an assumption seems to me to prevent our perceiving the full effect of the reduction of rate in the enhanced value.

I contend that the difference between the results given by the Annuity and the Postponement Formulas for the value of a contingent reversion is due to two causes:

(1) The employment of 4 per-cent for the annuities and for $P_{xy}^{\ 1}$ in the expression $\frac{P^1_{xy}(1+a_{xy})}{P_{xy}^{\ 1}(1+a_{xy})}$.

(2) The reduction of insurance, and consequent purchase of a part of the reversion by a net formula.

The problem, then, is to analyze the increase of value which the Postponement Formula yields, and ascertain in what proportions these two elements contribute thereto. Let us take the test case above mentioned—a reversion to £1,000 contingent on a life aged 30 surviving one aged 50.

The value given by the Postponement Formula (*J.I.A.*, xxx, p. 543) is	£260
The value given by the Annuity Formula (*J.I.A.*, xxx, p. 543) is	197
Showing a difference of	£63

To see how much of this is due to the rate of interest, let us bring the expression for the postponed premium into line with the Annuity Formula in this respect. As the latter assumes 5 per-cent as the investment rate throughout, and $3\frac{1}{2}$ per-cent as the commutation rate, we shall effect the desired change by writing, for the postponed premium,

$$\frac{P^1_{xy}(1+a_{xy})_{3\frac{1}{2}}}{A_{xy5}^{\ 1}}.$$

This brings out a value of £244 and an insurance of £637. We thus see that the change in the rate of interest accounts for £16 of the increased value instead of £5. Now, as regards the second element of difference, the Annuity Formula requires an insurance of £1,000; the Postponement Formula one of £637. The difference is £363. This the Postponement Formula purchases by the net formula $A_{xy(5)}^{1}$. Its value on this basis is £118. Under the Annuity Formula it would be bought on the same basis as the rest of the reversion, namely, at the rate of £197 per £1,000; and its value would thus be £71. The difference between £118 and £71 is £47; and this, added to the £16 already accounted for, makes £63—the exact difference between the values by the two methods.

This analysis shows us precisely what is done by the Postponement Formula. Apart from the differences arising from the rates of interest (which, as incidental and tending to obscure the issue, had better be disregarded) the reversion is made absolute only to a partial extent; and the balance is bought without any insurance cover, or, in other words, by a net formula.

I might just add that, on trying another case, that of a reversioner aged 40 and a life-tenant aged 60, in which the difference between the values is £39, I find that £15 thereof (or 38 per-cent) is due to the element of interest, and £24 to the reduction of insurance.

A similar analysis of the difference between the two methods in their bearing on reversionary life interests is a more intricate matter, and I present the results of my attempts in that direction with some diffidence.

So far as I am able to resolve into its component elements the process involved in the Postponement Formula:

$$\sqrt{v_5} \times \frac{1-d_5(1+a_{xy})_{3\frac{1}{2}}}{\dfrac{P_x(1+a_{xy})_{4\frac{1}{2}}}{(a_x-a_{xy})_{4\frac{1}{2}}}+d_5},$$

its effect is this:

(1) To purchase that part of the reversionary annuity which represents the excess of the postponed annual premium over the ordinary annual premium by the *net* formula $(a_x-a_{xy})_{4\frac{1}{2}}$.

(2) To purchase that part of the reversionary annuity which represents the ordinary annual premium by

the Annuity Formula with $4\frac{1}{2}$ per-cent instead of $3\frac{1}{2}$ per-cent as the commutation rate; *i.e.*, by the formula $\frac{1}{P_x+d_5}-(1+a_{xy})_{4\frac{1}{2}}$.

(3) To purchase the balance by the usual Annuity Formula $\frac{1}{P_x+d_5}-(1+a_{xy})_{3\frac{1}{2}}$.

An instance will make my meaning clearer. I take the ages of 30 for reversioner and 50 for life-tenant, with a reversionary life interest of £100 per annum. If we refer again to the *Journal*, vol. xxx, we see that the effect of the Postponement Formula, as applied to this benefit, is to give a value of £321, taking a policy for £857 (p. 546) the ordinary annual premium on which would be £2 per-cent, and the equivalent premium on which, postponed till after the expiry of the joint-life status, is £6. 18*s*. 1*d*. per-cent per annum (p. 539).

(1) The excess of the postponed premium over the ordinary premium is £4. 18*s*. 1*d*. per-cent per annum, which on a policy of £857 = £42·03 per annum; and $42{\cdot}03\times(a_x-a_{xy})_{4\frac{1}{2}}$ . . . = 203.7

(2) The ordinary premium is £2 per-cent per annum, which on a policy of £857 = £17·14 per annum; and $17{\cdot}14\times\left\{\frac{1}{P_x+d_5}-(1+a_{xy})_{4\frac{1}{2}}\right\}$ = 49·9

(3) The balance of income is £40·83 and $40{\cdot}83\times\left\{\frac{1}{P_x+d_5}-(1+a_{xy})_{3\frac{1}{2}}\right\}$ . . . . . = 75·2

Giving a total of . . 328·8

This total of £328·8, multiplied by $\sqrt{v_5}$, gives £320·8, which practically agrees with Mr. Paterson's value.

I have applied the same method of analysis to several of the other cases, with similar coincidence of result. It would appear, therefore, to represent correctly the nature of the process.

The important fact elicited by these investigations is, that the Postponement Formulas obtain their larger results by making the purchase, whether of capital or income in reversion, partially upon net formulas without the protection of insurance. We are

asked, in fact, to enter upon a speculative business. When dealing with the contingent reversion, we are to make only a part of it absolute by insurance, and to speculate in the balance. When dealing with the reversionary life interest, we are to forego premiums during the joint lives, and to speculate in a deferred annuity, to begin on the reversioner's death, of equivalent value to the premiums so foregone. In both cases the doctrine hitherto accepted as essential, that contingent reversionary investments be protected by insurance, is abandoned. From this standpoint I think we may fairly claim a reply to the plea by which these proposals were so skilfully and forcibly recommended for acceptance. The assumption underlying our opposition is not the acceptance of any "axiom" that premiums must be paid in advance (for there is no such axiom) any more than the assumption underlying the advocacy of these methods is one of mere postponement. What I think we are entitled to say is that the "postponement" of premiums is the *investment* of premiums, and has no shadow of a claim to be preferentially treated in comparison with other investments of the same class. With this provision we may cheerfully accept the principle, and it would be easy to frame postponement formulas which should involve for the investor no sacrifice of protection or profit, were any practical purpose to be answered thereby. The Postponement Formulas as they stand, however, are of a very different character, and I can see no valid reason for stopping in the progress towards net formulas at the particular point selected. Were the principle implied in the proposed method conceded, it might at no distant day be argued that an investor who is willing to defer any part of his compensation for a contingent advantage may as well so defer the whole; that if £363 of a contingent £1,000 may be purchased without a policy, the remaining £637 is a good investment on the same terms; and so we should arrive at the "sweet simplicity" of A^1_{xy} for contingent reversions, $a_x - a_{xy}$ for reversionary life interests, and I would add (for the principle is precisely identical) a_x for life interests in possession.

It must, in fairness, be remembered that one important consideration may be urged as likely to prevent matters ever being carried to such an extreme; the fact that so long as any life insurance, even of inadequate amount, is called for by the investor, medical selection of the reversioner's life is secured. It is unlikely, however, that this would prove a barrier to the discontinuance of insurance, should that course on other grounds

be advocated. Either such a chance along with the others would be cheerfully thrown upon the broad back of the doctrine of average, or a preliminary medical test, unconnected with the issue of a policy, would be imposed.

It would be foolish not to admit that this class of securities, like all others, must feel the force of a falling rate of interest, and rise in value. Those of us who are much interested in the market for absolute reversions, know well how that force has been felt there. We must make up our minds for the prospect that contingent reversionary interests will not in the future be quite so profitable as they have been in the past. While we may foresee such a tendency, however, and adapt ourselves to it as far as is necessary, we need not voluntarily accelerate or intensify it.

Although such investments must doubtless become *absolutely* less remunerative, there is not the slightest cause why they should recede from the *relative* position which they have hitherto held among our securities; for the reasons which have always, and justly, been held to warrant our looking to them for something higher than our average rate are not in the smallest degree weakened by the changes of the money market, but are likely, owing to complicated titles and possibilities of adverse legislation, to become weightier than ever. The methods of valuation which I have ventured to recommend, appear to give scope for adapting ourselves to changing financial conditions without sacrifice of any security or any reasonable profit.

In concluding this paper, I venture, at the risk of being held guilty of a very abrupt transition, to call attention to a point of law, affecting a large number of contingent reversionary interests, to which reference was made in our discussion of 27 March 1893. The question, whether or not a power of attorney, granted on creation of a base fee by a tenant-in-tail in remainder for the purpose of enabling the grantee, on the life-tenant's death, to enlarge the base fee into a fee simple, is irrevocable and valid, is of the very highest importance to insurance companies, as frequent purchasers of reversions accompanied by such a power. Upon the belief that such powers are valid, large transactions are constantly being entered into, with policies which are adequate for the protection of the security if the belief be well grounded, but far from adequate otherwise. Under these circumstances, the doubt raised by Mr. Burridge during the discussion above referred to (*J.I.A.*, xxx, p. 552) is a matter of the gravest possible moment. If, as he then stated, it becomes on the death of the

life tenant "necessary to produce the reversioner and to complete "the security by obtaining his personal signature to the disentailing "deed", the whole aspect of the case is changed; and not only is the ground cut from under our feet as regards past transactions, but future dealings on the former basis are not prudently practicable.

The valuable letter of Dr. Sprague (*Insurance Record* 1893, p. 184) has probably done much to set at rest the extremely disquieting sensations produced by the speech of Mr. Burridge, and has dealt with the matter so fully as to leave little to be desired.

In view of the exceeding importance of the subject, however, I may perhaps just give in a few words my reasons for believing the view hitherto held to be justified.

The question resolves itself into two parts: as regards transactions carried out prior to 1 January 1883, and as regards those on or after that day (the latter being governed by the Conveyancing Act of 1882).

As regards the former, we have the conveyancing practice according to which in such cases the power of attorney was expressly stated to be irrevocable and granted for valuable consideration. Presumably this would hold good; at least the onus of proof appears to lie upon those who question it. The only argument they allege, so far as I can see, is that the signature of the disentailing deed is a purely personal matter. It is difficult, however, to see how this applies to such a deed more than to any other. The "Fines and Recoveries Act", under which base fees and enlargements are now created, has no provision which would impart a "purely personal" character to disentailing deeds, other than is possessed for instance by a deed to convey a fee simple; and no one would contend that the latter could not be executed by power of attorney.

I happen to know that, in the year 1879, the opinion of eminent counsel was taken upon the point. I mention this because it has a special interest as being prior to the Conveyancing Act of 1882 (to which the opinion cited by Dr. Sprague was subsequent) and as bearing therefore on cases under the old practice, regarding which, rather than later ones, such doubt as may exist appears to arise. It is a most unqualified opinion that a power of attorney for valuable consideration, contained in a deed creating a base fee, to enable a mortgagee or grantee to enlarge it into a fee simple on death of the life-tenant is

irrevocable, and sufficient to enable the attorneys or any of them, during the life of the grantor and after his estate has become an estate in possession, to enlarge the base fee into a fee simple.

Counsel also advised that the power of attorney, in a deed properly framed, would remain valid under a sale by the mortgagee or grantee and could be made available by a purchaser; foreclosure, however, by putting an end to the deed, would also put an end to the covenant to bar the entail, and hence invalidate the power of attorney.

As regards the second class of cases, namely those which come under the provisions of the Conveyancing Act of 1882, it is difficult to conceive it possible that any doubt should exist. Section 8 of that Act is accurately epitomized by its marginal summary, "effect of powers of attorney, for value, made absolutely irrevocable." In three sub-sections, which need not be quoted here as they are readily available and doubtless familiar to most of us, the donee or purchaser is absolutely protected from revocation of the power in any conceivable event.

If in face of such a clause the power of attorney we are now considering could be held inoperative, it would be difficult to say what language means; and no statute, however explicit, could safely be relied upon.

In bringing to a close these observations on contingent reversionary interests, I can only again express the hope that, notwithstanding their somewhat fragmentary character, they may stimulate a helpful interchange of opinion upon a subject of great and growing importance.

Discussion.

The President (Mr. A. Hendriks) having called upon the referees to open the discussion,

Mr. T. G. C. Browne thought the prominence which the question of valuing reversionary transactions had been given in the Institute was somewhat out of proportion to the percentage which the transactions bore to other investments. No doubt the subject was attractive to the actuary from the fact that it exercised his ingenuity, especially in the case of contingent reversions where one was always desirous to reduce the inevitably high charges, but it might also arise from the fact that even in the case of the actuary the gambling element, so common in human nature, had not been altogether eliminated. Mr. Warner's paper divided itself into three parts. Mr. Warner first stated his own views as to how these interests should be valued; in the next part he

criticized the paper recently read by Mr. Paterson; and then, thirdly, he discussed a very interesting legal question in connection with base fees. As regards the second part, he (Mr. Browne) thought that Mr. Warner had finally disposed of Mr. Paterson's ingenious attempt to obtain twenty-one shillings change out of a sovereign. As regards the third question, the legal question, he (Mr. Browne) always felt that it was risky for a body of laymen to discuss such a purely technical legal point. If there was any respectable legal opinion which doubted the efficiency of powers of attorney in the matter of base fees, offices should not lean on them until an authoritative decision was obtained from the House of Lords. It was the opinion of a gentleman who had had a very large experience of such transactions, that they must look upon a lawsuit as tacked on to every reversion. Turning to Mr. Warner's views on the valuation of contingent reversionary interests, as one who had never accepted Dr. Sprague's investment formulas as suitable to the circumstances of the great majority of offices purchasing reversions, he (Mr. Browne) looked upon Mr. Warner's combined formulas as a step in the right direction. His only regret was that Mr. Warner had not reverted altogether to Mr. Jellicoe's formulas. In the discussion on Dr. Sprague's paper in February 1888, he (Mr. Browne) advocated adherence to the annuity formula, but using, instead of the Carlisle Annuity Tables, Government Tables, which discriminated the sexes, and he would now give further reasons for taking that view. When Dr. Sprague first introduced the investment formulas in 1868 he seemed to contemplate the case of a reversionary society, presumably with the greater part of its funds invested in such securities, or that of an office doing a large reversionary business. He said, "Assume then that the reversionary company buys its reversions at prices found from the 6 per-cent single premiums, then if the number is sufficiently large they will on the average fall in at such times as will return to the society the cost, together with 6 per-cent compound interest." Besides the question of the number of the transactions, they had also that of their equality in amount, and that was one of the great difficulties they had to deal with. Mr. Warner had referred to the point as of great importance. Now, his (Mr. Browne's) contention was that the investment formulas of Dr. Sprague assumed a state of things that did not exist in the great majority of offices purchasing reversions. They had not a sufficiently large number on the books, and the inequality of the amount of the reversions destroyed any prospect of an average being obtained. On referring to the last published *Blue Book* he found that the total amount of life interests and reversions purchased of the offices making returns amounted to £3,736,000, which only amounted to $1\frac{3}{4}$ per-cent of the total investments of the insurance companies, thus showing that in the aggregate these transactions could affect the rate of interest very little. It was not possible to find out how much of that sum consisted of life interest purchases, which did not come within the class of investment they were considering. Forty-eight out of 86 offices had some life interests and reversions; 12 offices had more than £100,000 each, and absorbed £2,851,000, or, roughly, three-fourths of the

business; six offices had more than £200,000, and absorbed more than £1,995,000, more than half the total business; 36 offices out of the 48 had therefore less than £100,000 invested in life interests and reversions, and 29 had less than £50,000. Let the case be considered of an office with about £50,000 invested in reversions alone. It meant probably that it had on its books perhaps 20 or 30 cases, and the great majority of those cases would be absolute reversions, and probably it would have from six to a dozen contingent reversionary transactions, varying very largely in amount, from, it might be, a few hundreds to perhaps a charge of £20,000. How, under those circumstances, was an average falling in to be expected which would ensure 6 per-cent on the investment; and was it justifiable, under those circumstances, to commute the half-dozen premiums at 6 per-cent? Mr. Warner had answered the latter question in the negative, but he (Mr. Browne) thought, although the argument might not be equally strong in dealing with interest, it was very strong indeed. It would be very interesting to ascertain the views of offices as to the maximum amount that they were prepared to take by way of a single reversionary charge. He thought they would find the practice differed enormously in that respect. Another argument favourable to Mr. Jellicoe's formulas was that it showed them very distinctly what he might term the mechanism of the transaction; and had those formulas been adhered to Mr. Paterson would not have had the trouble of elaborating his fallacy, nor Mr. Warner that of exposing it.

Mr. A. G. Mackenzie thought Mr. Warner had proved that if Mr. Paterson's proposals were carried out, they would in some measure be granting insurances at cost price, and purchasing reversions at a price which would be costly. As to the merits of the several methods discussed, a great deal must depend upon the individual circumstances of the offices which were entering into the transactions. Mr. Jellicoe's method was certainly appropriate to an office that only entered into those transactions occasionally. In the case of an office, such as Dr. Sprague had mentioned in one of his papers, which actually purchased from another company an annuity to provide the premiums and the interest, they had surely a complete investment method. Whether an annuity was purchased or set up in the books, a certain profit was assumed on the annuity part of the transaction. He understood that Dr. Sprague's method was suggested to him owing to the pressure of competition from reversionary societies, and it might be applicable to offices that entered into those transactions on a wholesale scale; but he agreed with Mr. Warner that the results must be carefully looked at when the ages of the reversioner and life-tenant approximated. The alternate methods which Mr. Warner submitted were well worth consideration. Of course it made a great difference in the results obtained by either Dr. Sprague's or Mr. Jellicoe's formula, or by any of Mr. Warner's methods, whether they altered the rate of interest or the mortality, or introduced other circumstances into the calculation. Mr. Warner had used throughout the Carlisle Table of Mortality, and Mr. Browne had pointed out the important defect that the Carlisle Table did

not separate male and female lives. It was a great testimony to the Carlisle Table that for so many years it had governed the practice of many companies in that matter; but he thought that some little trouble might be taken to investigate the mortality of life-tenants on the books of life offices, separating male and female lives. Such a table would help them in many cases. The Government Annuity Table might be used where they knew the life-tenant was a good life; but the Government annuity standard would be too high for general application, for selection did not enter into those transactions, so far as the life-tenant was concerned, to such a large extent as it did in the case of Government annuities. The rate of interest should undoubtedly depend upon the nature of the security, and also upon the rate which was being earned by the purchasing office upon the investments which it was making at the time. There was a tendency at present toward large offices, which might find it somewhat more difficult in the future to obtain a satisfactory rate of interest than smaller offices, on account of the large amounts coming in and requiring immediate investment. Where an office could only readily find new investments earning $3\frac{1}{2}$ per-cent the extra addition of 1 per-cent would be attractive. He would not put his trust in any one formula for all particular cases. He would not regard a formula as a fetish, but thought that in those matters they must supplement their actuarial knowledge in all cases by as much knowledge as possible of the particular transaction, in every case adding thought to theory and discrimination to doctrine.

Mr. J. E. Faulks said, like Mr. Browne, he divided the paper into three parts but not quite in his way; first, the valuation of contingent reversions; secondly, the valuation of reversionary life interests; and, thirdly, a discussion on the legal point. With regard to the second and third divisions, he would offer one suggestion with reference to Mr. Burridge's remark, on which the latter part of the paper was founded. Conveyancers, he believed, did not insert in those mortgage deeds merely a power of attorney. That power was accompanied by a covenant that the reversioner should himself execute a deed, so that, if the power of attorney was found insufficient, the covenant might be relied upon. Might it not be the case that Mr. Burridge's remark had reference to a case in which there was no power of attorney in the deed, but merely a covenant? Under such circumstances it was perfectly explicable that the reversioner himself would have to be produced to execute the deed, and in such a case, if he did not do so, the Court would probably in the end execute it for him. As regards the valuation of contingent reversions, Mr. Warner's first formula mentioned the commutation rate. He would like to be quite clear what was meant by the commutation rate; was it merely a rate of interest employed for the purpose of the formula, or was it the rate which the purchasing office would employ in every case for the commutation of its annual premiums? He apprehended it was the latter; and in that case the formula reduced to the familiar form $A_{xy_{\prime\prime}}-A'^{1}_{xy_{\prime\prime}}$, a formula which showed the transaction in its simplest light. The reversion was first made absolute by effecting the necessary assurance at a single premium, and then valued as an

ordinary absolute reversion on the joint lives. As regards lending on contingent reversions, some time ago he had enquired at several offices as to their practice, and found that some would not advance upon such reversions by way of ordinary mortgage at all. He doubted whether in arriving at that decision sufficient regard had been paid to the present state of legal procedure. Foreclosure was, he understood, not a formidable matter nowadays; it was granted practically as a matter of course, and without undue expense or delay. Under those circumstances might it not be defensible to advance on contingent reversions by way of ordinary mortgage? In the case of such loans, the premiums on the policy might be either single or annual. The calculations should be made as if the single premium were going to be paid, and the amount advanced fixed with reference to the value so brought out. Then, if an annual premium were selected, the office should be in such a position that if the premium fell into arrear it would at once be able to commute the premium under the policy, and tack the commutation money to the amount of the mortgage. A clause enabling that to be done could be inserted in the mortgage deed, and such a clause had actually been used in practice.

Mr. D. A. BUMSTED thought that Mr. Warner had completely disposed of Mr. Paterson's fallacy. He was rather surprised to hear one of the speakers say that what Mr. Warner called the "Investment Formula" had been invented by Dr. Sprague in 1868 on account of the competition of reversionary companies. He thought that rather hard on reversionary companies. The formula was in use before Dr. Sprague was born, and Mr. Jellicoe introduced his formula with the view to modify the investment formula. The paper under discussion was somewhat illogical. In writing it Mr. Warner seemed to him to have first calculated his values by the three formulas given in his paper, and to have made certain assumptions as to premium and interest which he thought correct. On working them out Mr. Warner found serious differences in the results. His own formula produced results which, as a practical man, he felt he must reject, but he merely set about to adjust his rates of interest so as to bring the values by his formula somewhere between those shown by the other two, and then pointed to his own method as the most practicable. With regard to the question of foreclosure, he (Mr. Bumsted) had had some experience of that not very long ago. One of the speakers had said there was now no delay. In the case he referred to the lawyer's bill was £100, and the foreclosure was delayed for six months, during which interest was accumulating.

Mr. A. H. BAILEY said that it not unfrequently happened that if the tenant-for-life died early the purchases were disputed, and when the matter came before the Court the one question asked was invariably, "What is the market price?" What they had to do was first to watch the market, and govern the rates of interest assumed in their calculations by the market prices. As regards the formulas of Mr. Jellicoe, called the annuity formulas, Mr. Jellicoe wanted to show how the purchase of reversions could become investments suitable for an individual. In old times if an absolute

reversion to consols were offered the general practice was to take consols at 85, and the values were worked out by Davies' Equitable Mortality at 6 per-cent interest. But if they watched the market they would find that the values had risen considerably since those days, and that the rate of interest varied to some extent according to the age of the tenant-for-life. If the tenant-for-life was above the age of 60 he (Mr. Bailey) did not think anything approaching 6 per-cent could now be obtained. If the tenant-for-life was young, say 45, then it was possible it might. Again, something depended upon the amount of the reversion, because the purchaser had to pay the law costs, and the expense of investigating a title was almost as great for a reversion of £100 as for £50,000. He agreed that the formulas in the paper were needlessly complicated. They reduced the contingent reversion to an absolute reversion by making it a reversion expectant on the failure of the joint lives, minus the actual premium charged for insuring against the contingency. So far as he knew, the usual way of carrying out these investments was to enter in the policy register a contingent policy at a single premium for the amount in reversion, to debit the account with the purchase money of the reversion and the single premium. The temptation of the insurance companies to make investments in reversions was that a higher rate of interest could be obtained than for investments suited to individuals. Mr. Browne had referred to the gambling element, but if he (Mr. Bailey) purchased a reversion to £1,000 and paid £500 for it the principal at all events was secured. A reversionary life interest was a much more difficult matter, but Mr. Jellicoe's formula was simpler than Mr. Warner's. The practical objection to it was that very rarely indeed when a reversionary life interest was purchased was an annuity bought during the joint lives, and the objection to the method was that an amount of insurance had to be effected which was not wanted in the early years. What he wanted to emphasize was the necessity of following the market and watching the prices given for these interests in the market.

Mr. A. G. Hemming said Mr. Warner assumed investors would get a much larger rate of interest on reversionary transactions than was actually the case. Again, he assumed that everybody employed the Carlisle Table in valuing reversions. He (Mr. Hemming) did not know whether such was the case, but possibly it was not so. One speaker had suggested that the experience of the lives involved in reversions might be obtained, and would be useful. The present might be an opportune time to get the necessary returns for such a purpose from the offices. Another speaker suggested that the Government Annuity Table, 1883, was not a suitable table to employ in valuing reversions, because there was not the same selection as when an annuity was taken out. But he thought there would certainly be a considerable selection against the office, especially where the difference in the ages of the tenant-for-life and the reversioner was small. Discussing the rate of interest that different offices earned on their funds, it had been argued that offices with large funds could not earn so much as offices with smaller funds. This was not necessarily the case. He had had an opportunity of

seeing a table showing the rate of interest earned from year to year in an office with funds of over 10 millions, and found that instead of decreasing in the last 30 years, with the increase of its funds, it had actually shown an increase. Mr. Warner had said, talking of the investment formula, "Regarded in its relation to the life assurance fund, such a practice involved the sacrifice for the benefit of the insured of the profit made by the company in the excess of its investment over its valuation rate." It seemed to him (Mr. Hemming) that the insured was entitled to his share of the profit made in this way, due precaution being taken to see that he did not get it twice.

Mr. G. F. HARDY was very much interested in Mr. Warner's analysis of the formulas brought before them by Mr. Paterson, which confirmed the conclusion that the high values for contingent reversionary interests given by Mr. Paterson's formulas were mainly due to the handing over to the vendor a great deal of the profit ordinarily attaching to the insurance part of the transaction. With regard to the general question of net values, to the use of which Mr. Browne had objected even in the case of absolute reversions, there was an obvious difference between the purchase of an absolute reversion under those formulas and that of an interest contingent on survivorship, not merely because in the former case they risked the loss of interest only, while in the latter they risked the capital invested, but because in the former case the loss was spread over a large number of years, whereas in the case of a contingent interest the loss fell entirely on the year in which the death took place. It was this sudden loss and the consequent fluctuation in the profits of the company which prevented them buying annuities and contingent interests uncovered by insurances. With respect to the use of what Mr. Warner had called investment formulas as applied to contingent reversions, they had already recognized the principles Mr. Warner advocated in the higher rate of interest they employed in purchasing such reversions. If in buying a contingent reversion made absolute by a contingent policy at annual premiums, they employed throughout a rate of $5\frac{1}{2}$ or 6 per-cent, they were getting a higher rate of interest than they would think of asking if they were buying an ordinary absolute reversion. This was a recognition of the fact that having to pay a premium for a large number of years they might be out of pocket in the matter of capital as well as that of interest. At the same time, Mr. Warner's suggestion met this feature of the case in a somewhat more scientific way. As to Mr. Browne's objections to net formulas for absolute reversions, they must remember that the market price of those investments was fixed by the large dealers, and offices which had not hitherto invested in those transactions, or had only done so to a small extent, must be content to take what business they could get on the old formulas, or must be prepared to risk something in individual cases until they got a sufficient number to make an average. In considering the question of an average of these transactions an important point seemed to him that the risk attaching to the purchase of an absolute reversion was exactly similar in kind to that which attached to the issue of every policy which the office might send out. In the one case a premature death

meant a profit, in the other a loss, so that even where the office had only a small number of absolute reversions on its books it was still justified in buying those reversions by the net formulas if it thought fit. In reference to the table to be used, the Carlisle Table was not stringent enough for female life-tenants, and he saw no objection to adopting in these cases the Government Annuity Female Table. There was, no doubt, some selection against the purchaser, and whatever selection there had been in the past it was likely to be increased in the future by the facilities now given for redeeming charges during a certain number of years. That gave a very considerable option to the owner of reversions which must have an effect on the rate of mortality, and which pointed to the use of the Government female table where the life-tenant was a female if these transactions were to continue to be profitable.

The President (Mr. A. Hendriks), in proposing the usual vote of thanks, agreed with Mr. Bailey that they must necessarily be very much guided by the market value of the transactions; but there was a certain amount of elasticity in each of these formulas, because he did not believe that Mr. Jellicoe or Dr. Sprague thought that the rate of interest on the mortality table to be employed would not vary with circumstances. They all knew that mortality had altered, and the rate of interest had altered, and the application of all formulas must alter according to the times in which they lived. There was no doubt a great deal of freshness in the ideas propounded that evening. At the same time he doubted whether anything that had been said was entirely without precedent. Take, for example, the single premium process. Whatever formula he adopted he invariably checked it by a calculation of that nature, because it always seemed to him that the real value of a contingent reversion consisted in first finding what the value would be if it were an absolute reversion, and then deducting from it anything which showed to the contrary, making it contingent, so that it might come upon the same basis as if it were absolute. The paper had given rise to a valuable discussion which he (the President) thought was a step towards the desideratum of an agreement as to the formula for common adoption. He did not say they ought not to compete, for they must necessarily compete, but that in their ideas as to the value at which reversions, and particularly contingent reversions, should be placed, they should not differ so widely as they had done up to the present.

Mr. Warner, in reply, said with regard to the composition of his two tables, alluded to by Mr. Bumsted, he (Mr. Warner) felt that a few words of explanation were necessary. In starting his investigation he took, as the classical deliverance on the subject, Dr. Sprague's paper of 1868 (*J.I.A.*, xiv, 417), and he based his results largely upon the tables which Dr. Sprague there gives. Dr. Sprague employed the Carlisle Tables throughout, and he (Mr. Warner) did so too, without expressing any opinion as to their suitability for female lives. The whole purpose of his paper was a comparison of results, and the Carlisle Table presented a mass of readily available joint-life values worked out. It was, therefore, the most convenient method to employ for purposes of comparison. As

regards the first table, he reproduced the figures given by Dr. Sprague as the results of his own formula and that of Mr. Jellicoe, with the sole exception of correcting the latter by the deduction of $\frac{1}{2}$ A_{xy}. When these were compared with the results by the combined formulas, it was seen that the latter gave in some cases smaller results than the annuity formulas. It was clear that the new results ought to lie between the two others, the suggested method being in fact a compromise between the existing ones. That this was not the effect shown by Table I was due to an accident (the use of 5 per-cent in the annuity and 6 per-cent in the investment formulas), and to rectify that and to show more closely what, in accordance with modern business conditions the results of the combined formulas would be, he framed his second table. He admitted that the first table had little except an academic value. He would direct attention rather to the second. As regards the investment formulas, he could not follow some of the objections urged against them. To his mind the two valid objections were those he had urged, the fact that they disposed of a part of the profit from surplus interest in the wrong quarter, and the fact that reversionary life interests generally involved amounts so large as rather to upset averages. But as regards Mr. Bailey's remarks they could do something a little more decided than either merely to "watch" or "follow" the market. They could do something to influence and to create it. To a large extent they themselves constituted the market; and by having definite views as to the profit that should be yielded to insurance companies by securities of the class in question, and acting upon these views, they might, besides watching and following the market, do much to influence it.

ACTUARIAL NOTE.

On the Treatment of Incomplete Years of Exposure in a Mortality Experience derived from Records of Assured Lives. By George J. Lidstone, F.I.A., *Assistant Actuary of the Alliance Assurance Company.*

In dealing with a mortality experience derived from the records of a life office, or other similar source, it is necessary to make some assumption with regard to the fractions of a year's exposure which are introduced in consequence of new entrants or withdrawals coming under or passing from observation during the currency of a year of risk.

The common assumption which is made in such cases—either rigidly as in the exact duration method and Mr. Meikle's life-year method, or by a process of average or approximation—is that the

total of such fractions of a year's exposure may be taken as representing the same number of lives at risk for a whole year. Thus if w lives are at risk for $\frac{1}{n}$th year, they are treated as equivalent to $\frac{w}{n}$ lives at risk for a complete year.

It has recently been pointed out that this plan introduces an error in the rate of mortality as deduced from the observations, and in the following note an attempt is made to determine mathematically the direction and magnitude of this error.

If we have l_x persons under observation at age x, and w withdraw at age $x+k$ $(k<1)$, the total number of deaths observed during the year of age x to $x+1$ will be

$$l_x - l_{x+k} \qquad \text{during the interval } k,$$

and $$(l_{x+k}-w)\frac{l_{x+k}-l_{x+1}}{l_{x+k}} \qquad \text{during the remainder of the year,}$$

giving a total of $$l_x - l_{x+1} - w\frac{l_{x+k}-l_{x+1}}{l_{x+k}}$$

$$= d_x - w\mathrm{C}, \quad \text{say.}$$

The number "exposed to risk" will be, by the method under consideration,

$$\mathrm{E}_x = l_x - w(1-k),$$

and the deduced rate of mortality ($=q'_x$, say) will be

$$q'_x = \frac{d_x - w\mathrm{C}}{l_x - w(1-k)}$$

$$= \frac{d_x}{l_x} - \frac{w\left(\mathrm{C} - \overline{1-k}\,\frac{d_x}{l_x}\right)}{l_x - w(1-k)}$$

$$= q_x - \frac{w}{\mathrm{E}_x}(\mathrm{C} - \overline{1-k}\,q_x),$$

where q_x is the *true* rate of mortality.

We have, then, to investigate the sign and value of $\mathrm{C}-(1-k)q_x$. Now we have

$$\mu_x = -\frac{1}{l_x}\frac{d}{dx}l_x$$

$$= -\frac{d}{dx}\log_e l_x,$$

whence $$\int \mu_x dx = -\log_e l_x + \text{constant},$$

and therefore $$-\int_k^1 \mu_{x+t}\,.\,dt = \log_e l_{x+1} - \log_e l_{x+k}$$
$$= \log_e \frac{l_{x+1}}{l_{x+k}}.$$

Therefore $$\frac{C}{q_x} = \frac{1-\dfrac{l_{x+1}}{l_{x+k}}}{1-\dfrac{l_{x+1}}{l_x}} = \frac{1-e^{-\int_k^1 \mu_{x+t}\cdot dt}}{1-e^{-\int_0^1 \mu_{x+t}\cdot dt}}.$$

This is of the form $\dfrac{1-e^{-X}}{1-e^{-Y}}$, which is $> = < \dfrac{X}{Y}$ as $\dfrac{X}{Y} < = > 1$.*

Now so long as μ is either constant or increasing, and k lies between 0 and 1, $\int_k^1 \mu_{x+t}\,.\,dt$ is obviously $< \int_0^1 \mu_{x+t}\,.\,dt$, but not $< (1-k)\int_0^1 \mu_{x+t}\,.\,dt$†, *i.e.*,

$$X < Y \text{ but not } < (1-k)Y$$

or $$\frac{X}{Y} < 1 \quad \text{,,} \quad < (1-k).$$

Now it has been shown above that if $\dfrac{X}{Y} < 1$, $\dfrac{1-e^{-X}}{1-e^{-Y}}$, which is equal to $\dfrac{C}{q_x}$, must be greater than $\dfrac{X}{Y}$, and it must therefore be

* This may be shown as follows:

$$\frac{1-e^{-X}}{1-e^{-Y}} = \frac{e^{X+Y}-e^{Y}}{e^{X+Y}-e^{X}} = \frac{X+Y-Y+\frac{1}{2}\{(X+Y^2)-Y^2\}+\ldots}{X+Y-X+\frac{1}{2}\{(X+Y^2)-X^2\}+\ldots}$$
$$= \frac{X+Y-Y}{X+Y-X} \times \frac{1+\frac{1}{2}(X+Y+Y)+\ldots}{1+\frac{1}{2}(X+Y+X)+\ldots}$$
$$= \qquad \frac{X}{Y} \qquad \times \frac{1+\frac{1}{2}(X+Y+Y)+\ldots}{1+\frac{1}{2}(X+Y+X)+\ldots}$$

where the second fraction is $> = < 1$ as $X < = > Y$, *i.e.*, as $\dfrac{X}{Y} < = > 1$.

† This will appear from a consideration of the fact that $\int_k^1 \mu_{x+t}.dt$ is $(1-k)$ times the average value of μ between the limits k and 1, while $(1-k)\int_0^1 \mu_{x+t}.dt$ is $(1-k)$ times the average value between the points 0 and 1. If μ is never decreasing the first average value can never be less than the second.

greater than $1-k$: consequently $C-\overline{1-k}q_x$ is positive, so long as μ does not at any point between x and $x+1$ decrease. The inequality becomes an equality, and the correction zero, when $k=0$ or 1.

The common assumption of an uniform distribution of deaths involves an increasing force of mortality, and we thus see that, on the hypothesis of either a constant value of μ or an uniform distribution of deaths during the year of life under observation, the error will be in the direction of under-rating the mortality.

We proceed to deduce an approximate expression for the amount of the error.

We have, by Taylor's theorem,

$$\mu_{x+t}=\mu_x+t\mu'_x+\frac{t^2}{2}\mu''_x \text{ nearly,}$$

where μ', μ'' . . . represent the successive differential coefficients of μ.

Hence $$\int\mu_{x+t}.dt=t\mu_x+\frac{t^2}{2}\mu'_x+\frac{t^3}{6}\mu''_x$$

and $$\int_k^1\mu_{x+t}.dt=(1-k)\mu_x+\frac{1-k^2}{2}\mu'_x+\frac{1-k^3}{6}\mu''_x.$$

Thus, C or $1-e^{-\int_k^1\mu_{x+t}.dt}$ will be equal to $1-e^{-(1-k)\mu_x}\cdots$

Expanding by the Exponential Theorem, we thus find

$$C=(1-k)\mu_x+\frac{1-k^2}{2}\mu'_x+\frac{1-k^3}{6}\mu''_x-\frac{\overline{1-k}^2\mu^2_x+(1-k)(1-k^2)\mu_x\mu'_x}{2}$$

Putting $k=0$, we have

$$1-e^{-\int_0^1\mu_{x+t}.dt}=q_x=\mu_x+\tfrac{1}{2}\mu'_x+\tfrac{1}{6}\mu''_x-\frac{\mu^2_x+\mu_x\mu'_x}{2}.$$

Hence we find, after reduction, that

$$C-(1-k)q_x=\frac{k-k^2}{2}(\mu'_x+\mu^2_x)+\frac{k-k^3}{6}\mu''_x+\frac{k^2-k^3}{2}\mu_x\mu'_x,$$

and finally the error in the rate of mortality will be the last quantity multiplied by $\frac{w}{E_x}$.

The coefficients are all zero when $k=0$ or 1, and their maximum values when k has a value between these limits are

found, by the methods of the Differential Calculus, to be as shown in the following table:

Coefficient	Value when $k=\frac{1}{2}$	Maximum Value of Coefficient	Corresponding Value of k
$\frac{k-k^2}{2}$	·125	·125	·500
$\frac{k-k^3}{6}$	·062	·064	·577 $=\frac{1}{\sqrt{3}}$
$\frac{k^2-k^3}{2}$	·062	·074	·667

The coefficient of the first and relatively most important term thus has its maximum value when $k=\frac{1}{2}$, and the values of the other coefficients when $k=\frac{1}{2}$ differ but slightly from their maximum values. We may therefore assume, for practical purposes, that the whole expression is a maximum when $k=\frac{1}{2}$.

The formula given above relates only to discontinuances at a given point of time, but from the manner in which it was derived it is evident that the resultant effect of the discontinuances in any given year of exposure will be the sum of the errors respectively introduced by the withdrawals at various points, so that we have for the total error

$$\frac{1}{\mathrm{E}_x}\left\{\Sigma w_{x+k}\left(\frac{k-k^2}{2}(\mu'_x+\mu^2_x)+\ldots\right)\right\},$$

which will necessarily be less than the total rate of discontinuances multiplied by the maximum error, *i.e.*,

$$\text{less than } \frac{\frac{1}{8}\Sigma w}{\mathrm{E}_x}(\mu'_x+\mu^2_x+\tfrac{1}{2}\mu''_x+\tfrac{1}{2}\mu'_x\mu_x).$$

An examination of the results of this formula seems to show that, in the case of an average or combined table such as the H^{M} or the $\mathrm{H}^{\mathrm{M}(5)}$ Table, the amount of the error is not of much practical significance. At the younger ages, where the ratio $\frac{\mathrm{E}_x}{\Sigma w\frac{1}{8}}$ will have its highest value (say about $1\frac{1}{2}$ per-cent), it will be multiplied by a very small quantity, since the force of mortality is small, and increasing comparatively slowly. At the higher ages the rate of withdrawal becomes very small, and the

higher values of the mortality function are thus counteracted. In the case of an analyzed or "select" mortality table, the same remarks will apply to the later years of assurance, but during the early years of assurance the error will be found to have a comparatively high value, since the maximum rate of withdrawal is coincident with a rapidly increasing force of mortality. For example, according to Mr. Sprague's Select Tables, age at entry 60, the force of mortality for the first few years appears to be increasing at the rate of about 1 per-cent per annum, so that if the rate of discontinuance be as much as 10 per-cent per annum, the maximum error will be about $\cdot 01 \times \cdot 010 \times \cdot 125 = \cdot 00012$. In this connection, it is interesting to notice that the error will be considerably less when the experience follows policy years than in the case of a calendar-year or life-year method, since under the first plan a large proportion of the withdrawals will take place at the end of a year of observation, and may consequently be neglected for our present purpose, the error in such cases being zero.

The formulas given above will evidently be made applicable to *new entrants* instead of withdrawals by the simple process of putting $-n_{x+k}$ for w_{x+k} (where n_{x+k} is the number of new entrants coming under observation at the expiration of an interval k from the commencement of the year of observation), provided that these new entrants are of the average age $x+k$. The error due to new entrants will therefore be in the direction of *over*-estimating the rate of mortality.

The Treatment of the Discontinuances in the New Mortality Experience.

[Abstract of Correspondence in the Insurance Press.]

Mr. G. H. Ryan wrote (16 February 1894, *Insurance Record*): "The question in dispute resolves itself into the relative merits of using the curtate or nearest duration in classifying the discontinuances according to policy-year of exit. The principal arguments in favour of the nearest duration method are the saving of labour and the avoidance of fractions, both of which advantages it certainly possesses. But if it can be shown that the curtate or, as it has been termed, the exact duration method follows the facts with greater fidelity, I do not think too much importance should be paid to what are, after all, mere matters of expense.

"Assuming for the moment that the vastly superior method of policy-years will be adopted by the Institute and Faculty of Actuaries, there are two points of difference in the rival methods—namely, the effect upon, first, the discontinuances themselves in any given policy-year (or the numerator of the expression representing the rate of discontinuance), and, secondly, the exposed to risk (or the denominator in that expression). The latter point is, I think, of comparatively little importance, though even here the displacement of the discontinuances during, say, the first five years of insurance might influence the exposed to risk and disturb the rates of mortality. But the correct registration of the discontinuances in their true policy-years is a very different matter. Everyone must admit that to arrange the deaths otherwise than according to their true policy-years would be inaccurate and would probably lead to confusing results, and yet the arguments are the same in the one case as in the other. During the first five policy-years the discontinuances are, moreover, very numerous, and ought to be treated with consideration. Within this period, the Connecticut Mutual Experience shows that 34,717 discontinuances occurred out of a total of 44,640. Now, tracing the effect of the exact duration and nearest duration methods on the allocation of the discontinuances in policy-years, the following statement will be useful:

Policy-year	Discontinuances actually occurring in these years will be placed by Exact Duration Method	Nearest Duration Method	
1	in policy-year 1	some in policy-year 1	others left out altogether
2	,, ,, 2	,, ,, 1	,, in policy-year 2
3	,, ,, 3	,, ,, 2	,, ,, ,, 3
4	,, ,, 4	,, ,, 3	,, ,, ,, 4
⋮	⋮	⋮	⋮

and so on.

"Here there is obviously no true average or balance of errors in the grouping, having regard to strict policy-years. The discontinuances are to some extent thrown back, and the error is *always in the same direction*. Nor do we even get the force of discontinuance by the nearest duration method, as, I think, Mr. King suggested, for the exposed to risk, which vary rapidly in the early policy-years, would require to be modified before that result could be obtained.

"From this chain of argument, I conclude that the nearest duration method does not give us what we want, and that its simplicity and convenience would be dearly purchased at the loss of accuracy, certainly in the case of the rate of discontinuance, possibly also in the case of the rate of mortality, during the early years of insurance."

Mr. J. Chatham wrote (16 February 1894, *Insurance Record*): "The only satisfactory method of treatment is to allocate the deaths and discontinuances to the policy-year in which they occur. The discontinued may be tabulated as if they passed out of observation at the end of the policy-year in which they discontinue, and the time between the actual date of discontinuance and the end of the policy-year may be calculated to the nearest

month, or to two or more decimal places, and the sum inserted to the nearest year and deducted from the number entering on the year so as to get the exposed to risk. This time, of course, will only require to be calculated in a proportion of the discontinued, as a number of them withdraw at the end of the policy-year. In the case of a policy on which one annual premium has been paid, it would be deducted at the end of the first policy-year, and therefore be reckoned in the discontinuances for that year. It is questionable whether it is worth while to take the days of grace into account; they may be set against the time which elapses between the date of the policy and the actual payment of the premium, which puts the office on the risk. The only valid objection taken to this exact duration method was the amount of labour which it will entail; but this has been much exaggerated, due no doubt to the table in which Dr. Sprague presented his results by that method. It must be borne in mind, however, that he was dealing with a comparatively small experience, which was terminated at a fixed period of time. Only two columns more than in the case of the nearest duration would be required to obtain the exact duration. This would give not only the exact rate of discontinuance, the importance of which cannot be over-estimated, but it would enable us to answer other questions. For instance, I do not think it is possible without it to determine accurately the rate of mortality. The rate of mortality goes up by leaps and bounds in the early years of insurance, and again at advanced ages; and if a considerable number withdraw before the end of the policy-year, it is clear that the rate of mortality may be materially affected. A larger number will be exposed to risk at the beginning of the policy-year when the mortality is light than at the end of it when the mortality is heavy. This is leaving out of account the question whether those who discontinue are better or worse than those who remain."

Dr. T. B. Sprague wrote (2 March 1894, *Insurance Record*): "I object to the phrase 'curtate duration' of a policy, used by Mr. Ryan. We all know what is meant by the curtate expectation of life, and we understand that it is half a year less than the true expectation; but Mr. Ryan seems to use his phrase as meaning the same thing as the exact (or true) duration of the policy; and such a use of the word curtate is not only inconvenient but clearly incorrect.

"I next observe that Mr. Ryan speaks of the discontinuances *in* any policy-year, and advocates 'the correct registration of the discontinuances *in* their true policy-years.' If the discontinuances, like the deaths, occurred at all periods of the policy-year, and were distributed pretty uniformly over that year, Mr. Ryan's argument would be conclusive; but I think he has not given sufficient weight to the fact that the discontinuances mostly occur at the end of the policy-year. Taking the ordinary case, when the premium is payable yearly, the lapses necessarily occur at the end of the year; and the surrenders will, in the great majority of cases, occur when a premium is about to fall due, that is to say, shortly before the close of the year. This being the case, our object is to ascertain the rate of

discontinuance, not *in* each policy-year, but *at the end of* each; and this consideration seems to me to invalidate Mr. Ryan's comparison of the effects of the exact duration and the nearest duration methods 'on the allocation of the discontinuances.' When Mr. Ryan remarks that the nearest duration method gives no true average or balance of errors, the error being 'always in the same direction', I think he cannot have examined the effect of that method with sufficient care. Taking the case of quarterly premiums, if three quarterly premiums have been paid on one policy and five on another, each policy would, according to the nearest duration method, be treated as if it had been in force for exactly a year. If six quarterly premiums had been paid on each of two policies, they would be treated as if the one had been in force for one year and the other for two years. In these and similar cases there can be no doubt that the method I advocate does give us a true average, and the errors in one direction fairly balance the errors in the other direction. The lapses in the first policy-year, after payment of premium for three or six months only, will, of course, require special treatment.

"As I suggested before, I may not have correctly understood Mr. Ryan, and I therefore invite him to give figures showing how 'the displacement of the discontinuances during, say, the first five years of insurance, might' (as he says) 'influence the exposed to risk, and disturb the rates of mortality.' In the experience of the Connecticut Mutual Office mentioned by Mr. Ryan I find that in the first insurance year there were 95,216 policies on male lives entered as 'exposed less died', and of these 14,070 are set down as discontinued, being 14·78 per-cent; and the corresponding percentages in the following years are 9·72, 8·19, 7·31, 5·69, 4·59, gradually decreasing down to ·30 for the twenty-third year; and from the description given at page 2 of the volume, all these discontinuances either actually took place or are held to have taken place at the ends of the respective years. What sort of error does Mr. Ryan suppose was introduced by the consistent use in this case of the nearest duration method?"

Mr. J. Chatham wrote (10 March 1894, *Post Magazine*): "In the exact duration method the discontinuances are tabulated in the policy-year in which they occur. In the nearest duration the discontinuances in the first half of any policy-year are relegated to the previous policy-year, and the discontinuances in the first half of the next policy-year are brought in to replace them. For instance, the discontinuances in the tenth year are made up of policies discontinued in the second half of the tenth and the first half of the eleventh. A difference, however, is made in the case of those who discontinue exactly at the middle of the policy-year, one-half of them only being relegated to the previous year. According to this method, therefore, the discontinuances for the first policy-year will consist of policies discontinued between 0 and $1\frac{1}{2}$ years—that is to say, for a period of 18 months—unless some adjustment is made; for the second policy-year, of policies discontinued between $1\frac{1}{2}$ and $2\frac{1}{2}$ years; and so on. Now, if the number of discontinuances in the first half of

every policy-year were the same, the nearest duration method would, after the first year, give correct results. To obtain this, it is obvious that the rate of discontinuance would require to increase, so that the smaller numbers at risk would yield the same number of discontinuances; but the opposite is the case in practice. The following figures, extracted from my essay, show the percentage in the Institute experience for the first 10 years of insurance:

Year	Percentage	Year	Percentage
0	2·7	6	2·4
1	6·8	7	3·4
2	4·9	8	1·9
3	4·1	9	1·5
4	3·2	10	1·5
5	2·8		

"I think these figures show that the nearest duration method will give us erroneous ideas of the rate of discontinuance, especially when half-yearly and quarterly policies are kept in view.

"The nearest duration method, again, prevents the rate of mortality from being determined with the accuracy attainable by the other method. A number will withdraw at the end of three months in any policy-year, a larger number at the end of six months, and a number again at the end of nine months, and, in addition, there will be a number between these periods. If, therefore, the rate of mortality is increasing rapidly, as in the early years of insurance and again at advanced ages, it is clear that the rate deduced from the unadjusted facts will be too small—in other words, if no withdrawals had taken place until the end of the year, the rate would have been heavier. By the exact duration method it will be possible to apply a correction which it would be impossible to do by the nearest duration.

"I do not think that the amount of labour involved in the two methods need have much weight, because there is not much difference between them in that respect. I speak from a practical acquaintance with the subject. During recent years I have made four mortality investigations, in two of which I used the nearest duration method, and in the other two the exact duration; and the extra labour involved in the latter when properly applied is so small as to be hardly worth while considering. It does, however, occupy more space, as it requires in the case of whole-life policies two extra columns."

Mr. G. H. Ryan wrote (16 March 1894, *Insurance Record*): "Dr. Sprague's admissions are, I venture to think, more significant than his criticisms. Let me remark, first of all, his statement in regard to my advocacy of the correct registration of the discontinuances in their true policy-years: 'If the discontinuances, like the deaths, occurred at all periods of the policy-year, and were distributed pretty uniformly over that year, Mr. Ryan's argument would be conclusive; but', &c. Then, secondly, his statement that 'the lapses in the first policy-year, after payment of premiums for three or six months only, will, of course, require special treatment.' These points should be kept steadily in mind:

their importance is not to be denied, as they confine discussion to narrower limits than it ranged over before.

"In connection with the disputed matters of phraseology, the terms employed by me bear their own interpretation to all who have studied Mr. Whittall's recent paper. The essential point of principle that I would strongly uphold is that the method of tracing our observations throughout policy-years should be strictly adhered to, both in regard to deaths and discontinuances. Let us follow the facts as closely as we can, and suffer them to convey their own lessons. I am, of course, fully aware that the discontinuances are not spread uniformly over the policy-year, and that is a circumstance that will have to be borne in mind when the resulting rates of discontinuance, derived by rigid adherence to the policy-year system, come to be employed for any purpose. But by such means we should at least obtain a function precisely similar in form to our rates of mortality, whereas by Dr. Sprague's proposed method of nearest durations we should have a function at present unknown to our system of notation. It is interesting to note that we now for the first time, from any speaker or writer on the subject, hear of the modification of the nearest duration method in the first policy-year, which Dr. Sprague assumes to be necessary. In his own recent paper, Dr. Sprague, so far from laying down the condition that the early discontinuances of the first year require to be specially dealt with, explains that he omitted them altogether. His words are: 'In the case of the existing, if $n+\delta$ is the time for which a policy has been in force, where n is the number of complete years and δ a fraction of a year, the duration has been taken as n, if $\delta<\frac{1}{2}$; and as $n+1$, if $\delta>\frac{1}{2}$; while, if $\delta=\frac{1}{2}$, the duration has been taken as n and $n+1$ alternately. *When a policy has been in force less than a year, so that $n=0$, precisely the same rule has been followed. A similar course of procedure was adopted in the case of the matured and the withdrawn.*' I have italicized the important words in this extract. The nearest duration method, if carried out strictly, would obviously leave out of account altogether certain discontinuances; but to avoid this difficulty Dr. Sprague now suggests it should be modified—in what manner he does not, however, state. Hence, by adopting his plan we not only modify the system of policy-years (which, by almost general agreement, is the best basis to employ in regard to the deaths), but that plan itself has to be modified as regards the observations in the first policy-year. I do not recognize the necessity of such a mode of procedure, nor can I agree that what we wish 'to ascertain is the rate of discontinuance at the end of each policy-year.' Not only would quarterly and half-yearly premium policies render such a result of doubtful significance, but the modern practice of extending the days of grace, either by payment of a fine or application of surrender-value, and the incidence of surrenders themselves, would tend to invalidate any conclusions to be drawn therefrom.

"Unfortunately I cannot comply with Dr. Sprague's request to produce figures showing how 'the displacement of the discontinuances in the first five years of insurance might influence the exposed to risk, and disturb the rates of mortality', for the necessary material is not

at hand. It could only be seen whether such a conjecture were well founded by arranging the rough observations according to the rival systems and then comparing the results. To some extent I based my opinion on the inference that the nearest duration method involved the exclusion of the discontinuances on some quarterly and half-yearly cases in the first year of insurance, which, according to Dr. Sprague's latest proposals, would not be correct. But in addition to this, seeing that the discontinuances in the early years of insurance form a rapidly-diminishing series I doubted whether the number of cases in which $\delta < \frac{1}{2}$ would be equal to the number of those in which $\delta > \frac{1}{2}$. Unless that were so, the exposed to risk would not be correctly deduced, and the rate of mortality would be affected."

REVIEW.

*Theories of Probability.**

The recently published paper of Dr. Sprague, *Probability and Chance*, read before the Actuarial Society, Edinburgh, in December, 1892, and the paper previously read before the same body by Professor Chrystal, of which it appears to have been the outcome, will be found of equal interest and importance to actuarial students, and seem to call for some notice in the *Journal*, more especially as they differ upon certain fundamental principles.

Professor Chrystal's essay is mainly concerned with the problem of inverse probabilities, so-called, as treated by Laplace and those who have followed him; but an important part is his discussion of the limits proper to the subject, and the definition to be given to the term "probability", both of which points are also discussed very fully by Dr. Sprague.

Laplace's view, as quoted by Professor Chrystal from the introduction to the *Théorie Analytique des Probabilités*, is as follows:

"The curve described by a simple molecule of air or of vapour is regulated in a manner as certain as the planetary orbits; there is no difference between the two cases save our ignorance.

"Probability relates in part to this ignorance, in part to our knowledge. We know that of three or a greater number of events, a single one must happen, but nothing leads us to believe that one of them will arrive rather than the others. *In this state of indecision* it is impossible for us to pronounce with certainty regarding the occurrence of the events. It is nevertheless probable that one of these events taken at pleasure will not happen; because we see several cases equally possible which exclude its appearance, while only one favours it.

"The theory of chances consists in reducing all events of the same kind (*du même genre*) to a certain number of equally possible cases, that is to say, such that we are equally undecided regarding their existence, and to determine the number of cases favourable to the event whose probability we seek. The ratio of

*1. *On some Fundamental Principles in the Theory of Probability:* by George Chrystal, M.A., LL.D. 1891.

2. *On Probability and Chance, and their connection with the business of Insurance:* by T. B. Sprague, M.A., LL.D., F.R.S.E., &c. 1894.

this number to the number of all possible cases is the measure of this probability, which is thus nothing but a fraction whose numerator is the number of favourable cases, and whose denominator is the number of all possible cases."

"There is", says Professor Chrystal, "in Laplace's view, a "confusion between two senses of the word 'Probability', which "although distinct are often more or less associated in point of fact. "In common speech we say that a single event is more or less "'probable', and by this word we indicate our own mental attitude "towards the event, an attitude that may be well or ill justified by "facts. When an actuary says that the probability that a man of "20 will live to be 60 is $\frac{59}{97}$, he is not, strictly speaking, *referring to* "*any one event at all*, but merely making an assertion to the effect "that out of any considerable number of men of 20 years of age "about $\frac{59}{97}$ will reach the age of 60. No one knows better than an "actuary that this statement *is a fact, established* (under certain "circumstances, and with certain limitations), *by experience*, and that "it has *nothing whatever to do with the mental attitude of anyone.* "Everyone will admit that we could never arrive at this result by "analyzing the event of a man of 20 reaching or not reaching the "age of 60 into cases regarding each of which we should be equally "undecided,—mentally suspended, as it were, like Buridan's ass "between the equal bundles of hay."

Taken literally in such a connection, Laplace's statement, no doubt, makes nonsense. His obscurity, however, is almost proverbial, and it is probable that he meant, not to explain how the probability of a given event might be deduced in practice; but how we can best represent to our mind an assigned probability such, *e.g.*, as $\frac{59}{97}$. The meaning of this fraction as measuring the probability Laplace would express by assuming 97 equally possible cases, of which 59 were favourable to the event.

Moreover, in the case supposed, the mental attitude of the actuary would certainly be involved in practice as is sufficiently shown by the fact that different actuaries would give different values to this probability according to the "circumstances and limitations" which they might suppose to attach to the particular case; in other words, whatever definition we may attach to the word "probability", it is clear that this particular probability cannot have a precisely determined value, but, as in all similar cases, is the subject of estimate only.

Both authors call attention to the objections to many of the current definitions of probability. Professor Chrystal's view, following the lines laid down in Mr. Venn's *Logic of Chance*, is as follows: "The notion of probability is always attached to a class "or series of events, which usually have more or less of other "attributes in common, but are always distinguished by this mark, "that certain phases of them, although not predicable with the "smallest certainty in any individual case, are predicable with more "or less uniformity in a certain proportion of cases in the long run. "The fundamental features of this series are statistical uniformity

"combined with irregularity of every conceivable kind in the "individual instance. The number of the events in the series must "be large. Its extension both as to space and time is arbitrary, "and in certain ideal cases infinite. It is in this last respect alone "that probability has anything to do with our mental attitude; we "may choose our standpoint, and this determines the probability to "which our knowledge may make a better or a worse approximation. "As the series is varied the probability alters. . . . We are thus led "to the following abstract definition of the *probability or chance of* "*an event.* If, on taking any very large number, N out of a series "of cases in which an event A is in question, A happens on pN "occasions, the probability of the event A is said to be p."

This definition Dr. Sprague objects to among other reasons as precluding the use of the word "probability", except in cases where the actual value of p had been determined from experience. As the professor himself admits, "there are some cases where the circum-"stances are so simple that the probability of the event can be "deduced without elaborate collecting and sifting of observations "merely from our definition of the circumstances under which the "event is to take place." A further objection is, that, of the two "fundamental features" of the series above mentioned, namely, "statistical uniformity" and "irregularity of every conceivable kind in the individual instance", the former only is embodied in the definition, which does not therefore exclude series in which the given event may be recurrent at regular intervals. In view, therefore, of these objections it may perhaps be better to invert the terms of the definition which might then run as follows: "If the probability "of an event A is p, then in any large number N out of a series of "independent cases in which the event A is in question, A will "happen on the average on pN occasions, and by taking N sufficiently "large the ratio of the number of occasions upon which the event "happens to the total number N may be made to differ from p by "as small a quantity as we please." We fail to see how any other meaning than this can be attached to the mathematical notion of probability, whether the actual value of p in any given case is estimated from *à priori* considerations or from experience; for, in the former case, if the statement do not hold true, this is a proof that the assigned value of p is erroneous.

This is in practical agreement with Dr. Sprague when he says, "If in tossing a coin the chances of head and tail turning up "are equal, then they must in the long run turn up equally often"; and again, "If we toss a coin a very large number of times, N, "we are not to suppose that it will fall head uppermost exactly "$\frac{N}{2}$ times, but only that the deviation from $\frac{N}{2}$ will be so small" (as compared with $\frac{N}{2}$) "that it may be neglected."

Dr. Sprague subsequently, however, appears to depart from this position, when, after quoting the above remark of Professor Chrystal, he says, "My own views with regard to this point are that *à priori* "considerations, when they are applicable, are a very much safer "guide than experience"; and again, "Independently of all experience,

"the chances of head and of tail turning up are equal, and the "probability of one of them, say head, turning up is one-half."

As illustrating this point, Dr. Sprague quotes two experiments from De Morgan and one of his own. In the first two, a coin was tossed until heads had appeared, in each case, 2,048 times, the number of tossings in the first experiment being 4,040 and in the second 4,092. The only conclusion that can be drawn from these two series of trials is that the common estimate of the probability of tossing head—namely, one-half—is very near the truth, the slightly larger fraction given by each series not differing from this estimate by a quantity larger than would naturally be expected. If, however, the first series of trials had been 100 times more numerous, and the proportion of heads thrown had remained the same, this would have been a conclusive proof that the true value of the probability, under the particular circumstances, was somewhat greater than one-half, all *à priori* considerations notwithstanding. It would seem preferable, therefore, to say that, in such cases, the latter are a *more convenient* guide than experiment, since to obtain a more accurate estimate a vast number of observations would be necessary.

To the illustrations quoted from De Morgan, Dr. Sprague adds an experiment of his own which is of considerable interest. He assumes that with an ordinary pack of cards, if a card is drawn at random, the probability of drawing an ace is precisely $\frac{1}{13}$; and with a view of testing how nearly actual experience would conform to theory in this case, he had two packs of cards shuffled and cut until 200 aces in all had been drawn, a result arrived at when the cards had been cut 2,173 times. The experiment in this case gives the probability of cutting an ace as nearly 1 in 11 instead of 1 in 13. Upon this result Dr. Sprague remarks: "This number of trials, "2,173, seems fairly entitled to be called large, but instead of giving "us the probability, as it should have done according to Professor "Chrystal's definition, it gives us a very insufficient approximation "to the real probability, 1 in 13, or ·07692. If we divide the "2,173 observations into four groups, each containing 50 aces, "the numbers in these groups are 626, 523, 480, 544 respectively, "the probability deduced in each case being less than that given by "theory. This example", he adds, "seems sufficient to prove that "if, as a matter of business, we had to estimate the probability of "drawing an ace from a pack of cards, our proper course would not "be to make a large number of trials and then be guided by the "results, but rather to examine carefully the constitution of the "pack for the purpose of determining (1) what proportion of "the cards are aces, and (2) whether each of these is as likely to "be drawn as any other card." In considering the result of this particular experiment, however, the conclusion we should be inclined to draw from it is different from that stated by Dr. Sprague. An important part of the theory of probabilities is taken up with the investigation of these departures or deviations from average or "expected" results; such deviations being as much the subject of calculation as the average results themselves. If the "real probability" of cutting an ace with these packs of cards had been exactly $\frac{1}{13}$, then it can be shown that the chance of drawing as many as 200 aces in 2,173 trials would be only about 1 in 170. There is a common

trick of cutting the aces in a new pack of cards, which depends for success upon the greater adhesiveness of the remaining cards, and it is not improbable that this fact would make the chance of drawing an ace somewhat greater than that of drawing any other card from a new pack, a conclusion which is very strongly supported by the result of Dr. Sprague's experiment.

Dr. Sprague draws a distinction between events that could conceivably be predicted and those which he considers could not; to the former of which he would restrict the use of the word "probability", employing the word "chance" in the case of the latter; in those cases, *i.e.*, where the event depends in part upon the "caprice" of an operator, as in the tossing of a coin. "This", he says, "is a matter which does not depend on invariable law, but on the "choice of the operator, which he may exercise in the most capricious "way. To say that this choice can be foreseen, even by an "omniscient being, is equivalent to saying that it is the necessary "consequence of an antecedent state of things; and this virtually "negatives the possibility of the operator having the power to make "a free choice on each occasion."

On this point it can only be said that the alternative proposition, namely, that the choice of the operator is *not* the necessary consequence of an antecedent state of things, is one which many of our readers will probably find themselves unable to accept, and they will be equally unable, therefore, to follow Dr. Sprague in the distinction he draws between "probability" and "chance." To the writer, Laplace's point of view appears to be the only sound one.

A short notice like the present cannot attempt to deal with many important points raised by the suggestive papers of Professor Chrystal and Dr. Sprague, a study of both of which will be found essential to every student of this interesting branch of mathematics.

G. F. H.

CORRESPONDENCE.

GRADUATION.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—After an interval of three years, the readers of the *Journal* may perhaps bear with some further observations on the well-worn subject of graduation.

Dr. Sprague has convinced us that graduation by a formula correct only to third differences distorts a table of mortality. On the other hand I think we may agree with him and Mr. Woolhouse in regarding the error as practically unimportant. Still, it is desirable to get rid of the error, and such is the object of the present communication, in which Δ will everywhere mean $\Delta\,U_0$.

If, by the formula in vol. xxv of the *Journal*, page 22, correct to

third differences, I graduate a series in seventh differences, I bring out successive results deficient as follows:

$$\begin{array}{llll} 6\cdot4\Delta^4 & +38\cdot4\Delta^5 & +107\cdot6\Delta^6 & +186\cdot0\Delta^7 \\ 6\cdot4 & +44\cdot8 & +146\cdot0 & +293\cdot6 \\ 6\cdot4 & +51\cdot2 & +190\cdot8 & +439\cdot6 \\ 6\cdot4 & +57\cdot6 & +242\cdot0 & +630\cdot4 \\ \&c. & \&c. & \&c. & \&c. \end{array}$$

These errors form a regular series in which the fourth differences vanish, and consequently, if graduated by the same formula or by one of similar capacity, they will be reproduced without alteration. Then, adding to the first graduation the graduated error (which is the same thing as the ungraduated error), I revert to the series in seventh differences with which I started.

Applying this to a mortality table, I graduate first the data, then the differences between the data and my results, and when the two are combined the work has been done correctly to the seventh order of differences.

Dr. Sprague's test (*J.I.A.*, xxix, 235) is a complete investigation of Mr. Woolhouse's formula as hitherto used, but in pursuance of what has now been written, I proceed to graduate the error or differences in l_x which Dr. Sprague demonstrates. I use my own formula above quoted because it is smoother than the other and more simple, while agreeing closely with it in result. If I may be allowed to repeat myself I would like to state more clearly than before how this formula is arrived at.

Let S represent the result of summing four times in fives.

Σ the result of summing thrice in fives, then in fours and twos.

U_c (being U_8) the central or ninth term of the seventeen in summation.

$S=\dfrac{(1+\Delta)^5-1}{\Delta}$ multiplied thrice by itself (*J.I.A.*, xxv, 246) expanding and multiplying.

$$S=625U_0+5{,}000\Delta^1+20{,}000\Delta^2+52.500\Delta^3+,\ \&c.$$

Compare $625U_c=625U_0+5{,}000\Delta^1+17{,}500\Delta^2+35{,}000\Delta^3+,\ \&c.$

$$\therefore\ 625U_c=S-2{,}500\Delta^2-17{,}500\Delta^3.$$

In the same manner we find

$$1{,}000U_c=\Sigma-3{,}750\Delta^2-26{,}250\Delta^3$$

and from these two equations

$$U_c=\frac{2\,\Sigma-3\,S}{125}$$

which is more intelligible when written thus:

$$U_c=\frac{2\,S_{5.5.5.4.2}-3\,S_{5.5.5.5}}{125}$$

The shortened working, which saves several columns, will be presently seen. (For explanation see *J.I.A.*, xxv, 23.)

$l_x - l^1_x$ (Dr. Sprague)	Fives: Three middle terms	Fives: Two outside terms	Fives: Three less Two	Fives	Fives	Fives	Last ÷ 125 or × ·008
64	...	...	...	...	...	...	...
73	...	...	...	...	...	...	...
80	241	157	84	...	...	...	...
88	261	170	91	...	...	...	...
93	278	178	100	488	...	...	...
97	288	184	104	512	...	...	...
98	291	182	109	526	2,539	...	...
96	283	175	108	520	2,488	...	...
89	263	158	105	493	2,329	10,999	88
78	227	133	94	437	2,039	9,474	76
60	175	98	77	353	1,604	7,259	58
37	106	53	53	236	1,014	4,323	35
9	21	− 3	24	85	273	686	5
− 25	− 79	− 67	− 12	− 97	− 607	− 3,570	− 29
− 63	−192	−135	− 57	− 304	−1,598	− 8,289	− 66
−104	−311	−206	−105	− 527	−2,652	−13,241	−106
−144	−429	−275	−154	− 755	−3,705	−18,122	−145
−181	−537	−338	−199	− 969	−4,679	−22,568	−181
−212	−627	−387	−240	−1,150	−5,488	−26,185	−209
−234	−689	−418	−271	−1,278	−6,044	−28,591	−229
−243	−714	−428	−286	−1,336	−6,269	−29,465	−236
−237	−696	−414	−282	−1,311	−6,111	−28,598	−229
−216	−633	−376	−257	−1,194	−5,553	−25,936	−207
−180	−529	−314	−215	− 992	−4,621	−21,619	−173
−133	−390	−236	−154	− 720	−3,382	−15,978	−128
− 77	−230	−146	− 84	− 404	−1,952	− 9,507	− 76
− 20	− 63	− 53	− 10	− 72	− 470	...	...
34	94	35	59	236	918	...	...
80	226	109	117	490	...	...	...
112	321	167	154	668	...	...	...
129	374	204	170	...	...	...	...
133	386	218	168	...	...	...	...
124	...	...	...	...	...	...	...
106	...	...	...	...	...	...	...

It will be seen that the quantities to be added to l^1_x agree very closely with those which Dr. Sprague shows to be wanting; and in any case where it would be of advantage to graduate the differences still remaining, the work could be carried to the utmost degree of exactness.

The test, therefore, establishes the applicability of formulas of this kind when the distortion is cured which Dr. Sprague has pointed out.

It remains to consider what effect this procedure has upon the formula in regard to adjustment of irregularities. The first application makes U_c, which I will now call U_0, equal to

$$\begin{aligned}&{\cdot}200U_0 + {\cdot}192(U_{-1}+U_{+1}) + {\cdot}144(U_{-2}+U_{+2}) + {\cdot}080(U_{-3}+U_{+3})\\&\quad + {\cdot}024(U_{-4}+U_{+4}) - {\cdot}016(U_{-6}+U_{+6}) - {\cdot}016(U_{-7}+U_{+7})\\&\quad - {\cdot}008(U_{-8}+U_{+8})\end{aligned}$$

obtained as follows, the terms of the numerator being differenced at commencement.

U_n	S_4	$S_{4,2}$	$2S_{4,2}$	S_5	$3S_5$	$2S_{4,2}-3S_5$*	Fives	Fives	Fives	Last $\div 125$ or $\times \cdot 008$	
...	...	...	...	...	...	...	...	...	−1	−·008	U_{-8}
...	...	...	...	...	...	...	...	...	−2	−·016	U_{-7}
...	...	...	...	...	...	...	...	−1	−2	−·016	U_{-6}
...	...	...	...	...	...	...	...	−1	0	0	U_{-5}
...	...	...	...	...	...	...	−1	0	3	·024	U_{-4}
...	...	...	...	...	...	...	0	2	10	·080	U_{-3}
...	1	1	2	1	3	−1	1	3	18	·144	U_{-2}
...	1	2	4	1	3	1	2	6	24	·192	U_{-1}
1	1	2	4	1	3	1	1	7	25	·200	U_0
...	1	2	4	1	3	1	2	6	24	·192	U_{+1}
...	...	1	2	1	3	−1	1	3	18	·144	U_{+2}
...	...	...	...	...	...	...	0	2	10	·080	U_{+3}
...	...	...	...	...	...	...	−1	0	3	·024	U_{+4}
...	...	...	...	...	...	...	...	−1	0	0	U_{+5}
...	...	...	...	...	...	...	...	−1	−2	−·016	U_{+6}
...	...	...	...	...	...	...	...	...	−2	−·016	U_{+7}
...	...	...	...	...	...	...	...	...	−1	−·008	U_{+8}

When, in like manner, the differences between the first and last columns have been expanded and the two sets of results combined, the completed operation makes U_0 equal to

$$
\begin{array}{ll}
\cdot 229696\ U_0 & \Delta - \cdot 005440 \\
+\cdot 224256(U_{-1}+U_{+1}) & -\cdot 067840 \\
+\cdot 156416(U_{-2}+U_{+2}) & -\cdot 089600 \\
+\cdot 066816(U_{-3}+U_{+3}) & -\cdot 072320 \\
-\cdot 005504(U_{-4}+U_{+4}) & -\cdot 014720 \\
-\cdot 020224(U_{-5}+U_{+5}) & -\cdot 010240 \\
-\cdot 030464(U_{-6}+U_{+6}) & +\cdot 010240 \\
-\cdot 020224(U_{-7}+U_{+7}) & +\cdot 017600 \\
-\cdot 002624(U_{-8}+U_{+8}) & +\cdot 012864 \\
+\cdot 010240(U_{-9}+U_{+9}) & -\cdot 004608 \\
+\cdot 005632(U_{-10}+U_{+10}) & -\cdot 003584 \\
+\cdot 002048(U_{-11}+U_{+11}) & -\cdot 001920 \\
+\cdot 000128(U_{-12}+U_{+12}) & -\cdot 000640 \\
-\cdot 000512(U_{-13}+U_{+13}) & \dots \\
-\cdot 000512(U_{-14}+U_{+14}) & +\cdot 000256 \\
-\cdot 000256(U_{-15}+U_{+15}) & +\cdot 000192 \\
-\cdot 000064(U_{-16}+U_{+16}) &
\end{array}
$$

This is correct to seventh differences, and the coefficients show the distribution of an irregularity occurring at U_0 and amounting to unity.

In constructing a formula to include in summation 15 terms only, we note that U_c is now U_7, and find that

$$375U_c = S_{5.5.5.3} - 1{,}250\Delta^2 - 7{,}500\Delta^3.$$

Also, bearing in mind that we are using two summations of different scope (U_0 in the shorter being U_1 in the longer when they are referred to the same centre),

$$125U_c = S_{5.5.5} - 375\Delta^2 - 2{,}250\Delta^3.$$

* In working, begin with this column (three middle terms of five, less two outside terms).

From these equations $U_c = \frac{10S_{5.5.5} - 3S_{5.5.5.3}}{125}$, which expands as follows:

U_n	S_3	$3S_3$	$10U_n - 3S_3$	Fives	Fives	Fives	Last ÷125 or ×·008	
...	...	...	...	...	...	− 3	−·024	U_{-7}
...	...	...	...	...	...	− 2	−·016	U_{-6}
...	...	...	...	...	− 3	0	0	U_{-5}
...	...	...	...	...	1	3	·024	U_{-4}
...	...	...	...	−3	2	7	·056	U_{-3}
...	...	...	...	+4	3	21	·168	U_{-2}
...	1	3	−3	+1	4	24	·192	U_{-1}
1	1	3	+7	+1	11	25	·200	U_0
...	1	3	−3	+1	4	24	·192	U_{+1}
...	...	...	...	+4	3	21	·168	U_{+2}
...	...	...	...	− 3	2	7	·056	U_{+3}
...	...	...	...	...	1	3	·024	U_{+4}
...	...	...	...	...	−3	0	0	U_{+5}
...	...	...	...	...	...	− 2	−·016	U_{+6}
...	...	...	...	...	...	− 3	−·024	U_{+7}

This formula is the exact equivalent of Woolhouse's. Indeed, it should claim no more than to be a ready means of obtaining Mr. Woolhouse's results; for it is he who has laid down the lines on which arithmetical graduation should proceed, and whose work I have imitated in the desire "to devise a method of adjustment as even and correct as that of Mr. Woolhouse and more easy in application" (*J.I.A.*, xxiv, 44). Certainly, the arrangement in black and white with which he left the practical part of his subject (*J.I.A.*, xxi, 45) was troublesome; and it did not admit of the check by addition which can be applied to the columnar arrangement. These objections were afterwards met by Mr. Ackland (*J.I.A.*, xxiii, 355), and the formula now given merely does his work by a shortened process, thus:

Age	d_x (H^M)	S_3	$3S_3$	$10d_x - 3S_3$	Fives	Fives	Fives	÷125 or ×·008
66	220	...	...	...	...	...	...	...
67	220	677	2,031	169	...	...	...	...
68	237	703	2,109	261	...	...	...	...
69	246	696	2,088	372	1,000	...	...	...
70	213	681	2,043	87	1,312	...	...	...
71	222	703	2,109	111	1,048	5,973	...	...
72	268	733	2,199	481	1,324	6,471	...	...
73	243	811	2,433	− 3	1,289	6,331	31,920	255·36
74	300	784	2,352	648	1,498	6,711	...	...
75	241	786	2,358	52	1,172	6,434	...	...
76	245	710	2,130	320	1,428	...	...	...
77	224	695	2,085	155	1,047	...	...	...
78	226	609	2,007	253	...	...	...	...
79	219	641	1,923	267	...	...	...	...
80	196	...	...	...	...	...	...	...

The two formulas are now equally short in lateral working (compare *J.I.A.*, xxv, 23), and the saving of labour is a material set-off against the added work of graduating the primary errors or differences.

When graduation by this formula is completed in the manner hereinbefore proposed, U_0 becomes equal to

$\cdot 220736\ U_0$	$\Delta - \cdot 000320$
$+\cdot 220416(U_{-1}+U_{+1})$	$-\cdot 017280$
$+\cdot 203136(U_{-2}+U_{+2})$	$-\cdot 184320$
$+\cdot 018816(U_{-3}+U_{+3})$	$-\cdot 022080$
$-\cdot 003264(U_{-4}+U_{+4})$	$-\cdot 010560$
$-\cdot 013824(U_{-5}+U_{+5})$	$-\cdot 013760$
$-\cdot 027584(U_{-6}+U_{+6})$	$-\cdot 007360$
$-\cdot 034944(U_{-7}+U_{+7})$	$+\cdot 048960$
$+\cdot 014016(U_{-8}+U_{+8})$	$-\cdot 004160$
$+\cdot 009856(U_{-9}+U_{+9})$	$-\cdot 006400$
$+\cdot 003456(U_{-10}+U_{+10})$	$-\cdot 002304$
$+\cdot 001152(U_{-11}+U_{+11})$	$-\cdot 001408$
$-\cdot 000256(U_{-12}+U_{+12})$	$-\cdot 000512$
$-\cdot 000768(U_{-13}+U_{+13})$	$+\cdot 000192$
$-\cdot 000576(U_{-14}+U_{+14})$	

This also is correct to seventh differences, but the formula is evidently less suited than the other to even graduation. The base of this is $-3U_{-1}+7U_0-3U_{+1}$; the base of the other is $-U_{-2}+U_{-1}+U_0+U_{+1}-U_{+2}$.

A section from a completed graduation of $H^M\ d_x$ by each of the two formulas will be as follows:—

First Formula (Higham): $U_0 = \dfrac{2S_{5.5.5.4.2}-3S_{5.5.5.5}}{125}$.

Age	d_x Ungraduated	First Graduation	Difference	Difference Graduated	Completed Graduation	Δ'	Δ''
60	1,840	1,747	93	− 5	1,742	+ 85	+ 5
61	1,860	1,831	29	− 4	1,827	+ 90	+ 1
62	1,910	1,916	− 6	1	1,917	+ 91	− 16
63	2,000	2,001	− 1	7	2,008	+ 75	− 9
64	2,060	2,074	− 14	9	2,083	+ 66	− 11
65	2,150	2,139	11	10	2,149	+ 55	− 17
66	2,200	2,195	5	9	2,204	+ 38	− 21
67	2,200	2,240	− 40	2	2,242	+ 17	+ 22
68	2,370	2,274	96	−15	2,259	+ 39	+ 13
69	2,460	2,320	140	−22	2,298	+ 52	+ 24
70	2,130	2,374	−244	−24	2,350	+ 76	− 1
71	2,220	2,439	−219	−13	2,426	+ 75	+ 1
72	2,680	2,500	180	1	2,501	+ 76	− 60
73	2,430	2,553	−123	24	2,577	+ 16	− 50
74	3,000	2,564	436	29	2,593	− 34	− 53
75	2,410	2,535	−125	24	2,559	− 87	− 20
76	2,450	2,464	− 14	8	2,472	−107	− 19

Second Formula (Woolhouse): $U_c = \dfrac{10S_{5.5.5} - 3S_{5.5.5.3}}{125}$.

Age	d_x Ungraduated	First Graduation *	Difference	Difference Graduated	Completed Graduation	Δ^1	Δ^2
60	1,840	1,747	93	− 4	1,743	+ 80	+ 14
61	1,860	1,828	32	− 5	1,823	+ 94	− 4
62	1,910	1,917	− 7	0	1,917	+ 90	− 7
63	2,000	2,001	− 1	6	2,007	+ 83	− 34
64	2,060	2,079	− 19	11	2,090	+ 49	+ 23
65	2,150	2,135	15	4	2,139	+ 72	− 33
66	2,200	2,199	1	12	2,211	+ 39	− 29
67	2,200	2,243	− 43	7	2,250	+ 10	+ 5
68	2,370	2,274	96	−14	2,260	+ 15	+ 82
69	2,460	2,307	153	−32	2,275	+ 97	− 66
70	2,130	2,383	−253	−11	2,372	+ 31	+ 71
71	2.220	2,427	−207	−24	2,403	+102	− 31
72	2,680	2,503	177	2	2,505	+ 71	− 30
73	2,430	2,554	−124	22	2,576	+ 41	−119
74	3,000	2,578	422	39	2,617	− 78	+ 28
75	2,410	2,527	−117	12	2,539	− 50	− 86
76	2,450	2,474	− 24	15	2,489	−136	+ 22

In graduating the differences it is convenient, for avoidance of negative signs, to add a constant at commencement and take it off afterwards. For instance: add 500 at outset, drop 2,000 in the first fives, and take off 100 at the end. And when there is much irregularity, it is well to postpone till after the first fives the differencing of the terms of the numerator.

I am not without hope that the foregoing may be of service to those whose skill qualifies them to use the graphic method. A clear and undistorted presentation of what a record of mortality does say must afford some assistance in the determination of what it meant to say.

I am, Sir,

Your obedient servant,

J. A. HIGHAM.

AUSTRALIAN MUTUAL PROVIDENT SOCIETY.

Appendices to the Opinion of Messrs. Hendriks, Bailey, and Hardy, *upon the Case submitted to them on behalf of the Society. Dated* 11 *September* 1893.

APPENDIX I.

On a method of Valuation that will produce a definite Reserve at the end of a fixed period, and where the rate of interest yielded upon the accumulations varies annually throughout such period.

[Prepared by Mr. R. P. Hardy.]

1. If i be constant and there be ${}_nV_x$ in hand in respect of an ordinary Assurance effected n years ago, and if it be desired that the

* Copied from Mr. Ackland.

Reserve should be ${}_{n+m}V_x$ at the end of m years hence, then P, the net annual Premium payable in advance for the period, can be determined from the following equation, namely:

$$P(1+{}_{|m-1}a_{x+n})={}_{|m}A_{x+n}+v^m{}_mp_{x+n}\cdot{}_{n+m}V_x-{}_nV_x \quad . \quad . \quad (1)$$

that is to say, the value of P for the term must equate to the sum of the values of the risk for the period and of an endowment of the Reserve to be eventually held, less the amount ${}_nV_x$ already in hand.

$$\text{Whence} \qquad P=\frac{{}_{|m}A_{x+n}+v^m{}_mp_{x+n}\cdot{}_{n+m}V_x-{}_nV_x}{1+{}_{|m-1}a_{x+n}} \quad . \quad . \quad . \quad . \quad (2)$$

If the values in equation (2) be homogeneous, then $P=P_x$, which is the uniform net annual Premium to be paid for the coming m years and thereafter.

2. Suppose that it be required to provide for a gradual strengthening, or reduction, of the existing Valuation-basis, to be completed at the end of m years, and further that the interest-yield be taken to rise or fall, or otherwise vary, within the period—both conditions being defined beforehand—then, in order to find P, the values corresponding to the new status must be employed in solving equation (2). Thus, for instance, if the Net-Premium Valuation-Reserve has to be gradually raised from a present basis of 4 per-cent to 3 per-cent, and if the interest-yield is to fall, by equal yearly steps over the same period, from the former to the latter rate, then ${}_{n+m}V_x$ must be taken at 3 per-cent, ${}_nV_x$ at 4 per-cent, and the values of ${}_{|m}A_{x+n}$, $1+{}_{|m-1}a_{x+n}$, and v^m must be specially calculated for the decreasing rate of interest to be yielded each year.

P so found will be the uniform net annual Premium payable for the m years, and the net Premium thereafter will be the ordinary P_x on the 3 per-cent basis.

3. The values involving the varying rate of interest may be found as follows.

If $i_1, i_2, i_3, \ldots . i_m$ be the rates of interest-improvement per unit in the first, second, third, ... and mth years respectively of the m years, and if i represent the net rate for the $(m+1)$th and all subsequent years, then the values of 1 may be thus typified.

Term to run		
p years after m	$m+p$	$(1+i_1)^{-1}(1+i_2)^{-1}\ldots(1+i_m)^{-1}(1+i)^{-p}$ $={}_mv'\times v^p$
Exact period .	m	$(1+i_1)^{-1}(1+i_2)^{-1}\ldots(1+i_m)^{-1}$ $={}_mv'$
Remainder of m after q	$m-q$	$(1+i_{q+1})^{-1}(1+i_{q+2})^{-1}\ldots(1+i_m)^{-1}$ $=\dfrac{{}_mv'}{(1+i_1)^{-1}(1+i_2)\ldots(1+i_q)}={}_{m-q}v'$

4. The value of an annuity on such interest-basis may be thus expressed.

First year of the working of the varying interest

$$ {}_{x+n}a'_{x+n} = \frac{{}_{x+n}N'_{x+n} - {}_{x+n}N'_{x+n+m}}{{}_{x+n}D'_{x+n}} + \frac{{}_{x+n}D'_{x+n+m}}{{}_{x+n}D'_{x+n}} \times a_{x+n+m} \quad . \quad . \quad (3) $$

Second year of the working of the varying interest

$$ {}_{x+n}a'_{x+n+1} = \frac{{}_{x+n}N'_{x+n+1} - {}_{x+n}N'_{x+n+m}}{{}_{x+n}D'_{x+n+1}} + \frac{{}_{x+n}D'_{x+n+m}}{{}_{x+n}D'_{x+n+1}} \times a_{x+n+m} \quad . \quad . \quad (4) $$

And so on, for the remaining years of the working of the varying interest.

Where $\quad {}_{x+n}N'_{x+n} = \Sigma_{x+n}D'_{x+n+1}$

$$ {}_{x+n}D'_{x+n} = l_{x+n} $$

$$ {}_{x+n}D'_{x+n+1} = (1+i_1)^{-1}l_{x+n+1} $$

$$ {}_{x+n}D'_{x+n+2} = (1+i_1)^{-1}(1+i_2)^{-1}l_{x+n+2} $$

a_{x+n+m} = value of an annuity upon the final and constant interest-basis, and for the age attained by the life when the interest has ceased to vary.

And, in like manner for other Annuity Benefits.

5. Similarly, the values of Assurances can be typified.

First year of the working of the varying interest

$$ {}_{x+n}A'_{x+n} = \frac{{}_{x+n}M'_{x+n} - {}_{x+n}M'_{x+n+m}}{{}_{x+n}D'_{x+n}} + \frac{{}_{x+n}D'_{x+n+m}}{{}_{x+n}D'_{x+n}} \times A_{x+n+m} \quad . \quad . \quad (5) $$

Second year of the working of the varying interest

$$ {}_{x+n}A'_{x+n+1} = \frac{{}_{x+n}M'_{x+n+1} - {}_{x+n}M'_{x+n+m}}{{}_{x+n}D'_{x+n+1}} + \frac{{}_{x+n}D'_{x+n+m}}{{}_{x+n}D'_{x+n+1}} \times A_{x+n+m} \quad . \quad . \quad (6) $$

And so on, for the remaining years of the working of the varying interest.

6. In order to illustrate the financial effect of employing a varying rate of interest, the following Tables have been prepared, showing the values of an Annuity and of an Assurance, temporary deferred and immediate, according to H^M mortality, when the interest commences at the rate of £3. 15*s*. per-cent per annum, and falls one shilling per-cent per annum until the 15th year, from and after which point (*i.e.* from the beginning of the 16th year) it remains constant at 3 per-cent per annum for the remainder of life.

7. By application of these Tables, samples of the Premiums (designated as P''_x) as found from equation (2), and payable for the term of 15 years, have been calculated, and are annexed hereto.

8. Measuring the liability by the A'_{x+n} of the same Tables, and valuing P''_x last referred to by the temporary annuities upon a like basis and further capitalizing P_x at 3 per-cent by the correlative deferred annuities, specimens of the resulting Reserves have been deduced, and will be found annexed hereto.

These specimens are shown for each fifth present Valuation-age and for each fifth entry-age from 25 to 50, and for each of the 15 years succeeding the commencement of the working. To facilitate comparison, the ordinary Reserves upon the H^M $3\frac{3}{4}$ and 3 per-cent basis are set by the side of the above—those of the former may be taken to represent the amounts that otherwise would have been held, and the latter those to which an approach is being made, and which are exactly reached at the end of the 15th year.

The specimen Reserves, therefore, when contrasted with those of the H^M $3\frac{3}{4}$ per-cent, measure the distance traversed towards the goal; and, when compared with those of the H^M 3 per-cent, they indicate the length of the journey yet to be made.

9. This method of Valuation is applicable to all classes of Benefit.

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Values of v'^n

n	i_n	$\log(1+i_n)$	$\log v'$	$\log v'^n$	n
1	·0375	0·01599	$\bar{1}$·98401	$\bar{1}$·98401	1
2	·0370	·01578	·98422	·96823	2
3	·0365	·01557	·98443	·95266	3
4	·0360	·01536	·98464	·93730	4
5	·0355	·01515	·98485	·92215	5
6	·0350	·01494	·98506	·90721	6
7	·0345	·01473	·98527	·89248	7
8	·0340	·01452	·98548	·87796	8
9	·0335	·01431	·98569	·86365	9
10	·0330	·01410	·98590	·84955	10
11	·0325	·01389	·98611	·83566	11
12	·0320	·01368	·98632	·82198	12
13	·0315	·01347	·98653	·80851	13
14	·0310	·01326	·98674	·79525	14
15	·0305	·01305	·98695	·78220	15

$100\ P''_x$

Net Annual Premiums by H[M] *Table payable at the Valuation-age* $(x+n)$ *and for* 15 *years from the commencement of the working of the varying interest (as above described), according to the entry-age* (x). *From and after age* $x+n+15$, *the ordinary Premium* (P_x) *will be payable for the remainder of life.*

$x+n$	$x=25$	$x=30$	$x=35$	$x=40$	$x=45$	$x=50$
30	1·6387	...	...	...	...	...
35	1·6728	1·8888	...	...	...	...
40	1·6996	1·9204	2·1953	...	...	...
45	1·7171	1·9428	2·2234	2·5823	...	...
50	1·7239	1·9541	2·2399	2·6053	3·0954	...
55	1·7201	1·9542	2·2447	2·6157	3·1129	3·7679
60	1·7062	1·9437	2·2380	2·6139	3·1170	3·7685
65	1·6841	1·9242	2·2217	2·6011	3·1086	3·7766
70	1·6565	1·8985	2·1982	2·5802	3·0911	3·7636

Values of Annuities and Assurances, according to the H[M] *Tables with interest at the rate of* £3. 15*s. for the first year and decreasing one shilling per-cent per annum until the end of the* 15*th year, when it remains constant at* 3 *per-cent for the remainder of life.*

m	$x+n=30$					
	Values of Annuities of 1			Values of Assurances of 1		
	$_{\vert 15-m-1}a'_{x+n+m}$	$_{15-m-1}a'_{x+n+m}$	a'_{x+n+m}	$_{15-m-1}A'_{x+n+m}$	$_{\vert 15-m-1}A'_{x+n+m}$	A'_{x+n+m}
0	10·255	8·713	18·968	·10063	·27132	·37195
1	9·723	9·111	18·834	·09743	·28369	·38112
2	9·163	9·522	18·685	·09386	·29654	·39040
3	8·575	9·951	18·526	·08992	·30986	·39978
4	7·958	10·395	18·353	·08557	·32372	·40929
5	7·311	10·857	18·168	·08079	·33807	·41886
6	6·634	11·336	17·970	·07551	·35300	·42851
7	5·926	11·836	17·762	·06964	·36853	·43817
8	5·186	12·355	17·541	·06314	·38471	·44785
9	4·413	12·894	17·307	·05603	·40150	·45753
10	3·604	13·456	17·060	·04828	·41899	·46727
11	2·760	14·038	16·798	·03996	·43711	·47707
12	1·879	14·640	16·519	·03107	·45589	·48696
13	·959	15·266	16·225	·02155	·47534	·49689
14	...	...	15·916	·01122	·49560	·50682

[In the original Tables the series is shown for every fifth value of $x+n$ from 25 to 70.]

Reserves for each 100 *of Assurance, according to the* H^M *Table.*

m	$x+n=50$						m
	$x=30$			$x=40$			
	$3\frac{3}{4}$ per-cent	Proposed basis	3 per-cent	$3\frac{3}{4}$ per-cent	Proposed basis	3 per-cent	
1	27·74	27·98	30·30	18·40	18·60	19·98	1
2	29·38	29·88	32·01	20·26	20·66	21·94	2
3	31·06	31·79	33·74	22·14	22·76	23·93	3
4	32·75	33·73	35·49	24·05	24·88	25·94	4
5	34·46	35·68	37·25	25·99	27·03	27·96	5
6	36·20	37·64	39·02	27·96	29·18	30·00	6
7	37·94	39·61	40·80	29·92	31·34	32·04	7
8	39·70	41·57	42·58	31·91	33·50	34·08	8
9	41·47	43·54	44·37	33·90	35·68	36·14	9
10	43·25	45·49	46·16	35·91	37·82	38·18	10
11	45·02	47·43	47·93	37·91	39·96	40·22	11
12	46·79	49·34	49·69	39·91	42·07	42·24	12
13	48·54	51·22	51·44	41·88	44·15	44·25	13
14	50·27	53·06	53·16	43·84	46·19	46·23	14
15	52·00	54·87	54·87	45·80	48·19	48·19	15

[The above extract is given to show the effect of the method. The original Tables range for every fifth value of x from 25 to 50, and for every like value of $x+n$ from 30 to 70.—ED. *J.I.A.*]

APPENDIX II.

On a method of calculating the Reversionary Bonus attaching to each Policy, according to the principles of distribution now in force in the Society.

[PREPARED BY MR. R. P. HARDY.]

1. There is no occasion to set out the last reserved value of each individual Policy, in order to ascertain the surface over which the excess of interest realized has to be taken for finding the cash appropriation in respect of such interest-profit on such values, such appropriation being converted into its reversionary equivalent. Exactly the same results can be arrived at by employing the following shortened process.

2. If j be the agreed rate of interest-profit, and B_n the total existing reversionary additions for the group of the common age $x+n$ at the point of distribution, then

$$b_n = jA_{x+n-1}B_n = \text{total cash appropriation in respect of the total existing additions at age } x+n.$$

whence $$j\frac{A_{x+n-1}}{A_{x+n}}B_n = \text{total reversionary additions corresponding to } b_n.$$

Hence, if $j\dfrac{A_{x+n-1}}{A_{x+n}}$ be set up in Crelle's Tables as a common factor, and each individual Bonus addition be multiplied into the same, the new reversionary Bonus in respect of that portion of the Surplus that arises from the interest-profit upon the last reserved value of the existing Bonus can be speedily ascertained and recorded. A perfect check will be obtained by multiplying the total of the existing additions by the common factor, and comparing such product with the cast of the column of the new additions just computed.

This process has to be performed for each present valuation-age, and it is applicable to all classes of Benefit.

3. To ascertain the interest-profit due to the last reserved value of the sum assured, taking j as the rate as before, we get

$$c_n = j({}_{n-1}Z_x + {}_{n-2}Z_{x+1} + {}_{n-3}Z_{x+2} + \ldots + {}_{n-(n-1)}Z_{x+n-2})$$

= total cash appropriation in respect of the total sums assured at age $x+n$.

$$\text{Whence } \frac{c_n}{A_{x+n}} = \frac{j}{A_{x+n}}({}_{n-1}Z_x + {}_{n-2}Z_{x+1} + \ldots + {}_{n-(n-1)}Z_{x+n-2})$$

= total reversionary addition corresponding to c_n.

Here, ${}_{n-1}Z_x$ represents the last reserved value of the total sums assured at entry-age x }
${}_{n-2}Z_{x+1}$ represents the last reserved value of the total sums assured at entry-age $x+1$ } the present Valuation-age being $x+n$
and so on.

Now, since the total reserved value of each group (for which we put the general symbol of ${}_nZ_x$) is made up of like elements, each individual composing part being merely a multiple of the fundamental unit ${}_nV_x$, it will be sufficient to prepare, for each value of $x+n$, a Table of the values of $\dfrac{j}{A_{x+n}}\,{}_{n-1}V_x$, and for each value of x from that of the oldest to that of the youngest Policy shown upon the Valuation-group. Care must be taken that ${}_{n-1}V_x$ follows strictly the data and principles employed in the last Valuation. This process is applicable to all forms of Benefit, the formula being suitably modified for each class of risk.

4. The Table as above can be prepared upon the basis of either £1, or £100, assured; and, if the latter unit be adopted, the new reversionary addition in respect of the interest-profit upon the last reserved value of the sum assured can be readily obtained by multiplying such unit by the number of hundreds assured by the Policy.

5. The total new Bonus addition due to the interest-profit will be the sum of the separately found amounts in respect of (1) the existing Bonus additions, and (2) the sum assured.

Such complete checks can be interposed at various stages of the work that there is no danger of any error arising affecting the results. It will be seen that a standard Table showing the new additions, or total additions, including the last, can be prepared for either publication or Office use.

[Appended to the original is a Table showing the working of the

distribution of profit according to the method in use by the Society, and presumably by other offices employing the same principle of apportionment, together with the shortened operation of the proposed method.—ED. *J.I.A.*]

6. The balance of the Surplus, which has to be divided in proportion to the "loading", has to be ascertained by discounting the reversionary sums last obtained, and deducting the same from the total Surplus recommended for division. Should, however, the gross divisible Surplus be apportioned beforehand into that attributable to the interest and that to the "loading", the Bonus due to the latter can be conveniently ascertained, in conjunction with that for the interest-profit on the sum assured, by modifying the formula given in 3, namely $\frac{j}{A_{x+n}}\left({}_{n-1}V_x+\frac{w}{j}L_x\right)$, where L_x=the amount of the "loading" per unit assured for the entry-age x, and w=the rate of distributable profit on such "loading."

7. As it is within the range of probability that exceptional losses may occur in any single year—for instance, the failure of some of the mortgage securities, or an excess of claims through an epidemic, or the setting up of a Suspense Fund against contingent liabilities of any kind—which might render it both fair to the more recent assured and generally politic to distribute such losses, or suspense, over both the excess-interest and the "loading", in accordance with the principles underlying the recommendations of the Actuarial Referees in 1873 in their before referred to answer to question No. 38. This, both as to the circumstances and manner of application, must remain for the discretion of the Management, and we can offer no further opinion upon the point. But, we suggest that it would always be desirable to make a rough estimate beforehand of the amounts of the divisible Surplus to be respectively assigned to the excess-interest and to the "loading"—and, we think that an approximation of sufficient nearness would be had by estimating the total last held Reserve in respect of the current contracts (including the value of Bonus additions last declared) by means of the following formula; putting Z_{n-1} and Z_n as the Reserves last and now held respectively, i as the Valuation-rate of interest, and P as the total net Premium income.

$$_{n-1}Z=\frac{_nZ+\frac{1}{2}\text{Expected Death Claims}}{1+i}-P$$

If this be done, the total cash to be appropriated as interest-profit can be closely found, and the balance attributable to the "loading" shown, from which latter w can be determined with sufficient accuracy; and, the formula given in article 6 hereof applied for bringing out, by a single operation, the reversionary bonuses resulting from the combined interest-profit and "loading"-profit attaching to the sum assured.

The above formula would have to be modified in the case of the Endowment-Assurances, by taking the whole of the matured claims in addition to one half of those expected to have fallen in by death. Speaking generally, it would be desirable to watch closely this latter suggested proportion of *one half*, which can be done by the Actuarial Staff.

11 *September* 1893.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1893, published in 1894.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	176,199,924	10,202,050	186,401,974
Adjustments: for one additional return (+49,725*l.*); for portion of Funds not taken over on transfer of Business (−19,663*l.* and −463*l.*); and for one return discontinued (−3,831*l.*) . .	+29,599	−3,831	+25,768
	176,229,523	10,198,219	186,427,742
Premiums	16,573,686	5,709,691	22,283,377
Consideration for Annuities . .	1,359,476	3,807	1,363,283
Interest and Dividends (less Tax)	7,206,828	335,289	7,542,117
Increase in value of Investments .	173,922	...	173,922
Fines, Fees, &c.	10,292	763	11,055
Capital Paid-up	48,051	8,197	56,248
Customs Timber Measuring, &c. .	3,620	...	3,620
Donations (Itinerant Methodists) .	2,242	...	2,242
Transfers from other Accounts .	116,279	67,017	183,296
Miscellaneous	28,168	1,775	29,943
	201,752,087	16,324,758	218,076,845

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	13,517,068	2,451,965	15,969,033
Cash Bonuses and Reduction of Premiums	959,450	146	959,596
Surrenders	937,971	15,909	953,880
Annuities	1,055,411	2,434	1,057,845
Commission	920,152	1,581,409	2,501,561
Expenses of Management . .	1,586,629	989,140	2,575,769
Bad Debts	2,612	148	2,760
Decrease in value of Investments .	121,497	468	121,965
Interest on Capital and Dividends and Bonuses to Shareholders .	427,664	466,999	894,663
Transfers to other Accounts . .	476,282	65	476,347
Capital withdrawn from Life Account (Sun of India, Limited) .	30,000	...	30,000
Miscellaneous	24,444	...	24,444
Balance* at the end of the Year .	181,692,907	10,816,075	192,508,982
	201,752,087	16,324,758	218,076,845

* This Balance includes the whole of the Life and Annuity Funds (£186,578,807), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1893).

LIABILITIES	Ordinary Companies	Industrial Companies	Total
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,254,995	859,528	12,114,523
Life and Annuity Funds	176,372,186	10,206,621	186,578,807
Fire Funds of Companies transacting Life Business	10,223,471	...	10,223,471
Marine Funds of Companies transacting Life Business	609,817	...	609,817
Reserve Funds	4,153,529	...	4,153,529
Other Funds	689,586	267,386	956,972
Profit and Loss Balances	2,516,894	...	2,516,894
Depreciation and Investment Balances	936,162	...	936,162
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims	3,510,883	36,004	3,546,887
Outstanding Accounts	462,043	16,414	478,457
Temporary Loans	148,852	...	148,852
	211,981,218	11,385,953	223,367,171

ASSETS	Ordinary Companies	Industrial Companies	Total
	£	£	£
Mortgages	84,480,920	328,733	84,809,653
Loans on Policies	9,754,200	27,626	9,781,826
,, Rates	20,369,932	4,292,147	24,662,079
British Government Securities	4,951,175	502,441	5,453,616
Indian and Colonial Government Securities	14,265,946	147,300	14,413,246
Foreign Government Securities	3,859,107	...	3,859,107
Debentures	24,766,416	1,740,125	26,506,541
Shares and Stocks	13,600,456	32,742	13,633,198
Companies' own Shares	603,491	442	603,933
Land and House Property and Ground Rents	13,977,961	3,254,013	17,231,974
Life Interests and Reversions	4,071,350	522	4,071,872
Loans on Personal Security	1,497,918	8,698	1,506,616
Agents' Balances and Outstanding Premiums	4,767,155	393,280	5,160,435
Outstanding Interest	2,062,828	115,903	2,178,731
Cash, Deposits, Stamps, &c.	8,740,338	152,904	8,893,242
Customs Timber Measuring Balances, &c.	1,973	...	1,973
Book-Room Grant (Itinerant Methodists)	50,000	...	50,000
Deficiencies, Preliminary Expenses, &c.	160,052	389,077	549,129
	211,981,218	11,385,953	223,367,171

Increase (+) *or* Decrease (−) *in the Chief Items of this Year's* Summary *compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
	£	£
Income.*		
Premiums	+2,007,825	+ 242,595
Consideration for Annuities	+ 262,606	− 523
Interest and Dividends (less Tax)	+ 587,916	+ 22,889
Net Result of Realization and Re-valuation of Investments	− 51,690	− 73
Outgo.*		
Claims	+1,120,946	− 85,296
Annuities	+ 249,177	+ 692
Surrenders	+ 114,195	+ 1,796
Commission	+ 167,705	+ 79,492
Expenses of Management	+ 253,044	+ 14,821
Liabilities.		
Paid-up Capital (including sundry Shareholders' Balances)	− 88,814	+ 4,612
Life and Annuity Funds	+5,872,666	+ 614,395
Assets.		
Mortgages (including Loans on Rates)	+1,881,657	+ 518,369
Life Interests and Reversions	+ 334,608	− 284
Loans on Policies	+ 230,464	+ 231
British Government Securities	− 346,921	− 111
Indian and Colonial Government Securities	+ 796,925	+ 28,770
Foreign Government Securities	+ 124,700	...
Debentures	+1,854,246	+ 47,570
Shares and Stocks	+ 61,986	− 138,991
Companies' own Shares	+ 21,204	+ 269
Land and House Property and Ground Rents	+ 729,861	+ 158,793
Loans on Personal Security	+ 40,950	− 4,771

* Including the figures of the "Gresham" for eighteen months.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 97, of which 86 have been classed as Ordinary, 7 as Industrial, and 4 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately. The Returns of the British Natural-Premium, the Clergy Pension, and the Pioneer are included for the first time.

During the year four names have been removed from the official List of Companies, namely, the Briton Medical and General, Limited; Crown; Leicester, Limited; and the Queen; two of which Companies have transferred their business, one has amalgamated, and the fourth is in process of liquidation; and one name has been added, namely, the Sun Life Assurance Company of Canada; in which case the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

Summary of the Assurances in Force, *as shown by the last Returns of the Companies.*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	718,226	350,228,970	90,080	58,795,362	808,306	409,024,332	19,861,400	389,162,932
Limited number of Premiums . .	29,776	17,161,792	5,382	2,531,000	35,158	19,692,792	688,672	19,004,120
	748,002	367,390,762	95,462	61,326,362	843,464	428,717,124	20,550,072	408,167,052
Endowments . .	1,922	431,506	8,038	1,397,082	9,960	1,828,588	6,000	1,822,588
Endowment Assurances . . .	326,168	56,451,995	30,173	9,296,673	356,341	65,748,668	979,016	64,769,652
Joint Lives . .	12,435	2,520,446	2,181	955,401	14,616	3,475,847	295,767	3,180,080
Last Survivor . .	1,114	901,597	1,233	1,331,308	2,347	2,232,905	320,392	1,912,513
Contingent . .	39	37,469	3,115	5,313,568	3,154	5,351,037	1,394,754	3,956,283
Issue . . .	6	14,189	980	3,745,961	986	3,760,150	1,045,070	2,715,080
Miscellaneous . .	238	239,920	4,413	5,304,990	4,651	5,544,910	1,212,076	4,332,834
	1,089,924	427,987,884	145,595	88,671,345	1,235,519	516,659,229	25,803,147	490,856,082
Annuities.								
Immediate . .	...	...	...	...	19,389	920,752	10,094	910,658
Deferred . .	...	...	...	...	6,565	192,786	12,422	180,364
	...	...	...	...	25,954	1,113,538	22,516	1,091,022

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.						£		£
Whole Term of Life	...	...	...	...	12,765,029	120,735,058	...	120,735,058
Limited number of Premiums . .	...	...	...	...	128	3,089	...	3,089
	...	...	...	...	12,765,157	120,738,147	...	120,738,147
Endowments . .	...	...	...	...	92,616	1,249,624	...	1,249,624
Endowment Assurances . . .	...	...	...	...	154,368	1,615,050	...	1,615,050
Joint Lives . .	...	...	...	...	201,413	3,194,883	...	3,194,883
	...	...	...	...	13,213,554	126,797,704	...	126,797,704
Annuities.								
Immediate . .	...	...	...	...	1	15	...	15

The above figures are based on Returns deposited for the most part during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of three companies, namely, the Co-operative, the Customs Fund, and the Northern, the amount of business at a more recent date has been included, but the figures of the six Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Application of Makeham's Modification of Gompertz's Expression for the Law of Mortality to the Practical Calculation of the Values of Survivorship Benefits. By Francis E. Colenso, M.A., F.I.A., *Actuary of the Eagle Insurance Company.*

[Read before the Institute, 30 April 1894.]

THE value, to the practical actuary, of Makeham's formula for the law of mortality, as a means of facilitating the calculation of joint-life benefits, is well established. Considerable use, for instance, is undoubtedly made of the tables appended to the second volume of the Institute *Text-Book*, in questions of every-day occurrence involving the computation by the H^{M} Tables of annuities on three or four lives. In like manner, where the employment of Carlisle annuities is desirable, the tables prepared in 1880 (see *J.I.A.*, xxii, 191), by Messrs. George King and G. F. Hardy, afford a ready means of obtaining results which are at once consistent, and substantially in agreement with values derived directly from the parent table. With regard, however, to the calculation of contingent survivorship benefits, it seems worthy of consideration whether something may not be done to render expressions based upon Makeham's formula better adapted than they are, in the shape in which they are usually presented, to meet the exigencies of practical requirements. In the United

States some attention has been given to this question. The table for male life based upon the Thirty American Offices' Experience lends itself, throughout nearly its whole extent,* to graduation by Makeham's formula, and the comprehensive treatise entitled *System and Tables of Life Insurance*† contains auxiliary tables designed to shorten the task of applying a well-known expression derived from that formula for the assurance value A^1_{uw}, u being a status of m, and w a status of n, joint lives.

One of the purposes of the following short paper is to bring under notice a method of employing Makeham's formula having, it is thought, not a little to recommend it for facility and directness of application. The method will be applied only to benefits other than those known as compound survivorship benefits, but some consideration will hereafter be also given to the latter class of benefit.

Makeham's expression for contingent survivorship assurances was communicated by him to the *Journal* in a letter dated July 1861,‡ and is reproduced, with a slight change in form and notation, in vol. ii of the *Text-Book*, p. 233, as follows:

$$\overline{A}^1_{uw} = \frac{K_x}{K_x+K_y}\overline{A}_{uw} - \frac{mK_y-nK_x}{K_x+K_y}\log_e s \,.\, \overline{a}_{uw}.$$

Here u denotes, as has been said, a status of m, and w a status of n, joint lives. These lives being, in the one case, (x_1), (x_2) . . . (x_m) say, and in the other (y_1), (y_2) . . . (y_n), K_x stands for $c^{x_1}+c^{x_2}+\ldots+c^{x_m}$, while K_y stands for $c^{y_1}+c^{y_2}+\ldots+c^{y_n}$, where c is the constant of Makeham's function, $l_x = ks^x(g)^{c^x}$.

The demonstration given in the *Text-Book* is as follows:

We have, using the symbol λ to denote $\log_e$,

$$\lambda\, {}_tp_u = mt\lambda s + K_x\lambda G$$

$$\lambda\, {}_tp_{uw} = (m+n)t\lambda s + (K_x+K_y)\lambda G$$

whence $$\frac{\lambda\, {}_tp_u - mt\lambda s}{\lambda\, {}_tp_{uw}-(m+n)t\lambda s} = \frac{K_x}{K_x+K_y}$$

and $$\lambda\, {}_tp_u = \frac{K_x}{K_x+K_y}\lambda\, {}_tp_{uw} + \frac{mK_y-nK_x}{K_x+K_y}t\lambda s.$$

* From age 10 to age 90.

† A treatise developed from the experience and records of Thirty American Life Offices, under direction of a Committee of Actuaries, by Levi W. Meech, Actuary in charge. Revised edition, 1886.

‡ *J.I.A.*, ix, 361.

Differentiating with respect to t, this becomes

$$\frac{d_t\,{}_tp_u}{{}_tp_u}=\frac{K_x}{K_x+K_y}\cdot\frac{d_t\,{}_tp_{uw}}{{}_tp_{uw}}+\frac{mK_y-nK_x}{K_x+K_y}\lambda s\,.\,dt\,;$$

and multiplying by v^t, and integrating,

$$-\int v^t\,{}_tp_w d_t\,{}_tp_u=-\frac{K_x}{K_x+K_y}\int v^t d_t\,{}_tp_{uw}-\frac{mK_y-nK_x}{K_x+K_y}\lambda s\int v^t\,{}_tp_{uw}\,.\,dt.$$

Finally, taking the integral between the limits 0 and ∞.

$$\bar{A}^1_{uw}=\frac{K_x}{K_x+K_y}\bar{A}_{uw}-\frac{mK_y-nK_x}{K_x+K_y}\lambda s\,.\,\bar{a}_{uw}\quad . \quad . \quad . \quad (1)$$

Mr. Meech, in the work above referred to, adopts the following method of solution*:

We have, slightly altering his notation,

$$\bar{A}^1_{uw}=-\int_0^\infty\frac{v^t l_{w+t}d(l_{u+t})}{l_u l_w}$$

$$=-\int_0^\infty(m\lambda s dt+\lambda g c^u dc^t)s^{(m+n)t}g^{(c^u+c^w)(c^t-1)}$$

where $\qquad l_u=l_{x_1}l_{x_2}\;.\;.\;.\;l_{x_m}$

$$c^u=c^{x_1}+c^{x_2}+\;.\;.\;.\;+c^{x_m}$$

and l_w and c^w have corresponding significations.

Interchanging u and w,

$$\bar{A}^1_{wu}=-\int_0^\infty(n\lambda s dt+\lambda g c^w dc^t)s^{(m+n)t}g^{(c^u+c^w)(c^t-1}\,.$$

Eliminating the last term of these equations, we get

$$c^w\bar{A}^1_{uw}-c^u\bar{A}^1_{wu}=-(mc^w-nc^u)\lambda s\,.\,\bar{a}_{uw},$$

and from this the required expression follows at once, in the form preferred by Mr. Meech,

$$A^1_{uw}=a'+b'A_{uw},$$

where a' and b' are put for the expressions

$$\frac{mc^{w-u}-n}{c^{w-u}+1}\cdot\frac{\sqrt{v}}{\delta}(-\lambda s)$$

and $\qquad \dfrac{m}{m+n}-\dfrac{mc^{w-u}-n}{c^{w-u}+1}\left(\dfrac{1}{m+n}-\delta^{-1}\lambda s\right)\qquad$ respectively,

* P. 293.

and the factor $\sqrt{r}$ is introduced in order to convert $\overline{A}^1_{uw}$ into A^1_{uw}.*

Mr. Meech remarks:—"The preceding expressions for a' "and b' have the singular property of remaining constant for the "duration of the given lives. Being easily tabulated, they appear "adapted for an important part in the system of survivorships." They share this property, it will be observed, with the coefficients of $\overline{A}$ and $\bar{a}$ in the equation (1), the fact being that

$$\frac{n(c^{x_1+t}+c^{x_2+t}+\ .\ .\ .)-m(c^{y_1+t}+c^{y_2+t}+\ .\ .\ .)}{c^{x_1+t}+c^{x_2+t}+\ .\ .\ .+c^{y_1+t}+c^{y_2+t}+\ .\ .\ .}$$

is independent of t.

The quantities a' and b' are tabulated by Mr. Meech for the case $m=n=1$, and a simple relation pointed out between their values in that case and in the case where $m=n=r$ say, *i.e.*, where the lives are equal in number.

The following simple and direct method of obtaining the general expression for $\overline{A}^1_{uw}$ may be here noted.

Let $c^u+c^w=(m+n)c^r$ and $c^u=mc^{r'}$.

Expressing μ_{u+t} in terms of Makeham's constants, we have

$$\overline{A}^1_{uw}=ml^{-1}{}_{uw}\int_0^\infty v^t l_{u+t\,.\,w+t}(-\lambda g\lambda c\,.\,c^{r'+t}-\lambda s)dt\,,$$

and it is evident that by a simple manipulation involving the introduction of any power of c, say c^h, we may write the integral,

$$ml^{-1}{}_{uw}\int_0^\infty v^t l_{u+t\,.\,w+t}\left\{-\lambda g\lambda c\,.\,c^{r'-h}c^{h+t}-\lambda s+c^{r'-h}\lambda s-c^{r'-h}\lambda s\right\}dt\,.$$

That is to say,

$$\overline{A}^1_{uw}=mc^{r'-h}l^{-1}{}_{uw}\int_0^\infty v^t\,.\,l_{u+t\,.\,w+t}\,\mu_{h+t}dt+m\lambda s(c^{r'-h}-1)\bar{a}_{uw}\,.$$

Since h may have any value we please, we may put it $=r$, and get

$$\overline{A}^1_{uw}=mc^{r'-r}l^{-1}{}_{uu}\int_0^\infty v^t\,.\,l_{u+t\,.\,w+t}\mu_{r+t}dt+m\lambda s(c^{r'-r}-1)\bar{a}_{uw}$$

that is, since $\mu_{u+t}+\mu_{w+t}=(m+n)\mu_{r+t}$,

$$\overline{A}^1_{uw}=\frac{mc^{r'-r}}{m+n}\cdot\overline{A}_{uw}+m\lambda s(c^{r'-r}-1)a_{uw}\quad .\quad .\quad .\quad .\quad .\quad (2)$$

the required relation.

* By a misprint, the slur has been retained in the result given on p. 294 of Mr. Meech's work.

Wherever, in fact, μ appears under the sign of integration, the integral can be at once written down in a form corresponding to (2).

Resuming the consideration of (1), the expression reduces in the case of three lives to

$$\bar{A}^1_{xyz} = \frac{c^x}{c^x+c^y+c^z}\bar{A}_{xyz} - \frac{c^y+c^z-2c^x}{c^x+c^y+c^z}\lambda s\,.\,\bar{a}_{xyz} \quad . \quad . \quad (3)$$

or, in the notation of (2),

$$= \{\tfrac{1}{3}c^{x-r}(1-3\delta^{-1}\lambda s) + \delta^{-1}\lambda s\}\bar{A}_{xyz} + \delta^{-1}\lambda s(c^{x-r}-1) \quad (4)$$

Expressing $\bar{A}_{xyz}$ in terms of $\bar{a}_{xyz}$, this becomes

$$\bar{A}^1_{xyz} = \tfrac{1}{3}c^{x-r} - \{c^{x-r}(\tfrac{1}{3}\delta - \lambda s) + \lambda s\}\bar{a}_{xyz}$$

$$= \text{say, } \underset{1}{\mathbf{f}}(c^{x-r}) - \underset{2}{\mathbf{f}}(c^{x-r})\bar{a}_{xyz} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Given, therefore, suitable tables of $\frac{1}{3}c^n$ and $\{\frac{1}{3}\delta c^n - \lambda s(c^n-1)\}$ for all required values of n, the operation of finding $\bar{A}^1_{xyz}$ by (5) may be regarded as fairly simple. The following figures exemplify the working out of $\bar{A}^1_{30.45.60}$ by this method:

Taking the *Text-Book* Tables as the basis, we have,

$$\begin{array}{rl}
\mu_{30}+\mu_{45}+\mu_{60} = & \cdot 04892 \;\therefore\; \mu = \cdot 01631 = \mu_{51} \text{ almost exactly.} \\
\text{at 4 per-cent } \delta = & \underline{\cdot 03922} \\
12) & \underline{\cdot 08814} \\
 & \cdot 007 \\
a_{51:51:51} = & \underline{7\cdot 764} \\
 & 7\cdot 757 \;\therefore\; \bar{a}_{51:51:51} = 8\cdot 257 \\
 & \qquad\qquad = \log^{-1} \;\; 0\cdot 91682
\end{array}$$

From a table of $\underset{2}{\mathbf{f}}(c^n)$ we find $\log \underset{2}{\mathbf{f}}(c^{-21}) = \bar{3}\cdot 52646$

$$\begin{array}{rr}
 & \bar{2}\cdot 44328 \\
=\log^{-1} & \cdot 02775 \\
\text{add } \frac{1}{3}\,c^{-21} & \underline{\cdot 04899} \\
\bar{A}^1_{30.45.60} = & \underline{\underline{\cdot 07674}}
\end{array}$$

In this example, the coincidence of $\bar{a}_{30.45.60}$ with $\bar{a}_{51.51.51}$ saves us from the interpolations which are in general necessary when accuracy is required.

The other cases of survivorship assurances involving three lives which are enumerated in ch. xiii of the *Text-Book* can, of

course, be evaluated by means of $\bar{A}^1_{xyz}$ and two-life survivorship assurances.

But the following process, it is thought, has the merit of reducing the operation of finding $\bar{A}^1_{xyz\ldots}$ to its simplest possible dimensions, while at the same time providing the means of directly computing the values of its companion benefits. The method will be illustrated by means of assurances involving only three lives, its applicability in the case of a larger number of lives being obvious.

Denoting common logarithms by log, and, as before, logarithms to the base e by λ, we have

$$\bar{A}^1_{xyz} = l^{-1}{}_{xyz}\int_0^\infty v^t . l_{x+t.y+t.z+t}(-\lambda s - \lambda g \lambda c\, c^{x+t})dt.$$

If, then we denote by $\bar{a}'_{xyz}$ what $\bar{a}_{xyz}$ becomes when $\frac{1}{1+i}$ is replaced by $\frac{c}{1+i}$, we can at once write

$$\bar{A}^1_{xyz} = -\lambda s \bar{a}_{xyz} - \lambda g . \lambda c\, c^x \bar{a}'_{xyz}$$

$$= -\lambda s \bar{a}_{xyz} + (\mu_x + \lambda s)\bar{a}'_{xyz} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The tabulation, therefore, of the three functions, $-\lambda s a_{xyz}$, $\log(\mu_x + \lambda s)$, and $\log \bar{a}'_{xyz}$, will give us at once the means of finding $\bar{A}^1_{xyz}$ by a very simple operation.

The annexed tables, in the preparation of which the most efficient assistance was rendered to the writer by Mr. J. F. E. Hall, A.I.A., are based upon the Carlisle annuity-values as graduated by the methods employed by Messrs. King and Hardy* to graduate Milne's values.

Having regard to the results obtained by these gentlemen by the application of what they call the "aggregate method" of deducing Makeham's constants, it has been thought that it would be worth while to calculate a set of annuity-values on the basis of one of the "aggregate" curves described by them on page 201 of their paper. The comparison which they there give of the deviations from Milne's unadjusted table of l_x that result from employing constants obtained by the "aggregate" method shows that of four curves "all gave sufficiently satisfactory results", and that there is not much to choose between them. The first of these curves, which embraces the Carlisle Table in

* *Vide J.I.A.*, xxii, 191.

one formula from age 15 to age 100, has been taken as the basis for a set of 3 per-cent annuities.

A description of the manner in which the values of $\bar{a}'_x$, a'_{xy}, and $\bar{a}'_{xyz}$, have been prepared will not be out of place here. Recourse has been had to the relation

$$\bar{a}'_{xyz\ldots} = \frac{\bar{a}_{xyz\ldots} - k}{nk'c^r},$$

where $k = \dfrac{1}{\delta - n\lambda s}$, $k' = \lambda g \lambda c k$, and $nc^r = c^x + c^y + \&c.$

The proof of this relation is as follows:

We have

$$\bar{A}_{xyz\ldots} = l^{-1}_{xyz\ldots}\int_0^{\infty} v^t . l_{x+t.y+t.z+t\ldots}(\mu_{x+t} + \mu_{y+t} + \&c.)dt.$$

or, $\quad 1 - \delta\bar{a}_{xyz\ldots} = -n(\lambda s \bar{a}_{xyz\ldots} + c^r \lambda g . \lambda c . \bar{a}'_{xyz\ldots})$

whence the relation in question.

As it was requisite to obtain tables of $-\lambda s . \bar{a}_x$, $-\lambda s . \bar{a}_{xx}$, $-\lambda s . \bar{a}_{xxx}$, and $(\mu_x + \lambda s)$, the expression for $\bar{a}'_{xxx\ldots}$ was put into the form

$$\frac{\bar{A}_{xxx\ldots} + n\lambda s \bar{a}_{xxx\ldots}}{n(\mu_r + \lambda s)},$$

and the quantities to be tabulated were

$$\frac{\bar{A}_x + \lambda s \bar{a}_x}{\mu_r + \lambda s}, \quad \frac{\bar{A}_{xx} + 2\lambda s . \bar{a}_{xx}}{2(\mu_r + \lambda s)}, \quad \text{and} \quad \frac{\bar{A}_{xxx} + 3\lambda s . \bar{a}_x}{3(\mu_r + \lambda s)}.$$

The continuous annuities corresponding to those tabulated according to the methods employed by Messrs. King and Hardy were first formed by means of $\Delta\bar{a} = \frac{1}{12}\Delta\mu$. The factors λs, $2\lambda s$, and $3\lambda s$, were then placed on the machine and multiplied into $\bar{a}_x$, $\bar{a}_{xx}$, and $\bar{a}_{xxx}$, and the products deducted from $\bar{A}_x$, $\bar{A}_{xx}$, and $\bar{A}_{xxx}$, respectively, these functions being obtained by means of the recently-published conversion tables.* A logarithmic process completed the construction.

It is easy to see that, if the accented annuity-values in the above formula for $\bar{A}^1_{xyz}$ be expressed in terms of ordinary continuous annuities, the usual formulæ in terms of Makeham's constants will be the result.

The various cases of survivorship assurances involving three

* Premium Conversion Tables, &c., by the late Howard J. Rothery and Gerald H. Ryan. London, C. & E. Layton.

lives which are enumerated in chapter xiii of the *Text-Book* admit of being treated in a similar manner.

Denoting the quantities $-\lambda s$ and $\mu_x+\lambda s$ by h and h', we have

$$\overline{\mathrm{A}}^2_{\underset{1}{x}yz}=h\bar{a}_{y|xz}+h'\bar{a}'_{y|xz}$$

$$\overline{\mathrm{A}}^2_{xyz}=h(\bar{a}_{z|xy}+\bar{a}_{y|xz})+h'(\bar{a}'_{z|xy}+\bar{a}'_{y|xz})$$

$$\overline{\mathrm{A}}^3_{xyz}=h\bar{a}_{\overline{yz}|x}+h'\bar{a}'_{\overline{yz}|x}$$

$$\overline{\mathrm{A}}^{\;1}_{\overline{xy}z}=2h\bar{a}_{xyz}+2h''\bar{a}'_{xyz},$$

$$\text{[where}\quad h''=\tfrac{1}{2}(\mu_x+\mu_y)+\lambda s=\mu_w+\lambda s]$$

$$\overline{\mathrm{A}}^1_{x:\overline{yz}}=h\bar{a}_{x:\overline{yz}}+h'\bar{a}'_{x:\overline{yz}}$$

$$\overline{\mathrm{A}}_{\overline{xy}.\underset{3}{z}}=h(\bar{a}_{y|xz}+\bar{a}_{x|yz})+h'(\bar{a}'_{y|xz}+\bar{a}'_{x|yz})$$

The annual premiums are obtained in these cases by means of the annuities, $\bar{a}_{xz}$, $\bar{a}_{\overline{yz}:x}$, a_x, $\bar{a}_{xyz}$, $\bar{a}_{\overline{yz}:x}$, and $\bar{a}_{\overline{xy}:z}$, respectively.

Ex.—To find $\overline{\mathrm{A}}^{\;1}_{\overline{30.45}:60}$

If we assume that we possess tables similar to those now submitted, but based upon the *Text-Book* 4 per-cent annuities, we have at once

$$
\begin{aligned}
\log \bar{a}'_{30:45:60} &= \log \bar{a}'_{51:51:51} = 1{\cdot}23630 \\
\log(\mu_{39\cdot9}+\lambda s) &= \bar{3}{\cdot}56467 \\
\bar{2}{\cdot}80097 &= \log^{-1}\ {\cdot}06324 \\
(-\lambda s)\ \bar{a}_{30:45:60} &= \quad {\cdot}05113 \\
 & \quad {\cdot}11437 \\
 & \qquad\quad 2 \\
\overline{\mathrm{A}}^{\;1}_{\overline{30:45}:60} &= \quad {\cdot}22874
\end{aligned}
$$

a result agreeing closely with that given on page 234 of the *Text-Book*.

Ex.—To find $\overline{\mathrm{A}}^1_{30:\overline{45.60}}$:

On the basis of the *Text-Book* annuities, we have, at 4 per-cent,

$$
\begin{array}{rlcr}
(-\lambda s)\bar{a}_{30:45}=(-\lambda s)\bar{a}_{39\cdot9:39\cdot9}= & {\cdot}07972 & \left\{\begin{array}{c}\text{corresponding}\\ \text{accented annuity}\end{array}\right\} & =45{\cdot}729 \\
(-\lambda s)\bar{a}_{30:60}=(-\lambda s)\bar{a}_{53\cdot1:53\cdot1}= & {\cdot}05753 & \text{,,} \quad \text{,,} & 21{\cdot}692 \\
 & {\cdot}13725 & & 67{\cdot}421 \\
(-\lambda s)\bar{a}_{30:45:60}=(-\lambda s)\bar{a}_{51:51:51}= & {\cdot}05113 & \text{,,} \quad \text{,,} & 17{\cdot}230 \\
(-\lambda s)\bar{a}_{30:\overline{45:60}}= & {\cdot}08612 & \text{,,} \quad \text{,,} & 50{\cdot}191 \\
50{\cdot}191\times(\mu_{30}+\lambda s)= & {\cdot}07479 & & \\
 & {\cdot}16091=\mathrm{A}^1_{30:\overline{45:60}} & &
\end{array}
$$

It will be noted that, when the two annuities requisite for obtaining $\overline{A}^1_{xyz}$ are written down, the values of $\overline{A}_{x\overset{1}{y}z}$ and $\overline{A}_{xy\overset{1}{z}}$ can be obtained by simply employing μ_y and μ_z respectively, instead of μ_x in the factor $(\mu+\lambda s)$. A very ready check on the work is thus afforded. And in a similar manner we can test the results obtained in the case of any other simple survivorship benefit.

Let us, for example, calculate $\overline{A}^1_{30:45:60}$, $\overline{A}_{30:\overset{1}{45}:60}$ and $\overline{A}_{30:45:\overset{1}{60}}$ by the annexed tables.

We have

$$\overline{A}^1_{30:45:60} = -\lambda s\bar{a}_{30:45:60} + (\mu_{30}+\lambda s)\bar{a}'_{30:45:60}$$
$$= \cdot 07436 \qquad + \cdot 0012371 \times 20 \cdot 200$$
$$= \cdot 09935$$

Substituting for the first factor of the second term $(\mu_{45}+\lambda s)$ and $(\mu_{60}+\lambda s)$ respectively we get,

$$\overline{A}_{30:\overset{1}{45}:60} = \cdot 17270$$
$$\overline{A}_{30:45:\overset{1}{60}} = \cdot 46142$$
$$\overline{A}_{30:45:60} \text{ should } \therefore = \cdot 73347$$

and this is the precise result obtained by entering the 3 per-cent conversion tables with $\bar{a}=9 \cdot 017$.

It may be mentioned that the corresponding values calculated on the basis of Messrs. King and Hardy's tables, are ·09839, ·17354, and ·46471 respectively, and that the application of formula 39*a* of the *Text-Book* to Milne's table brings out the values ·097, ·160, and ·457 respectively.

On a comparison between the three last values and those which have been derived from the annexed tables, the considerable divergence in the results presented in the case of $\overline{A}_{30:\overset{1}{45}:60}$ will not escape attention. The figures will not, however, surprise those who have studied the precise nature of the defects resulting from Milne's manipulation of the data that form the basis of his table. As is very well known, the fullest light has been thrown by Mr. George King (*J.I.A.*, xxiv, 186), upon the genesis of Milne's l_x column, and on page 202 of the paper cited, its author indicates the character of the corrections required in Milne's curve. It so happens that the values of μ_x at and about age 45 are particularly irregular; so much so that $\mu_{52}<\mu_{45}$ and μ_{50} substantially $=\mu_{41}$. Any graduation then which produces a more natural progression in the values of μ_x may be expected to result in a value of $\overline{A}_{30:\overset{1}{45}:60}$ materially larger, but nevertheless more

suitable, it is submitted, for adoption in practice, than the corresponding value computed directly from Milne's table.

Compound Survivorship Annuities and Assurances.

Inasmuch as the values of all benefits of this nature can be expressed in terms of compound survivorship annuities of the form $\bar{a}_{\overset{1}{yz}|x}$ and simple benefits, interest naturally centres round this function. It is probable that in the whole range of the calculus of life contingencies, no function has attracted more attention from actuarial mathematicians, relatively to its small commercial importance. In a well-known paper by Mr. George King (*J.I.A.*, xxvi, 276), the remark occurs:—"Compound "survivorship annuities and assurances have always been a source "of trouble and weariness to actuaries." Mr. King's purpose was to exemplify the great importance, in practice, of formulæ of interpolation applied to the approximate evaluation of definite integrals. The epoch-making contribution to the literature of this subject which had proceeded from the pen of Mr. G. F. Hardy (*J.I.A.*, xxiv, 95) was justly characterized by Mr. King as destined to revolutionize our methods of dealing with complicated problems. And it will be conceded that, in so far as regards such monetary values as are required from time to time in practice, the formula in question, and in particular that which is generally referred to as 39(*a*) of the *Text-Book*, which Mr. King obtained by an amplification of Mr. Hardy's analysis, effectually relieves the actuary from all anxiety on the subject of complex benefits. But functions of the form $a_{\overset{1}{yz}x}$ remain, nevertheless, a subject of interest to the mathematician. Attempts to resolve the general expression $\Sigma v^t {}_tp_{x\,t}q_{\overset{1}{yz}}$, or the corresponding definite integral, into expressions involving the values of simple benefits, have probably occupied the attention of most students of the Institute, and may be classed with efforts to square the circle or trisect a given angle. The special shapes assumed by the function when the l_x column is made to conform to a mathematical law invite attempts at integration with more promise of success. As is well known, the assumption of Gompertz's law enables us to write $\bar{a}_{\overset{1}{yz}|x} = \mathrm{Q}_{\overset{1}{yz}}\bar{a}_{yz|x} = \dfrac{c^z}{c^y+c^z}\bar{a}_{yz|x}$. But if we assume a law which more satisfactorily embraces the l_x column, such as Makeham's modification of Gompertz's law, a difficulty presents

itself which, so far as the writer is aware, has not as yet been overcome. Mr. Makeham, indeed, in the letter to which reference has been made, seems to intimate that he had discerned the possibility of arriving at a solution of the problem; but nothing that has appeared in the *Journal* with reference to his expression for the law of mortality has redeemed the function $\bar{a}_{y\overset{1}{z}|x}$ from its character of being thoroughly intractable.

Confining our attention to the continuous function $\bar{a}_{y\overset{1}{z}|x}$, which we may denote by ϕ, while we denote by ψ its companion function $\bar{a}_{\overset{1}{y}z|x}$, so that $\phi+\psi=\bar{a}_{yz|x}=\chi$ say, we may throw the general expression for it into either of two alternative forms.

$$\phi=l^{-1}{}_{xyz}\int_0^\infty v^t.\,l_{x+t.y+t.z+t}\cdot\mu_{z+t}\cdot\bar{a}_{x+t}dt \quad . \quad . \quad . \quad (1)$$

or $$\phi=l^{-1}{}_{xyz}\int_0^\infty v^t.\,l_{x+t\,t}q_{y\overset{1}{z}}dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By means of Makeham's expression for μ, we may write (1) in the form

$$\phi=c^{z-w}\,\tfrac{1}{2}\chi+\lambda s\,(c^{z-w}-1)\mathrm{U} \quad . \quad . \quad . \quad . \quad (3)$$

where $\mathrm{U}=l^{-1}{}_{xyz}\int_0^\infty v^t.\,l_{x+t.y+t.z+t}\cdot\bar{a}_{x+t}\cdot dt$, and will be recognized as the value of an annuity on x increasing during the joint lives xy. Here $c^w=\frac{1}{2}(c^y+c^z)$.

Similarly $$\psi=c^{y-w}\tfrac{1}{2}\chi+\lambda s(c^{y-w}-1)\mathrm{U}$$
$$=(2-c^{z-w})\tfrac{1}{2}\chi+\lambda s(1-c^{z-w})\mathrm{U},$$

which merely expresses the relation $\psi=\chi-\phi$, and gives us no means of eliminating U.

In like manner the relations which can be obtained by varying the index of c in accordance with the method explained above, or by merging c with v, when not identities, cease to be so merely through the introduction of new quantities which resist integration equally with U. Thus it will be seen that, as U remains constant for changes in the variables y and z provided the relation $c^y+c^z=$ a constant, be maintained,

$$\mathrm{U}=\mathrm{L}^t{}_{z=w}\frac{\phi-c^{z-w}\frac{1}{2}\chi}{\lambda s(c^{z-w}-1)}.$$

This fraction assumes the form $\frac{0}{0}$ when $z=w$, and in search

for the limit we must differentiate above and below. In this way we get

$$\lambda s \lambda c \mathrm{U} = \left(\text{limit } \frac{d\phi}{dz}\right) - \lambda c \tfrac{1}{2}\chi.$$

Here $$\frac{d\phi}{dz} = \left(\frac{d\phi}{dz}\right) + \left(\frac{d\phi}{dy}\right)\frac{dy}{dz} \text{ and } \frac{dy}{dz} = -c^{z-y}.$$

Differentiating, with regard to y and z respectively, the integral expressing ϕ, we have

$$\left(\frac{d\phi}{dy}\right) = \mu_y \phi - l^{-1}{}_{xyz}\int_0^\infty v^t . l_{x+t.y+t.z+t} . \mu_{y+t} . \mu_{z+t} . \bar{a}_{x+t} . dt,$$

and $$\left(\frac{d\phi}{dz}\right) = \mu_z \phi + \bar{\mathrm{A}}_{xy\overset{1}{z}} - \mu_z \bar{a}_x + \text{(the same integral)}.$$

Denoting this last integral by H_{yz}, and proceeding to the limit, we find

$$\lambda s \lambda c \mathrm{U} = \bar{\mathrm{A}}_{xw\overset{1}{w}} - \mu_w \bar{a}_x - \lambda c \tfrac{1}{2}\chi + 2\mathrm{H}_{ww}.$$

But in the absence of any method of evaluating H_{ww} we are no better off than we were before.

From the alternative form for the integral expressing ϕ, we have

$$\phi = \mathrm{Q}_{y\overset{1}{z}} \bar{a}_x - \tfrac{1}{2} c^{z-w} \bar{a}_{xyz} - \lambda s (c^{z-w} - 1)\mathrm{W},$$

where $$\mathrm{W} = l^{-1}{}_{xyz}\int_0^\infty v^t . l_{x+t.y+t.z+t} . \bar{e}_{y+t.z+t} . dt,$$

and if we eliminate ϕ from this and (3) we obtain a relation of some little interest:

$$\mathrm{U} + \mathrm{W} = \bar{e}_{yz} . \bar{a}_{xyz}.$$

Returning to (3), it will be seen that the numerical value of the factor $(c^{z-w} - 1)$ in the coefficient of U is always less than unity. An approximate value may therefore be adopted for U, the error in which is multiplied by a quantity that is always less than λs in numerical value.

Thus let $\mathrm{U} = \bar{a}_k . \bar{a}_{xyz}$, where $\bar{a}_k$ is an unknown quantity. Differentiating $\bar{a}_k . \bar{a}_{xyz}$ and $l^{-1}{}_{xyz}\int_0^\infty v^t . l_{x+t.y+t.z+t} . \bar{a}_{x+t}\, dt$ with respect to x alone, we get

$$\{(\mu_k + \delta)\bar{a}_k - 1\}\bar{a}_{xyz}\frac{dk}{dx} + \bar{a}_k(\mu_x \bar{a}_{xyz} - \bar{\mathrm{A}}^{1}_{xyz}) = (\mu_x + \delta)\bar{a}_k . \bar{a}_{xyz} - \bar{a}_{xyz}.$$

From this, if we make the assumption that $\frac{dk}{dx}=1$, we find

$$\mu_k = \frac{A^1_{xyz}}{a_{xyz}} = (\mu_x + \delta)\frac{a'_{xyz}}{a_{xyz}} - \lambda s \quad . \quad . \quad . \quad . \qquad (4)$$

This will be found to give a very tolerable approximation to ϕ as long as x is not a very old life.

Thus $a_{45.\overset{1}{60}\,30}$, according to the calculation by formula 39(a) of the *Text-Book*, p. 266, $=6{\cdot}9957$, and by the above approximation $=7{\cdot}0012$.

It may be useful if we here consider a professed solution of the problem before us which has been incautiously admitted into the second part of Mr. Meech's work. On page 294, with a slightly-varied notation, we have the following demonstration:

$$\phi = -\int_0^\infty \frac{v^t . l_{y+t} . l_{x+t} . dl_{z+t} . \bar{a}_{x+t}}{l_{xyz}}$$

$$= \int_0^\infty dA_{x+t.y+t.\overset{1}{z+t}} . a_{x+t}$$

$$= b'\int_0^\infty d\bar{A}_{x+t.y+t.z+t} . \bar{a}_{x+t} .$$

"Here the differentiation refers to the variable t; and b' "enters by the substitution of $a'+b'\bar{A}$ from the previous "investigation. Interchanging y and z,

" $$\psi = b''\int d\bar{A}_{x+t.y+t.z+t} . \bar{a}_{x+t} .$$ "

Hence, by elimination of the integral,

$$\phi = \frac{b'}{b'+b''}\chi .$$

The expression for b' is given in the form

$$\tfrac{1}{3} + (\tfrac{3}{2}c^{z-u} - 1)(\tfrac{1}{3} - \delta^{-1}\lambda s),$$

where $2c^u = c^x + c^y + c^z$.

This is easily transformed into a shape in which it will be recognized as the coefficient of $\bar{A}_{xyz}$ in (5) *supra*.

Thus let $\qquad 3c^r = c^x + c^y + c^z.$

Then $\qquad c^u = \tfrac{3}{2}c^r \qquad \therefore \quad \tfrac{3}{2}c^{z-u} = c^{z-r}.$

Substituting, we get:

$$b' = \tfrac{1}{3}c^{z-r}(1-3\delta^{-1}\lambda s) + \delta^{-1}\lambda s$$

and

$$b'' = \tfrac{1}{3}c^{y-r}(1-3\delta^{-1}\lambda s) + \delta^{-1}\lambda s.$$

The correctness of the above solution depends upon the assumption that

$$d\overline{\mathrm{A}}_{x+t.y+t.\overset{1}{z+t}} = -\,v^t\frac{l_{x+t.y+t}.dl_{z+t}}{l_{xyz}}.$$

Integrating both sides of this equation between the limits t and ∞, we have

$$\overline{\mathrm{A}}_{x+\infty.y+\infty.\overset{1}{z+\infty}} - \overline{\mathrm{A}}_{x+t.y+t.\overset{1}{z+t}} = {}_t\overline{\mathrm{A}}_{x.y.\overset{1}{z}}.$$

Now the expression for $\overline{\mathrm{A}}_{x+t.y+t.\overset{1}{z+t}}$,

$$\frac{c^{z+t}}{c^{x+t}+c^{y+t}+c^{z+t}}\overline{\mathrm{A}}_{x+t.y+t.z+t} + \lambda s(c^{z-r}-1)\bar{a}_{xyz},$$

reduces to $\dfrac{c^z}{c^x+c^y+c^z}$ when $t=\infty$.

This fraction, then, is the limiting value of $\overline{\mathrm{A}}_{x+t.y+t.\overset{1}{z+t}}$, as t increases indefinitely. Thus the above assumption is inadmissible.

It may, perhaps, be instructive to bring into view here the elementary considerations involved in the process of differentiating functions which are presented in the form of definite integrals.

In the application of the integral calculus to life contingencies we are almost entirely concerned, so far as the researches of mathematicians have extended, with definite integrals. The transcendental forms introduced under the sign of integration by means of Gompertz's hypothesis, or of Makeham's modification of that hypothesis "are all alike intractable. None of them "admit of independent summation or integration, excepting by "infinite series."*

When, therefore, we write

$$l^{-1}{}_x\!\int v^t l_{x+t}.dt = \mathbf{f}(x) + \mathrm{C},$$

we know nothing of the general form of $\mathbf{f}(x)$. We only know that, if t_1 and t_2 are particular values of x,

$$\mathbf{f}(t_2) - \mathbf{f}(o) - \{\mathbf{f}(t_1) - \mathbf{f}(o)\} = {}_{t_2}\bar{a}_x - {}_{|t_1}\bar{a}_x,$$

* Woolhouse, *J.I.A.*, xv, 402.

the function $\bar{a}_x$ being our fundamental integral, the numerical values of which are, for practical purposes, tabulated for particular cases, like the values of Euler's and other definite integrals.

It is an observation lying on the threshold of the subject, that the value of a definite integral is a function, not of the variable with respect to which the integration is effected, but of the limits of integration, and of other quantities which, entering into the composition of the function integrated, are, like those limits, treated as constants during the process. It is with reference to such quantities only that we can perform the operation of differentiation upon definite integrals, and some confusion may arise, in certain cases, through our making a symbol like "t" do double duty, *i.e.*, both as the variable of integration and as one of the limits of the summation.

The rules appertaining to differentiation under the sign $\int$ are as follows:*

If
$$u=\int_a^b \phi(x,\, a)dx,$$

the limits a and b being independent of a,

$$\frac{du}{da}=\int_a^b \frac{d\phi(x,\, a)}{da}dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But if a and b are not independent of a,

$$\frac{du}{da}=\int_a^b \frac{d\phi(x,\, a)}{da}dx+\phi(b,\, a)\frac{db}{da}-\phi(a,\, a)\frac{da}{da} \quad . \quad . \quad (6)$$

If, then, we write

$$\bar{a}_x=l^{-1}{}_x v^{-x}\int_x^\infty v^x l_x dx,$$

instead of using a stricter form,

$$\bar{a}_x=l^{-1}{}_x v^{-x}\int_x^\infty v^t l_t dt,$$

and differentiate with regard to x, the variable of $\bar{a}_x$, we have, from (6),

$$\frac{d\bar{a}_x}{dx}=(\mu_x+\delta)\bar{a}_x-l^{-1}{}_x v^{-x}(v_x l_x)$$
$$=(\mu_x+\delta)\bar{a}_x-1,$$

but only as a consequence of our tacitly treating $v^x l_x dx$ under the

* Bertrand, *Calcul Intégral*, ch. viii.

sign as constant, or, rather, replacing that expression by $v^t l_t dt$, where t is independent of x.

Let us now apply the above rules to the differentiation of $\overline{\mathrm{A}}_{x+t \cdot y+t \cdot \overset{1}{z+t}}$, ($=\overline{\mathrm{A}}$ say), with regard to t. We may in the first place express this function by

$$l^{-1}\int_0^\infty v^\tau l_{x+t+\tau} \ldots . \mu_{z+t+\tau} d\tau,$$

using obvious abbreviations. Hence, if we write $\Sigma\mu$ for $\mu_{x+t}+\mu_{y+t}+\mu_{z+t}$ we obtain by (5),

$$\frac{d\overline{\mathrm{A}}}{dt}=\mathrm{A}\Sigma\mu - l^{-1}\int_0^\infty v^\tau l_{x+t+\tau} \ldots . \mu_{z+t+\tau}\Sigma\mu d\tau$$

$$+ l^{-1}\int_0^\infty v^\tau l_{x+t+\tau} \ldots . \frac{d}{dt}\mu_{z+t+\tau} d\tau .$$

Of the two integrals in this expression the latter can be resolved, by integration *per partes,* into

$$\delta\overline{\mathrm{A}} - \mu_{z+t} + l^{-1}\int_0^\infty v^\tau l_{x+t+\tau} \ldots . \mu_{z+t+\tau}\Sigma\mu d\tau,$$

whence
$$\frac{d\overline{\mathrm{A}}}{dt}=(\Sigma\mu+\delta)\overline{\mathrm{A}}-\mu_{z+t}.$$

By expressing $\overline{\mathrm{A}}$, however, in the form

$$l^{-1}\int_t^\infty v^{\tau-t} l_{x+\tau . y+\tau . z+\tau} . \mu_{z+\tau} d\tau,$$

and applying rule (6) we can at once write

$$\frac{d\overline{\mathrm{A}}}{dt}=(\Sigma\mu+\delta)\overline{\mathrm{A}}-\mu_{z+t}.$$

Returning to the American solution of the problem before us, it may be said, briefly, that the device employed in obtaining it depends upon the assumption that $-\overline{\mathrm{A}}=$ the *indefinite* integral $l^{-1}\int v^t l_{x+t . y+t . z+t} . \mu_{z+t} . dt$ plus a constant. It is only on this assumption that the differential of $\overline{\mathrm{A}}$ can be equated to the element which is integrated.

It may be added that when the solution is applied to obtain the value of $\bar{a}_{45:\overset{1}{60}\,30}$ at 4 per-cent by the *Text-Book* tables, that value works out 8·3023 instead of 6·9957, the result which follows the employment of the actuary's *vade mecum,* formula 39(a).

The above considerations may, it is hoped, throw some little light on the difficulties involved in attempts to integrate ϕ.*

Reference has been made to the small importance of this function in practice. In a well-known monograph—one of two which are dear to the hearts of students—to be found in vol. x of the *Journal*, entitled "Solutions of the Compound Survivorship Problems", Mr. Makeham gives, as an illustration of the class of cases which a table of ϕ would be calculated to meet, an example in which a son is entitled to an estate in the event of his surviving his father, and is desirous of providing an annuity of £500 to his (the son's) wife, to commence at his death in the event of his dying before his father. Mr. Makeham adds:—"When it is "considered how frequently cases of the above description must "occur, and at how little cost a provision can be secured against "the contingency to which the wife is exposed, it must be a "matter of surprise that assurances of the kind referred to are "not more frequently effected. The explanation is probably to "be found in the fact that the general public are but little aware "with what precision the science of life contingencies, when the "requisite tables have been computed, is capable of meeting the "various complications in which contingent life interests are "frequently involved."

Having regard to the special schemes and prospectuses that are pressed upon their notice, the public have certainly, at the present day, no reason to complain that they are left in ignorance of the capabilities of actuarial science. The writer of this paper has seldom come across enquiries for compound survivorship annuities. He remembers a conspicuous case, however, in which a large annuity of this character, purchased in the early part of the century, proved a source of great loss to the grantors, and it would be interesting to know what the general experience of the offices is in respect of this particular class of contract.

* *i.e.*, to express ϕ in terms of fundamental definite integrals of *one* variable, such as $\bar{a}_{xxx}$.

APPENDIX.

Carlisle Table.

Age	μ_x	$\Delta\mu_x$	$\frac{1}{\Delta\mu_x} \div 10^5$	Age	μ_x	$\Delta\mu_x$	$\frac{1}{\Delta\mu_x} \div 10^5$
15	·00856	·00003	·33333	58	·02421	·00152	·00658
16	·00859	·00003	·33333	59	·02573	·00168	·00595
17	·00862	·00004	·25000	60	·02741	·00183	·00546
18	·00866	·00004	·25000	61	·02924	·00201	·00498
19	·00870	·00004	·25000	62	·03125	·00220	·00455
20	·00874	·00005	·20000	63	·03345	·00241	·00415
21	·00879	·00005	·20000	64	·03586	·00264	·00379
22	·00884	·00005	·16667	65	·03850	·00289	·00346
23	·00890	·00006	·16667	66	·04139	·00317	·00315
24	·00896	·00007	·14286	67	·04456	·00348	·00287
25	·00903	·00007	·14286	68	·04804	·00380	·00263
26	·00910	·00009	·11111	69	·05184	·00417	·00240
27	·00919	·00009	·11111	70	·05601	·00457	·00219
28	·00928	·00010	·10000	71	·06058	·00500	·00200
29	·00938	·00010	·10000	72	·06558	·00549	·00182
30	·00948	·00012	·08333	73	·07107	·00600	·00167
31	·00960	·00013	·07692	74	·07707	·00659	·00152
32	·00973	·00014	·07143	75	·08366	·00721	·00139
33	·00987	·00016	·06250	76	·09087	·00790	·00127
34	·01003	·00017	·05882	77	·09877	·00866	·00115
35	·01020	·00019	·05263	78	·10743	·00949	·00105
36	·01039	·00020	·05000	79	·11692	·01039	·00096
37	·01059	·00023	·04348	80	·12731	·01139	·00088
38	·01082	·00024	·04167	81	·13870	·01247	·00080
39	·01106	·00027	·03704	82	·15117	·01367	·00073
40	·01133	·00029	·03448	83	·16484	·01498	·00067
41	·01162	·00033	·03030	84	·17982	·01640	·00061
42	·01195	·00035	·02857	85	·19622	·01798	·00056
43	·01230	·00039	·02564	86	·21420	·01970	·00051
44	·01269	·00042	·02381	87	·23390	·02158	·00046
45	·01311	·00047	·02128	88	·25548	·02365	·00042
46	·01358	·00051	·01961	89	·27913	·02590	·00039
47	·01409	·00056	·01786	90	·30503	·02839	·00035
48	·01465	·00061	·01639	91	·33342	·03110	·00032
49	·01526	·00067	·01493	92	·36452	·03407	·00029
50	·01593	·00074	·01351	93	·39859	·03733	·00027
51	·01667	·00080	·01250	94	·43592	·04090	·00024
52	·01747	·00089	·01124	95	·47682	·04482	·00022
53	·01836	·00096	·01042	96	·52164	·04910	·00020
54	·01932	·00106	·00943	97	·57074	·05379	·00019
55	·02038	·00116	·00862	98	·62453	·05894	·00017
56	·02154	·00127	·00787	99	·68347	·06458	·00015
57	·02281	·00140	·00714	100	·74805	...	...

Carlisle Table—3 *per-cent.*

Age	$\bar{a}_{xxx}$	$-\Delta\bar{a}_{xxx}$	$\bar{a}'_{xxx}$	$-\Delta\bar{a}'_{xxx}$	Age	$\bar{a}_{xxx}$	$-\Delta\bar{a}_{xxx}$	$\bar{a}'_{xxx}$	$-\Delta\bar{a}'_{xxx}$
15	16·205	·107	127·409	5·509	58	7·027	·281	12·915	·837
16	16·098	·112	121·900	5·276	59	6·746	·278	12·078	·791
17	15·986	·116	116·624	5·094	60	6·468	·276	11·287	·748
18	15·870	·121	111·530	4·890	61	6·192	·272	10·539	·706
19	15·749	·126	106·640	4·712	62	5·920	·267	9·833	·666
20	15·623	·131	101·928	4·544	63	5·653	·264	9·167	·627
21	15·492	·136	97·384	4·364	64	5·389	·258	8·540	·591
22	15·356	·142	93·020	4·201	65	5·131	·253	7·949	·556
23	15·214	·147	88·819	4·035	66	4·878	·247	7·393	·522
24	15·067	·152	84·784	3·883	67	4·631	·241	6·871	·490
25	14·915	·158	80·901	3·729	68	4·390	·234	6·381	·460
26	14·757	·164	77·172	3·589	69	4·156	·228	5·921	·431
27	14·593	·169	73·583	3·453	70	3·928	·220	5·490	·403
28	14·424	·175	70·130	3·316	71	3·708	·213	5·087	·377
29	14·249	·181	66·814	3·186	72	3·495	·205	4·710	·352
30	14·068	·187	63·628	3·058	73	3·290	·197	4·358	·328
31	13·881	·193	60·570	2·936	74	3·093	·189	4·030	·307
32	13·688	·198	57·634	2·819	75	2·904	·182	3·723	·285
33	13·490	·205	54·815	2·707	76	2·722	·173	3·438	·265
34	13·285	·211	52·108	2·598	77	2·549	·166	3·173	·247
35	13·074	·216	49·510	2·492	78	2·383	·157	2·926	·229
36	12·858	·222	47·018	2·390	79	2·226	·150	2·697	·213
37	12·636	·228	44·628	2·290	80	2·076	·142	2·484	·197
38	12·408	·233	42·338	2·195	81	1·934	·134	2·287	·183
39	12·175	·239	40·143	2·103	82	1·800	·127	2·104	·169
40	11·936	·244	38·040	2·012	83	1·673	·120	1·935	·156
41	11·692	·249	36·028	1·927	84	1·553	·112	1·779	·144
42	11·443	·254	34·101	1·843	85	1·441	·106	1·635	·134
43	11·189	·259	32·258	1·762	86	1·335	·099	1·501	·123
44	10·930	·263	30·496	1·685	87	1·236	·093	1·378	·113
45	10·667	·267	28·811	1·609	88	1·143	·087	1·265	·105
46	10·400	·271	27·202	1·537	89	1·056	·081	1·160	·096
47	10·129	·273	25·665	1·466	90	·975	·076	1·064	·089
48	9·856	·277	24·199	1·398	91	·899	·070	·975	·081
49	9·579	·280	22·801	1·332	92	·829	·066	·894	·075
50	9·299	·282	21·469	1·269	93	·763	·062	·819	·069
51	9·017	·283	20·200	1·208	94	·701	·057	·750	·064
52	8·734	·284	18·992	1·148	95	·644	·053	·686	·058
53	8·450	·285	17·844	1·092	96	·591	·049	·628	·053
54	8·165	·286	16·752	1·037	97	·542	·047	·575	·049
55	7·879	·285	15·715	·983	98	·495	·043	·526	·044
56	7·594	·284	14·732	·933	99	·452	·040	·482	·041
57	7·310	·283	13·799	·884	100	·412	...	·441	...

Carlisle Table—3 *per-cent.*

Age	$(-\lambda s\bar{a}_{xxx})$	$\log_{10}(\mu_x+\lambda s)$	$\log_{10}\bar{a}'_{xxx}$	Age	$(-\lambda s\bar{a}_{xxx})$	$\log_{10}(\mu_x+\lambda s)$	$\log_{10}\bar{a}'_{xxx}$
15	·13363	$\bar{4}$·49734	2·10520	58	·05795	·20309	·11109
16	·13274	·53706	·08600	59	·05563	·24276	·08200
17	·13182	·57669	·06679	60	·05333	·28242	·05258
18	·13086	·61637	·04739	61	·05106	·32209	·02280
19	·12987	·65600	·02792	62	·04882	·36176	0·99269
20	·12883	·69566	·00829	63	·04661	·40143	·96223
21	·12775	·73536	1·98849	64	·04444	·44110	·93146
22	·12663	·77503	·96858	65	·04231	·48076	·90031
23	·12546	·81471	·94851	66	·04022	·52043	·86882
24	·12425	·85437	·92831	67	·03819	·56010	·83702
25	·12299	·89404	·90795	68	·03620	·59977	·80489
26	·12169	·93369	·88746	69	·03427	·63943	·77240
27	·12034	·97336	·86678	70	·03239	·67910	·73957
28	·11894	$\bar{3}$·01305	·84590	71	·03058	·71877	·70646
29	·11750	·05273	·82487	72	·02882	·75844	·67302
30	·11601	·09240	·80365	73	·02713	·79810	·63929
31	·11447	·13207	·78226	74	·02551	·83777	·60531
32	·11288	·17173	·76068	75	·02394	·87744	·57089
33	·11124	·21139	·73890	76	·02245	·91710	·53631
34	·10955	·25105	·71690	77	·02102	·95677	·50147
35	·10781	·29072	·69569	78	·01965	·99644	·46627
36	·10603	·33039	·67226	79	·01835	$\bar{1}$·03607	·43088
37	·10420	·37007	·64961	80	·01712	·07577	·39515
38	·10232	·40973	·62673	81	·01595	·11544	·35927
39	·10040	·44940	·60361	82	·01484	·15512	·32305
40	·09843	·48907	·58024	83	·01380	·19477	·28668
41	·09641	·52874	·55664	84	·01281	·23444	·25018
42	·09436	·56840	·53277	85	·01188	·27411	·21352
43	·09226	·60807	·50864	86	·01101	·31378	·17638
44	·09013	·64774	·48424	87	·01019	·35344	·13925
45	·08796	·68740	·45956	88	·00943	·39311	·10209
46	·08576	·72707	·43460	89	·00871	·43278	·06446
47	·08353	·76674	·40934	90	·00804	·47244	·02694
48	·08127	·80641	·38380	91	·00742	·51211	$\bar{1}$·98900
49	·07899	·84608	·35795	92	·00683	·55178	·95134
50	·07668	·88575	·33181	93	·00629	·59145	·91328
51	·07436	·92542	·30535	94	·00578	·63111	·87506
52	·07202	·96508	·27857	95	·00531	·67078	·83632
53	·06968	$\bar{2}$·00475	·25149	96	·00488	·71045	·79796
54	·06733	·04442	·22407	97	·00447	·75011	·75967
55	·06497	·08409	·19631	98	·00408	·78978	·72099
56	·06262	·12375	·16826	99	·00372	·82945	·68305
57	·06028	·16342	·13985	100	·00339	·86912	·64444

Discussion.

The President (Mr. A. Hendriks) said Mr. Colenso's paper was an attempt to reduce to practical problems the mathematical data found in the papers of Gompertz and Makeham. This had not been very much studied in practice because these problems did not occur very frequently. People who insured were usually satisfied to provide for their own condition, and probably the generation that came next. The paper came very aptly, because they had now in discussion the proposition of the Chancellor of the Exchequer to tax severely successive interests, and this might from the insurance point of view lead to a greater application of compound survivorship problems. Not only might owners of large estates wish to secure the inheritance intact for the next heir, but actuaries might be called upon to define at what period this proposal would bring the estate into a negative value. It certainly might lead to proposals being placed before insurance companies going further into succession than had hitherto been attempted, and the paper would help actuaries to arrive at a proper solution.

Mr. A. W. Sunderland was sure they felt grateful to Mr. Colenso for this interesting paper, as practical men apart from actuaries who took a theoretical interest in their work. The author had produced a new tool for the calculation of survivorship benefits, and it was necessary that they should be prepared to make use of the tables and formulas provided, though this might mean an increase of practical work. The first part of the paper dealt with formulas which scarcely required discussion. The main formula was that numbered (1) and the demonstrations were presented in a variety of ways. For his part he was inclined to prefer the *Text-Book* demonstration, which appeared simple and easy. Mr. Meech's solution was, he presumed, put by its author in a particular form for a particular purpose, which it apparently failed to achieve. The main interest from a practical point lay perhaps in equation (6) and in the tables. The extent to which the tables would assist them in their labours could only be tested by time and experiment. The second part of the paper contained some interesting investigations illustrating the way in which the theoretical actuary searched for formulas amenable to calculation. A portion of the paper was devoted to a demonstration of the error into which Mr. Meech had fallen, but it would have been more interesting if the author had directly followed Mr. Meech's investigation into the value of the function ϕ, and shown exactly where there was error. Instead, he had restricted himself to showing by calculating certain particular values that the relation which Mr. Meech thought he had established led to erroneous results. The following considerations had made clear to him (Mr. Sunderland) the error into which Mr. Meech had fallen. Differentiating with respect to t the equation

$$\bar{A}_{x+t.y+t.z'+t}=a'+b'\bar{A}_{x+t.y+t.z+t},$$

we obtain

$$d\bar{A}_{x+t.y+t.z'+t}=b'd\bar{A}_{x+t.y+t.z+t} \quad . \quad . \quad . \quad . \quad (A)$$

Here the expression on the left of (A) meant the small difference in

the survivorship premiums for ages $x+t$, $y+t$, $z+t$, and ages $x+t+dt$, $y+t+dt$, $z+t+dt$. But in Mr. Meech's equation

$$\phi=\int_0^\infty d\bar{A}_{x+t.y+t.z'+t}\bar{a}_{x+t} \quad . \quad . \quad . \quad . \quad . \quad (B)$$

the expression $d\bar{A}_{x+t.y+t.z'+t}$ had an entirely different meaning. It was, of course, an element in a summation of an infinite number of small quantities the sum total of which is $\bar{A}_{x+t.y+t.z'+t}$, but there was no reason for equating it to the expression of similar notation in (A), and, in fact, in order to see what it meant they must go back a step in the reasoning by which Mr. Meech obtained the equation (B).

Mr. G. F. Hardy, as one interested in the mathematical side of their studies, welcomed the paper as an important contribution to that department of their work. He referred more particularly to the formulas given at the end of the first part of the paper. With the single exception of the always unpractical compound survivorship problem they there had a set of results embracing all cases of survivorships which could arise involving three lives, given in a very compact and homogeneous form, so that, by the aid of the tables appended to the paper involving the ingenious idea of annuities at negative rates of interest, they could readily get any numerical result desired. The author had scarcely made enough of those results, and it would have been worth while to give a numerical example in the case of each formula, showing exactly how the tables were to be applied in practice. He thought it would be convenient to give at the head of the tables the value of h and h' used throughout in the six equations; h was a constant equal to ·00825 and $h'=\mu-h$. These were two important factors, on which the whole of the formulas hinged, and it would be convenient to have them immediately before them in using the tables. A great deal of interest centred round what the author had to say about the compound survivorship annuity. It was disappointing, but not surprising, that Mr. Colenso had not offered them an exact solution of this problem; in fact, his efforts in that direction had merely resulted in destroying the one professed solution extant. This negative result was satisfactory to him (the speaker) because he once wasted a good deal of time in a similar attempt to reduce the expression to a practical form, the result of his investigation then being to convince him that there was no solution to be obtained on the lines of Makeham's hypothesis. The approximate solution given in the paper, judging by the numerical example to which it was applied by the author, was surprisingly accurate. This accuracy was said to depend very much on the age of the life, but it would be interesting to have further information on that point. Reference was made in the paper to the comparative results brought out by the Carlisle Tables as adjusted by Makeham's formula and those deduced from the unadjusted table. It was well known the irregularities in the Carlisle Table rendered it very ill adapted to adjustment by Makeham's hypothesis. Some persons would lay the blame on the hypothesis, others, including the author, with whom he quite agreed, upon the Carlisle Table. In all probability, in the case given, the result brought out when Makeham's formula was used was

nearer the truth than that given by the unadjusted table. One of the main objections brought against Makeham's formula was that it could not be applied to adjust select tables. By a simple modification of the formula, however, it could be made applicable to this purpose. All that was necessary was to add terms involving the duration of assurance. If, for example, they wrote

$$\mu_{[x]+t}=A+Hx+Bc^{x+t}+\phi(t)+\psi(t)c^{x+t},$$

they retained the principle of uniform seniority, and he had found, on roughly adjusting Dr. Sprague's Select Tables by this process, that premiums were obtained nearly identical with those of the original tables.

Mr. George King wished to say how desirable it was to have occasional theoretical papers brought before the Institute. He was sure that the capabilities of Makeham's formulas were not yet exhausted. Mr. Hardy had just shown that there were some most important discoveries yet to be made, and he hoped that that gentleman would see his way to develop more fully the new formula, and to show how he reached the results. One great difficulty in Makeham's formula had been that they could not see how it could be applied to select tables. At one time he (Mr. King) took a good deal of interest in the formula, and had written much about it, but that seemed to him a great difficulty, more particularly as at the present day select tables were likely to take a far higher place than they had done in the past. One great advantage of formulas of approximate summation had been that they could make use of select tables, and, moreover, they often found in these complex functions the lives were subject to different conditions. One might be male and another female, subject to different rates of mortality, and the formulas of approximate summation could be employed with perfect facility. It made no difference to them what description of mortality was in question. That, however, did not take away from the value of such investigations as the present. The rarity of cases of compound survivorship in practice had been remarked upon. In his official work he had never seen a case of compound survivorship benefit, but in consulting practice, he had several times come across questions involving the compound status, and had been obliged to solve them by applying a formula of approximate summation. It was not, therefore, so purely theoretical as might at first sight be supposed. It was quite true that Makeham's formula could only be applied to some tables, and it no doubt admirably adapted itself to what might be called mixed tables of assured lives. These mixed tables presented a somewhat peculiar curve, and that curve fitted Makeham's formula in a wonderful way. If, however, the formula was applied to such a table as the $H^{M(5)}$, which really represented more accurately the true rate of mortality in an undisturbed population, it did not work so well, and they did not get the same practical results. That took away to a considerable extent one's hope that in practice very great use would be made of the formula. It was not always the fault of the formula that it did not apply. Reference had been made to the Carlisle Table, and it had been shown that the results worked out from

that table graduated by Makeham's formula were not very harmonious with those from the ungraduated table. It must, however, be remembered that the Carlisle Table as they had it did not necessarily represent the rates of mortality in Carlisle. It was not got by a method which gave rates of mortality at various ages, but was only Mr. Milne's idea of the rates formed by him by drawing certain curves; and, as he (Mr. King) had shown on another occasion, at one point where the graduation was most faulty Milne had not carried out to the full extent his own plan of drawing curves. At another part of the table Milne had made a dip in the curve which produced a perfect graduation in the final table; but in the period from 45 to 60 he failed to make a similar dip, where the original data seemed to show it was required. If they re-drew the curve, following strictly the rule, they got a decidedly better graduated Carlisle Table, and one closer to the original facts than Milne's own table, and such a re-graduated table would fit much better to Makeham's formula.

Mr. J. F. E. Hall wished to add a few remarks bearing on the practical application of Mr. Colenso's method of calculating survivorship premiums. At the end of Mr. Makeham's communication to the *Journal*, already referred to in the paper, was given, among other things, a short table of annuity-values on one life, on two lives of equal ages and on three lives of equal ages, based on the Carlisle mortality, with interest at 4 per-cent. As an example of the application of these tables to the calculation of survivorship benefits, Mr. Makeham took the case of $\bar{A}^1_{28.30.33}$, the common age being 30·515, and the resulting value ·1524. By Mr. Colenso's tables the same lives were equal to three others, each aged 30·5; and the resulting value was ·17822. The rate of interest involved being 3 per-cent it was necessary to resort to some means of approximation in order to obtain a ready comparison with Mr. Makeham's result. It would perhaps be sufficient if they used the ratio between the Carlisle 3 per-cent and 4 per-cent annuities at age 30, and that equalled ·862, which, multiplied into ·17822, gave ·1536 as the approximate value at 4 per-cent. Any demonstration of the accuracy of Mr. Colenso's method was, of course, superfluous, but it was interesting to note the close agreement between the two values.

Mr. R. P. Hardy feared that many of them were scarcely fit to criticize the mathematical portion of Mr. Colenso's paper. He would, however, like briefly to deal with the matter in its practical aspects. The author had, by one of those adaptations of prior labour which almost amounted to independent discovery, supplied them with a means of arriving more rapidly and more certainly at some of those complicated results that by their rarity found them in a state of unpreparedness. He, therefore, thought Mr. Colenso was entitled to their thanks, whether they looked upon the question from the theoretical or the practical side. They often, in consulting practice, were called upon to value an estate or reversionary interest where the interests had to be strictly set out, and he had frequently to fall back upon an approximation. He had shuddered

sometimes as to what would be the effect if he was called up in the witness-box to explain the slender grounds upon which that computation was based. These formulas were also useful for certain occasional market purposes, for estimating the value of the benefit of survivorship in a reversion. That was a duty that he had had to perform, and which again he would now be able to perform with far greater ease and accuracy than hitherto. He was pleased to think that Mr. Colenso's paper would find a permanent place in their records.

Mr. F. T. M. Byers said that Mr. Colenso had succeeded in reducing Makeham's formula for a contingent assurance to a more workable shape. With regard to formula 5, Mr. Colenso assumed as necessary a table of $F_2c^{n'}$. He would suggest a table of c^n only, as being sufficient and also more general, for instance, the introduction of $\frac{1}{3}$ in the example would limit the use of such a table to three-life calculations—in fact, they would require a separate table of the $F_2c^{n'}$ for every number of lives as the F_1 and F_2 depend on the number of lives, but given a table of c^n, which is independent, these functions would be easily calculated. With regard to another of Mr. Colenso's formulas, the plan of introducing c^n in forming a' seemed a happy idea. All that was wanted were the tables of $\bar{a}$ and a'. To make a table of $ls\bar{a}$, as Mr. Colenso proposed, was scarcely necessary, ls being constant, and the table $\bar{a}$ in its simple form would be serviceable for many other purposes. To use Mr. Colenso's formulas seemed to him to require less machinery in the way of tables than he claimed for them, and, with the necessary tables, they might be very useful in survivorship combinations where more than two lives were concerned.

Mr. Colenso, in reply, thanked the referees for their remarks. The main formulas of his paper were those in which he had thrown into practical and symmetrical shape expressions for survivorship assurances. He had illustrated the method by means of three lives, but it would lend itself equally readily to more than three, and he had, as a matter of fact, nearly completed a set of four-life tables to supplement those now submitted. He was satisfied that, whether for the purpose of working out some of the problems that were presented in connection with consulting practice, or whether it was desired to verify figures quoted in connection with assurance business, the method was a useful one. It might be true that many of these three-life and four-life assurances did not come to birth, but it was also true that premiums of all sorts had frequently to be calculated. He had been encouraged to pursue the subject further.

The late W. S. B. Woolhouse.

The numerous valuable contributions of the late Wesley Stoker Barker Woolhouse to the pages of this *Journal* have placed our readers under such considerable obligations that his lamented death in August 1893, calls for some acknowledgment of the debt which actuarial students will always owe to him. Though active to the last, Mr. Woolhouse almost seemed to belong to the past generation of scientific men who were instrumental in laying the very foundations of modern actuarial knowledge. In much of his work he was a pioneer whom successive labourers in the same fields of enquiry were to follow, as a guide and preceptor. Though it is a truth worthy to be borne in mind by all interested in the development of the actuary's special department of research, that profound mathematical ability is not synonymous or co-terminous or correlative with actuarial science, it nevertheless provides its essential basis and foundation; and it needed the great mathematicians who were so active in their contributions to the literature of life-contingencies some thirty years ago to plan the groundwork of the science.

Few more remarkable instances of innate mathematical ability are to be met with in the records of the past than that of the late Mr. Woolhouse. Born in 1809, he attracted the notice of his teachers by his extraordinary gifts while yet in his teens; and, when nineteen years of age only, he produced treatises on "Geometry of Two Dimensions", and "Analytical Dynamics." Thenceforth his triumphs in the field of pure mathematics were frequent and conspicuous; and the reputation he had made for himself received a well-deserved recognition, when in 1830, at the age of twenty-one, he was appointed deputy-superintendent of the *Nautical Almanack* Office. It would be swerving too much from the objects to which this *Journal* is devoted, to dwell upon Woolhouse's brilliant succession of papers on mathematical and astronomical subjects—studies which might have occupied him throughout his life had not a difference of opinion between him and his official chief led to his taking office as actuary of the International Loan Fund in 1839. After this, it is natural to find him appearing as an author on actuarial topics. His well-known "Investigation of the Mortality in the Indian Army" appeared in 1839, and some five years later he was entrusted with

the manipulation of the data and the graduation of the tables of the Seventeen Offices' Experience. By this time his position as an expert actuary was firmly established, and his career as an active force in the advancement of the science of life-contingencies was fairly started.

It is with no object of following that career, step by step, that the present note is written, but rather with the endeavour of tracing Woolhouse's influence on the condition of actuarial knowledge in these days. Our readers will see from the list of his contributions appearing in the *Index to the First Twenty Volumes* how numerous and varied were the subjects on which he wrote. Of the purely mathematical essays we propose to say little. A deep knowledge of the highest branches of the science, extreme neatness and dexterity in analysis, and great inventive skill in applying theory to the elucidation or illustration of practical matters, are the chief characteristics of these writings; but it would ill become us to try to assign to him the position in the world of mathematics which of right belongs to him. It is, however, worthy of remark that he seemed to recognise the limitations of the legitimate application of mathematical laws to actuarial matters. He was, apparently, no advocate of the use of the Theory of Errors in this direction—a branch of study to which recent writers, both in this country and America, have given much thought and labour—for learned as his purely scientific writings were in this path of knowledge, the records of this *Journal* fail to show that he adopted these methods in any of his practical investigations. It is now futile to discuss whether he considered that as there is so fundamental a distinction to be drawn between mortality statistics and observations of a strictly homogeneous character, such as the drawing of balls from a bag at random, and as the laws of error could only remove the single imperfection or defect due to the *number* of the facts observed, leaving all other imperfections untouched, the application of this method served little useful purpose; but the fact itself, in a mathematician of Woolhouse's type, is at least significant.

It is easier—though the task is not free from difficulty—to deal with his claims as one of the most original and brilliant exponents of the actuarial theory. And in this connection, we may first of all notice his attraction to the subject of the graduation of mortality tables, wherein his name subsequently became a household word. His examination of the tables deduced

from Indian Military Experience and the Seventeen Offices' Experience brought home to him the necessity for removing the accidental irregularities in the original facts, and he developed the rough and simple plan of correcting the second (or fourth) differences of the observations by means of formulas given by him (*J.I.A.*, xii, 139-142), which, we believe, has been termed, rather crudely, the Method of Differences. It is a long step from this to the great method of graduation which goes by his name, and was applied by him to the Twenty Offices' Experience. The marked superiority of the later plan is in itself an abiding proof of the progress actuarial science had made in the meantime. Much has been written on this famous system of graduation, and its entire groundwork and *raison-d'être* have been so abundantly discussed by the author and his redoubtable antagonist, Dr. Sprague, in recent numbers of this *Journal*, that we are absolved from any necessity to traverse once more the well-worked field of argument. But it is fitting to call attention to the great improvements in the practical application of Woolhouse's method, which we owe to Mr. T. G. Ackland (*J.I.A.*, xxiii, 352), and Mr. G. F. Hardy (*J.I.A.*, xxiii, 351). These gentlemen showed with extreme ingenuity how Woolhouse's formula, hitherto regarded as somewhat untractable in operation, could be applied by a simple self-checking columnar process to any body of facts; and they thereby earned the gratitude of students and those who were to be employed in the actual work of graduation. Mr. Ackland also made a highly useful suggestion for doubling-back the over-lapping results at the extremities of the table, thus preserving the sum total of the original facts. In connection with the system of Woolhouse, reference must also be made to the eminently practical series of papers by Mr. J. A. Higham on new methods of graduation, which may be held to be based upon or suggested by Woolhouse's writings. In fact, it would be difficult to name any subject which has engaged closer attention in the professional mind and called forth wider research than the method of graduation devised by the subject of this memoir.

It would, however, be far from the truth to say that it is to this effort of his that the profession counts itself Woolhouse's chief debtor. His early paper on "An Improved Theory of Annuities and Assurances" (*J.I.A.*, xv, 95), wherein he propounded the continuous method, is a well-marked stage in the advance of actuarial science, and has had a profound influence on its

development. His masterly papers on "Interpolation, Summation, and the Adjustment of Numerical Tables" (*J.I.A.*, xi, 61, 301; xii, 136) will also be borne in mind in connection with his earlier contributions to actuarial science; and his quick appreciation of the merits and power of Makeham's Formula for expressing the Law of Mortality is to the credit alike of his insight and his generosity. Passing on, we must rest satisfied with brief mention of his practical paper "On the Construction of Tables of Mortality" (*J.I.A.*, xiii, 75), which is of historical value as explaining the methods adopted in working-up the Seventeen Offices' Experience; of his elaborate investigation into the subject of Half-Yearly and Quarterly Annuities (*J.I.A.*, xvii, 171); of his brilliant and most comprehensive treatment of "Integration by means of Selected Values of the Function" (*J.I.A.*, xxvii, 122); and finally of his novel, ingenious and elegant paper "An Easy Method of getting out a Rough Estimate Valuation of a Whole-Life Assurance Business" (*J.I.A.*, xxvii, 433).

Our references to Woolhouse's writings have no pretensions to completeness. From the full list, a sufficient number of papers have, however, doubtless been cited to show the extreme value and importance of his contributions. Woolhouse was a prominent member of a small group of distinguished authors on actuarial subjects, whose services to the Institute of Actuaries in its early years were inestimable; and he deserves to be held in grateful remembrance by generations of students as yet unborn. Luminous and powerful his writings invariably were, but their chief characteristic is, perhaps, thoroughness. "Rare "as epic song is the man who is thorough in what he does. And "happily so; for he subjugates us, and makes us bondsmen to "his ashes." Bondsmen we are, and our successors will be; and actuarial science need suffer no check in its steady advance, and, indeed, know no bounds, if men of such high powers of mind and strength of purpose can be found in the future to devote their time and talents to its cause.

G. H. R.

Tables of the Values of Endowment Assurance Policies (H^{M} $2\frac{1}{2}$ *per-cent*). *Constructed on the Arithmometer by* Arthur Digby Besant, B.A., A.I.A., *of the Clerical, Medical and General Life Assurance Society.*

The recent publication of Messrs King and Whittall's Valuation Tables suggested the idea that Mr. Carment's little volume of Endowment Assurance Policy-Values might be usefully extended to include the $2\frac{1}{2}$ per-cent results, and the following tables give the values for ages of attainment 50, 55, 60 and 65.

The method of construction exactly follows that sketched out by Mr. Carment (*J.I.A.*, xxii, 369), the temporary annuity-values being taken from Messrs. King and Whittall's Tables, and the reciprocal of the annuity-due being set up to seven significant figures on the Arithmometer. Continued multiplication by the differences of the annuities then gave the policy-values, which have been arranged in the familiar form.

Every care has been taken to ensure accuracy. The reciprocals of the annuities were obtained by actual division and checked independently by Oakes's Tables.

An effective check over the correctness of the working at each age was afforded by the final value, from the relation ${}_{n-1}V_{x\overline{n|}} = 1 - \dfrac{1}{1+a_{x:\overline{n-1|}}}$, and in every case this gave an absolutely correct result to the last decimal place. In order to ensure that the figures had been correctly entered on the sheets, the columns were added and checked from the summation formula

$$\Sigma_{m}V_{x\overline{n|}} = m - \frac{\Sigma(1+a_{x+1:\overline{n-2|}})}{1+a_{x:\overline{n-1|}}}.$$

The results agreed very closely.

In 39 cases the results of the verification formula agreed exactly with the cast of the columns, in 64 cases the difference was 1 in the third decimal place, in 20 cases the difference was 2, in 9 cases it was 3, and in 2 cases only was it as much as 4. In these two latter cases the ages were reworked and found to be correct.

Finally the proof-sheets, having been carefully called over, have been cast and agreed with the original calculations.

In the hope that they will prove of general service, I gladly offer these tables as a humble contribution to the literature of the Institute.

VALUES OF ENDOWMENT ASSURANCE POLICIES.

H^M $2\frac{1}{2}$ *per-cent—Death or 50.*

Duration	20	21	22	23	24	25	26	Duration
	2·690	2·802	2·920	3·048	3·186	3·338	3·503	
1	2·139	2·217	2·321	2·464	2·621	2·779	2·941	1
2	4·308	4·486	4·727	5·020	5·327	5·638	5·954	2
3	6·529	6·839	7·224	7·659	8·112	8·567	9·043	3
4	8·831	9·281	9·802	10·375	10·964	11·571	12·204	4
5	11·221	11·802	12·455	13·157	13·889	14·644	15·449	5
6	13·688	14·396	15·172	16·010	16·881	17·798	18·782	6
7	16·226	17·053	17·959	18·929	19·953	21·039	22·210	7
8	18·827	19·778	20·810	21·925	23·109	24·372	25·740	8
9	21·493	22·566	23·737	25·003	26·354	27·803	29·370	9
10	24·222	25·427	26·743	28·169	29·696	31·333	33·113	10
11	27·022	28·367	29·835	31·428	33·133	34·972	36·958	11
12	29·899	31·391	33·019	34·780	36·677	38·709	40·909	12
13	32·858	34·504	36·293	38·237	40·316	42·551	44·985	13
14	35·904	37·706	39·670	41·786	44·057	46·514	49·186	14
15	39·038	41·007	43·137	45·435	47·916	50·598	53·529	15
16	42·269	44·398	46·701	49·199	51·893	54·821	58·015	16
17	45·587	47·883	50·378	53·078	56·005	59·182	62·656	17
18	48·997	51·478	54·167	57·089	60·252	63·694	67·451	18
19	52·516	55·183	58·085	61·231	64·645	68·355	72·406	19
20	56·141	59·014	62·131	65·516	69·185	73·173	77·534	20
21	59·890	62·970	66·317	69·944	73·876	78·158	82·846	21
22	63·762	67·063	70·642	74·520	78·731	83·323	88·342	22
23	67·768	71·292	75·111	79·255	83·760	88·666	94·058	23
24	71·906	75·663	79·736	84·160	88·963	94·223	26	
25	76·183	80·186	84·528	89·235	94·375	25		
26	80·609	84·871	89·485	94·513	24		41	
27	85·194	89·718	94·641	23		42	10·347	
28	89·938	94·759	22		43	11·714		
29	94·872	21		44	13·473		87·214	8
	20		45			85·848	74·914	7
				15·816	84·089	72·233	63·087	6
			19·096					
				81·745	68·783	59·142	51·656	5
4	…	…	78·467	64·184	54·065	46·490	40·621	4
3	…	…	57·752	47·298	39·841	34·277	29·958	3
2	…	…	37·834	30·978	26·110	22·474	19·639	2
1	…	…	18·583	15·225	12·840	11·053	9·653	1
			45	44	43	42	41	

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or* 50—(continued).

Duration	27	28	29	30	31	32	33	Duration
	3·683	3·879	4·093	4·329	4·588	4·877	5·199	
1	3·104	3·285	3·475	3·695	3·942	4·221	4·537	1
2	6·287	6·646	7·042	7·492	7·997	8·567	9·204	2
3	9·544	10·096	10·707	11·397	12·171	13·037	14·016	3
4	12·886	13·640	14·476	15·417	16·465	17·646	18·958	4
5	16·321	17·286	18·356	19·552	20·892	22·379	24·038	5
6	19·853	21·039	22·348	23·816	25·439	27·244	29·277	6
7	23·489	24·899	26·463	28·194	30·112	32·263	34·678	7
8	27·230	28·879	30·690	32·695	34·933	37·435	40·261	8
9	31·087	32·967	35·034	37·338	39·902	42·783	46·028	9
10	35·047	37·168	39·515	42·122	45·039	48·306	51·994	10
11	39·118	41·502	44·134	47·070	50·344	54·020	58·158	11
12	43·318	45·969	48·909	52·179	55·833	59·924	64·528	12
13	47·646	50·588	53·841	57·465	61·504	66·025	71·120	13
14	52·121	55·358	58·943	62·926	67·365	72·339	77·948	14
15	56·743	60·292	64·215	68·571	73·429	78·879	85·014	15
16	61·524	65·390	69·663	74·411	79·712	85·646	92·362	16
17	66·465	70·660	75·300	80·462	86·212	92·684		
18	71·570	76·112	81·141	86·722	92·973			
19	76·853	81·760	87·183	93·232				
20	82·326	87·604	93·467					
21	87·989	93·682						
22	93·878							

Duration	40	39	38	37	36	35	34	Duration
	9·255	8·361	7·617	6·986	6·444	5·974	5·562	
							91·999	15
						91·587	84·301	14
					91·117	83·495	76·900	13
				90·575	82·571	75·713	69·747	12
			89·945	81·508	74·354	68·192	62·842	11
		89·200	80·271	72·790	66·412	60·932	56·169	10
9	88·307	78·810	70·970	64·364	58·746	53·916	49·712	9
8	77·058	68·820	61·981	56·230	51·337	47·127	43·463	8
7	66·242	59·164	53·303	48·369	44·168	40·557	37·422	7
6	55·788	49·843	44·917	40·763	37·230	34·205	31·573	6
5	45·697	40·836	36·802	33·402	30·523	28·056	25·916	5
4	35·945	32·120	28·949	26·287	24·029	22·108	20·427	4
3	26·508	23·685	21·357	19·397	17·749	16·337	15·106	3
2	17·376	15·531	14·007	12·733	11·655	10·743	9·930	2
1	8·548	7·636	6·898	6·268	5·748	5·300	4·889	1

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or 55.*

Duration	20	21	22	23	24	25	26	27	Duration
	2·287	2·370	2·456	2·548	2·648	2·755	2·870	2·995	
1	1·720	1·770	1·846	1·950	2·060	2·176	2·288	2·396	1
2	3·460	3·583	3·760	3·970	4·191	4·415	4·630	4·841	2
3	5·241	5·463	5·742	6·060	6·384	6·705	7·019	7·346	3
4	7·089	7·410	7·794	8·209	8·627	9·042	9·466	9·900	4
5	9·003	9·425	9·904	10·409	10·916	11·437	11·962	12·524	5
6	10·984	11·498	12·062	12·653	13·261	13·878	14·526	15·220	6
7	13·020	13·619	14·265	14·953	15·652	16·386	17·160	17·991	7
8	15·105	15·783	16·522	17·297	18·109	18·962	19·867	20·838	8
9	17·231	18·000	18·823	19·706	20·632	21·611	22·649	23·761	9
10	19·410	20·260	21·188	22·180	23·226	24·333	25·506	26·755	10
11	21·631	22·582	23·616	24·723	25·891	27·127	28·431	29·830	11
12	23·914	24·968	26·112	27·337	28·628	29·989	31·436	32·993	12
13	26·258	27·420	28·678	30·020	31·431	32·928	34·526	36·242	13
14	28·669	29·940	31·311	32·768	34·310	35·951	37·701	39·595	14
15	31·145	32·527	34·009	35·591	37·270	39·057	40·977	43·056	15
16	33·688	35·177	36·780	38·494	40·312	42·261	44·359	46·631	16
17	36·292	37·899	39·629	41·476	43·451	45·570	47·852	50·315	17
18	38·967	40·697	42·556	44·554	46·691	48·987	51·452	54·108	18
19	41·717	43·573	45·577	47·731	50·038	52·509	55·158	58·020	19
20	44·544	46·540	48·695	51·012	53·487	56·134	58·981	62·052	20
21	47·460	49·603	51·917	54·394	57·037	59·873	62·920	66·208	21
22	50·470	52·767	55·236	57·875	60·700	63·727	66·982	70·506	22
23	53·580	56·028	58·653	61·466	64·474	67·700	71·181	74·957	23
24	56·784	59·384	62·178	65·167	68·366	71·808	75·530	79·575	24
25	60·083	62·847	65·810	68·983	72·389	76·062	80·042	84·373	25
26	63·486	66·415	69·555	72·928	76·555	80·477	84·731	89·366	26
27	66·993	70·094	73·427	77·012	80·879	85·063	89·610	94·566	27
28	70·608	73·898	77·437	81·252	85·371	89·836	94·691	27	
29	74·347	77·836	81·598	85·656	90·045	94·806	26		
30	78·217	81·924	85·921	90·239	94·913	25			
31	82·235	86·170	90·420	95·012	24				
32	86·408	90·589	95·105	23					
33	90·751	95·191	22						
34	95·274	21							
	20								

Duration		50	49	48	47	46	45	44	Duration
		19·257	15·984	13·640	11·879	10·504	9·401	8·497	
								89·064	10
							88·160	78·598	9
						87·057	76·829	68·548	8
					85·682	74·670	65·948	58·891	7
				83·920	71·979	62·775	55·494	49·595	6
			81·577	68·532	58·820	51·346	45·430	40·639	5
4	...	78·303	63·946	53·755	46·177	40·344	35·733	31·988	4
3	...	57·540	47·015	39·556	34·006	29·744	26·368	23·622	3
2	...	37·600	30·748	25·888	22·279	19·506	17·310	15·507	2
1	...	18·442	15·088	12·719	10·954	9·604	8·525	7·633	1
		50	49	48	47	46	45	44	

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or* 55—(continued).

Duration	28	29	30	31	32	33	34	35	Duration
	3·128	3·271	3·425	3·592	3·773	3·970	4·187	4·425	
1	2·505	2·632	2·756	2·913	3·081	3·269	3·472	3·693	1
2	5·072	5·316	5·589	5·904	6·249	6·627	7·036	7·475	2
3	7·688	8·074	8·498	8·980	9·504	10·075	10·687	11·360	3
4	10·377	10·906	11·488	12·140	12·846	13·606	14·437	15·355	4
5	13·138	13·818	14·561	15·384	16·268	17·234	18·293	19·459	5
6	15·977	16·810	17·716	18·707	19·784	20·964	22·255	23·694	6
7	18·894	19·882	20·948	22·120	23·399	24·797	26·343	28·066	7
8	21·889	23·029	24·267	25·630	27·113	28·751	30·564	32·583	8
9	24·957	26·260	27·680	29·237	30·946	32·833	34·923	37·237	9
10	28·108	29·584	31·187	32·957	34·903	37·051	39·416	42·028	10
11	31·348	32·998	34·805	36·799	38·990	41·396	44·040	46·970	11
12	34·677	36·521	38·541	40·767	43·201	45·869	48·811	52·063	12
13	38·112	40·159	42·400	44·856	47·537	50·484	53·727	57·313	13
14	41·658	43·916	46·376	49·065	52·009	55·239	58·795	62·743	14
15	45·321	47·787	50·469	53·407	56·618	60·142	64·036	68·364	15
16	49·095	51·773	54·692	57·882	61·370	65·212	69·463	74·199	16
17	52·981	55·884	59·043	62·495	66·284	70·461	75·094	80·259	17
18	56·989	60·121	63·529	67·266	71·371	75·908	80·945	86·567	18
19	61·120	64·489	68·168	72·205	76·651	81·568	87·034	93·136	19
20	65·379	69·006	72·971	77·331	82·136	87·458	93·374	35	
							34		
21	69·782	73·682	77·956	82·656	87·844	93·591			
22	74·342	78·536	83·134	88·198	93·788	33			
23	79·074	83·578	88·523	93·969	32			36	
24	83·989	88·825	94·136	31			37	4·688	
25	89·105	94·290	30			38	4·980		
		29			39	5·305		92·873	18
26	94·433			40			92·582	86·052	17
	28		41		5·670	92·256	85·482	79·503	16
		42		6·083					
	43		6·556		91·891	84·846	78·665	73·209	15
		7·103		91·478	84·131	77·730	72·114	67·151	14
	7·742		91·005	83·322	76·679	70·892	65·809	61·314	13
		90·458	82·396	75·490	69·518	64·310	59·733	55·677	12
11	89·819	81·326	74·130	67·965	62·626	57·968	53·865	50·225	11
10	80·075	72·557	66·187	60·721	55·984	51·843	48·190	44·936	10
9	70·719	64·132	58·541	53·741	49·570	45·919	42·685	39·805	9
8	61·729	56·021	51·174	47·000	43·367	40·173	37·344	34·830	8
7	53·075	48·206	44·059	40·481	37·350	34·598	32·166	29·998	7
6	44·736	40·658	37·177	34·157	31·512	29·193	27·136	25·308	6
5	36·683	33·359	30·503	28·021	25·851	23·943	22·255	20·768	5
4	28·894	26·279	24·026	22·073	20·354	18·848	17·530	16·371	4
3	21·340	19·408	17·748	16·295	15·018	13·915	12·953	12·109	3
2	14·009	12·748	11·649	10·687	9·852	9·137	8·516	7·961	2
1	6·903	6·279	5·730	5·258	4·849	4·507	4·199	3·927	1
	43	42	41	40	39	38	37	36	

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or* 60.

Duration	20	21	22	23	24	25	26	27	28	Duration
	2·008	2·073	2·140	2·210	2·286	2·367	2·454	2·547	2·645	
1	1·436	1·462	1·516	1·599	1·691	1·778	1·859	1·930	2·008	1
2	2·877	2·955	3·091	3·264	3·410	3·605	3·753	3·899	4·057	2
3	4·319	4·507	4·730	4·984	5·235	5·465	5·686	5·908	6·146	3
4	5·879	6·123	6·424	6·751	7·064	7·363	7·658	7·957	8·282	4
5	7·471	7·792	8·161	8·550	8·930	9·300	9·669	10·051	10·478	5
6	9·116	9·506	9·936	10·386	10·831	11·275	11·724	12·205	12·725	6
7	10·806	11·252	11·745	12·260	12·776	13·294	13·838	14·409	15·033	7
8	12·527	13·035	13·590	14·171	14·760	15.370	16·000	16·672	17·397	8
9	14·284	14·853	15·472	16·123	16·801	17·494	18·222	18·991	19·812	9
10	16·076	16·707	17·395	18·132	18·890	19·676	20·497	21·359	22·283	10
11	17·904	18·602	19·373	20·187	21·035	21·911	22·821	23·782	24·814	11
12	19·771	20·551	21·397	22·298	23·232	24·194	25·199	26·265	27·412	12
13	21·693	22·546	23·475	24·469	25·476	26·530	27·636	28·813	30·086	13
14	23·658	24·594	25·604	26·668	27·772	28·923	30·137	31·435	32·847	14
15	25·677	26·692	27·779	28·927	30·125	31·379	32·710	34·143	35·689	15
16	27·745	28·835	30·005	31·243	32·549	33·907	35·367	36·930	38·612	16
17	29·857	31·028	32·285	33·618	35·025	36·517	38·102	39·797	41·612	17
18	32·019	33·275	34·625	36·064	37·590	39·203	40·916	42·738	44·687	18
19	34·233	35·580	37·033	38·589	40·232	41·967	43·803	45·755	47·839	19
20	36·506	37·953	39·519	41·187	42·948	44·802	46·763	48·846	51·078	20
21	38·845	40·493	42·079	43·861	45·736	47 710	49·797	52·022	54·408	21
22	41·259	42·925	44·712	46·604	48·594	50·690	52·914	55·287	57·839	22
23	43·745	45·520	47·413	49·417	51·524	53·751	56·119	58·653	61·378	23
24	46·302	48·182	50·183	52·299	54·533	56·899	59·422	62·123	65·038	24
25	48·926	50·911	53·022	55·261	57·628	60·143	62·827	65·713	68·831	25
26	51·616	53·709	55·939	58·306	60·817	63·488	66·350	69·432	72·755	26
27	54·374	56·583	58·938	61·444	64·106	66·949	70·000	73·281	76·828	27
28	57·206	59·538	62·028	64·680	67·508	70·534	73·778	77·275	81·063	28
29	60·119	62·583	65·215	68·027	71·032	74·244	77·697	81·428	85·475	29
30	63·121	65·724	68·512	71·496	74·689	78·094	81·773	85·756	90·081	30
31	66·216	68·972	71·928	75·085	78·464	82·097	86·020	90·273	94·916	31
32	69·418	72·338	75·462	78·809	82·400	86·269	90·454	95·014	28	
33	72·735	75·821	79·130	82·682	86·501	90·623	95·107	27		
34	76·168	79·435	82·944	86·717	90·782	95·194	26			
35	79·731	83·193	86·919	90·929	95·275	25				
36	83·435	87·110	91·067	95·351	24					
37	87·295	91·197	95·421	23						
38	91·324	95·488	22							
39	95·553	21								
	20									

Duration	55	54	53	52	51	50	49	48	47	Duration
	19·501	16·222	13·872	12·103	10·721	9·619	8·712	7·953	7·308	
									90·253	12
								89·608	80·984	11
							88·849	79·726	72·154	10
						87·942	78·245	70·311	63·694	9
					86·837	76·474	68·142	61·291	55·575	8
				85·459	74·319	65·549	58·463	52·634	47·768	7
			83·689	71·630	62·393	55·083	49·175	44·311	40·244	6
		81·340	68·178	58·456	50·967	45·038	40·243	36·288	32·973	5
4	78·061	63·594	53·401	45·834	40·003	35·379	31·635	28·536	25·955	4
3	57·196	46·688	39·243	33·721	29·459	26·070	23·316	21·054	19·172	3
2	37·319	30·491	25·656	22·074	19·297	17·075	15·288	13·821	12·593	2
1	18·276	14·947	12·592	10·848	9·477	8·393	7·527	6·807	6·209	1
	55	54	53	52	51	50	49	48	47	

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or* 60—(continued).

Duration	29	30	31	32	33	34	35	36	37	Duration
	2·749	2·860	2·978	3·104	3·240	3·386	3·544	3·715	3·901	
1	2·091	2·178	2·275	2·395	2·510	2·645	2·782	2·923	3·081	1
2	4·223	4·403	4·615	4·845	5·088	5·353	5·624	5·915	6·239	2
3	6·402	6·692	7·009	7·361	7·729	8·120	8·532	8·980	9·478	3
4	8·643	9·035	9·469	9·938	10·426	10·951	11·512	12·125	12·813	4
5	10·936	11·440	11·987	12·571	13·186	13·852	14·569	15·362	16·256	5
6	13·292	13·904	14·560	15·265	16·015	16·829	17·717	18·704	19·800	6
7	15·704	16·421	17·193	18·026	18·916	19·893	20·966	22·144	23·445	7
8	18·169	18·996	19·891	20·858	21·904	23·056	24·310	25·683	27·186	8
9	20·690	21·635	22·659	23·774	24·987	26·312	27·751	29·314	31·021	9
10	23·274	24·343	25·508	26·783	28·162	29·662	31·281	33·038	34·952	10
11	25·925	27·130	28·449	29·882	31·427	33·098	34·901	36·854	38·991	11
12	28·654	30·007	31·477	33·069	34·778	36·623	38·611	40·774	43·143	12
13	31·471	32·969	34·592	36·339	38·213	40·234	42·422	44·806	47·423	13
14	34·371	36·016	37·788	39·693	41·734	43·945	46·341	48·960	51·835	14
15	37·354	39·143	41·065	43·130	45·352	47·760	50·380	53·243	56·400	15
16	40·415	42·348	44·423	46·660	49·071	51·692	54·544	57·675	61·130	16
17	43·554	45·634	47·874	50·291	52·905	55·746	58·852	62·266	66·024	17
18	46·770	49·009	51·422	54·032	56·857	59·941	63·316	67·017	71·103	18
19	50·075	52·480	55·078	57·890	60·946	64·286	67·935	71·947	76·384	19
20	53·473	56·057	58·848	61·881	65·183	68·783	72·728	77·074	81·887	20
21	56·975	59·745	62·749	66·016	69·567	73·449	77·712	82·416	87·631	21
22	60·586	63·560	66·789	70·295	74·116	78·301	82·906	87·992	93·660	22
23	64·322	67·513	70·971	74·735	78·846	83·358	88·326	93·845		
24	68·192	71·603	75·310	79·353	83·775	88·635	94·017			
25	72·197	75·818	79·822	84·164	88·920	94·175				
26	76·353	80·262	84·524	89·186	94·321					
27	80·674	84·861	89·432	94·457						
28	85·178	89·662	94·583							
29	89·878	94·701								
30	94·812									
	29	30	31	32	33	34	35	36	37	

Duration	46	45	44	43	42	41	40	39	38	Duration
	6·752	6·268	5·842	5·466	5·132	4·833	4·565	4·323	4·102	
									93·458	21
								93·238	87·238	20
							92·996	86·808	81·311	19
						92·728	86·336	80·682	75·633	18
					92·429	85·813	79·990	74·812	70·184	17
				92·095	85·230	79·225	73·911	69·180	64·944	16
			91·718	84·577	78·371	72·913	68·077	63·764	59·894	15
		91·293	83·843	77·415	71·800	66·856	62·467	58·544	55·014	14
13	90·809	83·013	76·340	70·553	65·493	61·031	57·060	53·499	50·304	13
12	82·068	75·124	69·151	63·968	59·429	55·417	51·835	48·631	45·751	12
11	73·741	67·566	62·253	57·636	53·585	49·993	46·792	43·925	41·336	11
10	65·763	60·313	55·619	51·534	47·937	44·757	41·918	39·360	37·051	10
9	58·107	53·339	49·226	45·636	42·486	39·696	37·190	34·931	32·884	9
8	50·744	46·617	43·048	39·945	37·217	34·788	32·603	30·624	28·828	8
7	43·649	40·122	37·085	34·443	32·107	30·025	28·141	26·432	24·871	7
6	36·792	33·853	31·321	29·107	27·148	25·393	23·799	22·341	21·011	6
5	30·175	27·793	25·731	23·929	22·326	20·884	19·562	18·351	17·250	5
4	23·778	21·916	20·306	18·893	17·632	16·485	15·429	14·463	13·593	4
3	17·574	16·212	15·031	13·992	13·052	12·195	11·402	10·684	10·041	3
2	11·553	10·666	9·896	9·209	8·585	8·013	7·487	7·012	6·600	2
1	5·699	5·268	4·886	4·545	4·232	3·949	3·684	3·455	3·258	1
	46	45	44	43	42	41	40	39	38	

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or* 65.

Duration	20	21	22	23	24	25	26	27	28	29	30	Duration
	1·815	1·868	1·923	1·980	2·041	2·107	2·176	2·250	2·327	2·409	2·495	
1	1·234	1·253	1·296	1·361	1·438	1·505	1·574	1·627	1·683	1·736	1·801	1
2	2·472	2·533	2·639	2·780	2·921	3·055	3·175	3·282	3·389	3·505	3·641	2
3	3·735	3·859	4·039	4·243	4·449	4·632	4·804	4·961	5·129	5·313	5·521	3
4	5·046	5·242	5·483	5·750	6·004	6·237	6·457	6·672	6·907	7·161	7·450	4
5	6·411	6·668	6·971	7·283	7·585	7·864	8·141	8·421	8·723	9·056	9·423	5
6	7·819	8·137	8·484	8·843	9·189	9·523	9·862	10·208	10·586	10·995	11·436	6
7	9·270	9·631	10·024	10·425	10·824	11·219	11·621	12·041	12·493	12·973	13·494	7
8	10·746	11·152	11·586	12·038	12·496	12·951	13·425	13·917	14·438	14·995	15·590	8
9	12·248	12·694	13·178	13·687	14·203	14·728	15·271	15·830	16·425	17·055	17·732	9
10	13·771	14·266	14·805	15·370	15·954	16·546	17·154	17·785	18·451	19·159	19·922	10
11	15·324	15·873	16·467	17·098	17·746	18·401	19·079	19·778	20·520	21·312	22·177	11
12	16·911	17·514	18·172	18·866	19·574	20·296	21·040	21·813	22·636	23·527	24·496	12
13	18·531	19·198	19·917	20·669	21·443	22·228	23·043	23·895	24·814	25·806	26·878	13
14	20·195	20·921	21·697	22·512	23·347	24·201	25·092	26·037	27·054	28·148	29·321	14
15	21·897	22·678	23·516	24·390	25·291	26·219	27·201	28·241	29·357	30·547	31·827	15
16	23·632	24·474	25·370	26·308	27·280	28·297	29·371	30·506	31·716	33·010	34·378	16
17	25·406	26·305	27·263	28·270	29·328	30·433	31·600	32·827	34·137	35·517	36·978	17
18	27·214	28·175	29·200	30·290	31·434	32·629	33·884	35·209	36·602	38·071	39·632	18
19	29·061	30·087	31·193	32·367	33·598	34·879	36·229	37·633	39·113	40·680	42·350	19
20	30·950	32·055	33·243	34·502	35·815	37·188	38·615	40·104	41·678	43·351	45·128	20
21	32·894	34·080	35·350	36·689	38·091	39·538	41·047	42·627	44·304	46·080	47·985	21
22	34·893	36·160	37·509	38·934	40·408	41·934	43·530	45·210	46·988	48·887	50·920	22
23	36·948	38·293	39·725	41·219	42·769	44·379	46·073	47·850	49·747	51·772	53·944	23
24	39·054	40·481	41·980	43·548	45·179	46·884	48·671	50·565	52·583	54·744	57·053	24
25	41·215	42·708	44·279	45·925	47·648	49·443	51·343	53·355	55·505	57·798	60·250	25
26	43·414	44·978	46·626	48·360	50·170	52·075	54·089	56·229	58·508	60·940	63·545	26
27	45·656	47·295	49·029	50·849	52·764	54·780	56·918	59·183	61·597	64·178	66·950	27
28	47·945	49·668	51·485	53·407	55·430	57·566	59·826	62·222	64·781	67·523	70·472	28
29	50·289	52·093	54·011	56·037	58·177	60·430	62·816	65·354	68·070	70·985	74·128	29
30	52·684	54·587	56·606	58·746	60·999	63·376	65·899	68·589	71·473	74·577	77·932	30
31	55·148	57·150	59·280	61·530	63·902	66·412	69·083	71·937	75·005	78·315	81·898	31
32	57·679	59·791	62·028	64·394	66·895	69·549	72·379	75·411	78·680	82·213	86·058	32
33	60·287	62·504	64·855	67·346	69·987	72·794	75·798	79·027	82·512	86·300	90·434	33
34	62·967	65·295	67·769	70·395	73·185	76·163	79·357	82·796	86·530	90·600	95·066	34
35	65·724	68·173	70·779	73·550	76·505	79·667	83·067	86·749	90·758	95·152	30	
36	68·565	71·145	73·893	76·825	79·960	83·322	86·958	90·908	95·234	29		
37	71·501	74·220	77·125	80·232	83·562	87·154	91·051	95·311	28			
38	74·538	77·412	80·489	83·786	87·339	91·186	95·385	27				
39	77·691	80·733	83·996	87·511	91·313	95·454	26					
40	80·971	84·196	87·673	91·431	95·520	25						
41	84·391	87·827	91·542	95·581	24							
42	87·978	91·648	95·638	23								
43	91·751	95·693	22									
44	95·746	21										
	20											

Duration		60	59	58	57	56	55	54	53	52	51	Duration
		19·916	16·631	14·270	12·489	11·095	9·973	9·048	8·273	7·614	7·046	
											90·515	13
										89·948	81·609	12
									89·288	80·509	73·195	11
								88·512	79·229	71·592	65·200	10
							87·588	77·725	69·727	63·118	57·574	9
						86·466	75·934	67·536	60·696	55·036	50·261	8
					85·072	73·758	64·925	57·852	52·084	47·286	43·232	7
				83·292	71·055	61·754	54·462	48·616	43·824	39·837	36·460	6
			80·931	67·602	57·815	50·345	44·483	39·759	35·886	32·660	29·916	5
4	..	77·644	63·024	52·782	45·231	39·464	34·914	31·246	28·238	25·724	23·580	4
3	..	56·651	46·110	38·697	33·229	29·030	25·717	23·044	20·846	19·009	17·433	3
2	..	36·821	30·034	25·263	21·720	19·001	16·355	15·113	13·690	12·495	11·458	2
1	..	17·975	14·703	12·381	10·658	9·338	8·291	7·444	6·749	6·162	5·644	1
		60	59	58	57	56	55	54	53	52	51	

Values of Endowment Assurance Policies.

H^M $2\frac{1}{2}$ *per-cent—Death or* 65—(continued).

Duration	31	32	33	34	35	36	37	38	39	40	Duration
	2·585	2·681	2·783	2·892	3·008	3·132	3·264	3·406	3·558	3.722	
1	1·874	1·951	2·042	2·132	2·222	2·323	2·424	2·537	2·663	2·816	1
2	3·788	3·953	4·131	4·307	4·494	4·691	4·899	5·132	5·403	5·711	2
3	5·753	6·001	6·261	6·530	6·809	7·108	7·431	7·803	8·222	8·687	3
4	7·762	8·090	8·439	8·796	9·173	9·582	10·038	10·550	11·118	11·737	4
5	9·812	10·225	10·658	11·109	11·591	12·127	12·718	13·373	14·087	14·867	5
6	11·907	12·401	12·924	13·476	14·080	14·746	15·473	16·266	17·133	18·052	6
7	14·042	14·623	15·243	15·912	16·640	17·436	18·296	19·235	20·234	21·299	7
8	16·223	16·896	17·629	18·418	19·271	20·194	21·193	22·257	23·394	24·613	8
9	18·454	19·236	20·084	20·993	21·967	23·024	24·142	25·338	26·621	28·008	9
10	20·750	21·643	22·606	23·631	24·734	25·904	27·147	28·482	29·925	31·477	10
11	23·111	24·116	25·191	26·339	27·551	28·840	30·216	31·703	33·301	35·044	11
12	25·538	26·650	27·843	29·095	30·421	31·837	33·358	34·993	36·774	38·710	12
13	28·025	29·251	30·543	31·905	33·351	34·906	36·569	38·377	40·342	42·487	13
14	30·577	31·898	33·295	34·773	36·353	38·042	39·871	41·855	44·018	46·368	14
15	33·174	34·596	36·104	37·710	39·419	41·268	43·265	45·438	47·796	50·360	15
16	35·822	37·351	38·982	40·711	42·573	44·582	46·761	49·120	51·682	54·476	16
17	38·525	40·172	41·922	43·798	45·814	47·997	50·354	52·908	55·688	58·727	17
18	41·293	43·055	44·945	46·969	49·153	51·507	54·049	56·812	59·826	63·126	18
19	44·122	46·019	48·052	50·237	52·585	55·117	57·859	60·845	64·108	67·691	19
20	47·031	49·066	51·253	53·596	56·114	58·838	61·794	65·018	68·552	72·442	20
21	50·020	52·204	54·543	57·050	59·753	62·682	65·866	69·349	73·175	77·395	21
22	53·100	55·430	57·927	60·611	63·511	66·659	70·092	73·856	77·997	82·589	22
23	56·265	58·748	61·415	64·289	67·400	70·787	74·490	78·555	83·052	88·054	23
24	59·521	62·168	65·018	68·095	71·436	75·082	79·075	83·482	88·372	93·839	24
25	62·877	65·701	68·747	72·045	75·636	79·561	83·883	88·667	94·003	40	
26	66·343	69·356	72·616	76·155	80·015	84·257	88·911	94·155	39		
27	69·931	73·150	76·642	80·441	84·607	89·198	94·297	38			
28	73·654	77·098	80·841	84·935	89·438	94·429	37				
29	77·527	81·214	85·243	89·664	94·553	36					
30	81·567	85·531	89·875	94·669	35						
31	85·802	90·072	94·778	34							
32	90·258	94·880	33								
33	94·976	32									
	31										

Duration	50	49	48	47	46	45	44	43	42	41	Duration
	6·552	6·119	5·734	5·390	5·079	4·798	4·542	4·308	4·095	3·900	
										93·660	23
									93·466	87·708	22
								93·253	87·330	82·084	21
							93·020	86·917	81·534	76·740	20
						92·763	86·465	80·932	76·026	71·643	19
					92·482	85·968	80·274	75·245	70·772	66·755	18
				92·172	85·422	79·548	74·389	69·820	65·734	62·058	17
			91·827	84·821	78·753	73·448	68·777	64·618	60·893	57·531	16
		91·442	84·153	77·877	72·416	67·629	63·395	59·618	56·227	53·157	15
14	91·009	83·406	76·904	71·278	66·371	62·049	58·223	54·801	51·719	48·922	14
13	82·566	75·815	70·015	64·984	60·574	56·687	53·239	50·145	47·354	44·814	13
12	74·591	68·601	63·444	58·948	55·003	51·520	48·422	45·638	43·119	40·820	12
11	67·011	61·720	57·143	53·147	49·635	46·526	43·760	41·266	39·003	36·934	11
10	59·782	55·122	51·087	47·558	44·448	41·692	39·236	37·015	34·997	33·162	10
9	52·850	48·780	45·252	42·156	39·426	37·002	34·839	32·879	31·110	29·492	9
8	46·188	42·670	39·613	36·927	34·554	32·443	30·560	28·864	27·326	25·922	8
7	39·768	36·765	34·153	31·854	29·817	28·007	26·407	24·958	23·647	22·429	7
6	33·564	31·048	28·857	26·922	25·209	23·701	22·365	21·159	20·047	19·019	6
5	27·558	25·503	23·709	22·123	20·735	19·511	18·435	17·441	16·532	15·678	5
4	21·732	20·111	18·699	17·465	16·382	15·436	14·589	13·811	13·088	12·400	4
3	16·067	14·865	13·836	12·933	12·149	11·449	10·833	10·256	9·710	9·180	3
2	10·556	9·773	9·104	8·525	8·007	7·555	7·155	6·767	6·390	6·042	2
1	5·206	4·818	4·503	4·212	3·962	3·741	3·546	3·340	3·156	2·980	1
	50	49	48	47	46	45	44	43	42	41	

Some Remarks on the Mortality among Persons with Consumptive Family History. By PROFESSOR HARALD WESTERGAARD.

[Extracted by permission from the *Economic Journal,* March 1894.]

ONE of the most interesting questions in mortality statistics is that of hereditary influences, but very little light has hitherto been thrown on this subject. What effect will the good or bad health of the present generation have on that of the future one? In which way will nature eliminate the fatal germs of a family disposition? Will they disappear through a gradual dying out of the persons with such a disposition, or will they in the course of time grow less dangerous till at last they have lost entirely their original character? All these questions have hitherto been left almost unanswered in spite of the great scientific and economic interest attached to them.

This paper offers some contributions to the solution of this difficult problem, choosing the most important of all the human diseases. In the first place, I will mention some previous attempts to solve the problems; in dealing with them I shall have an opportunity of showing the difficulties in dealing with the question itself.

After referring to the important contribution made by Dr. Reginald E. Thompson in his interesting work: "The different Aspects of Family Phthisis" (*J.I.A.*, xxiv, 443), the author proceeds:—

Notwithstanding the results which might be derived from observations like Dr. Thompson's, it must be taken for granted that these results are few and meagre in comparison with the great labour required to compile the data. If we want to know the effect of family phthisis it is necessary to know the number of persons who have been exposed to risk. This remark applies to another recently-published investigation by Mr. Manly, in the *Journal of the Institute of Actuaries,* July 1892: "An attempt to measure the extra risk arising from a consumptive family history, when the life proposed for assurance is physically sound and healthy."

The author compiles the statistical data from the experience of a life office, thus being enabled to observe the number of deaths taking place among a certain number of persons. The probability of dying can thus be calculated if the data are otherwise reliable, and on these values extra premiums covering

the risk arising from applicants with a consumptive family history can be safely based.

Though the basis thus is thoroughly correct, grave objections can be made to the application of these principles. The main objection is, that the author almost entirely leaves out of consideration the casual deviations from the averages, which frequently appear when the numbers are not very great. To show this I shall quote some of the conclusions at which Mr. Manly arrives. When there is evidence that a brother or sister died from consumption and the applicants are perfectly healthy, he finds a close correspondence to the Institute of Actuaries H^{M} Table, the total number of expected deaths being 94 and the actual number of deaths 95. When one of the parents died of consumption, he finds an excess of mortality, namely, 42 actual cases of death and 34 expected. A similar deviation appears in the cases where the mother died of childbirth (or both mother and a sister), which is generally considered to arise from consumption, when no other evidence is forthcoming, the actual number of deaths being 69 against 55 as expected. The author then proceeds to construct graduated mortality tables on the two latter groups of facts, and on these tables are based the extra premiums for applicants with a consumptive family history.

It is evident from an examination of these numbers that this is too small a basis for safe conclusions as to the influence of a family disposition as regards health. If some few unexpected cases of deaths take place in a class of persons exposed to risk, the number of deaths may easily swell from 34 to 42, as in the group where one of the parents died of phthisis. Even from a practical point of view, without referring to the theory of statistics and the laws of error, it would appear very hazardous to build on these numbers. The deviation is 23 to 24 per-cent of the expected number, but nobody can tell if the average mortality in future will bear the same relation to the H^{M} Table. A single death more or less would cause an increase or decrease of 3 per-cent. And this preliminary result will be corroborated if we make use of the results of the Theory of Statistics. As measure for the casual deviations from the average I shall, in the following, use the mean error. According to the theory of probabilities the mean error is $\sqrt{m p q}$, m being the number of observations, p the probability of the event, and $q=1-p$, the probability that this event shall not take place. In the present case we may leave q out of consideration, this fraction

being very near 1, and we thus find $\sqrt{mp}$ as an approximated value of the mean error. This again is evidently the square root of the expected number of deaths. If we expect 34 deaths the mean error will be about 6, in the case of 55 expected deaths we shall have 7 to 8. According to the theory of probabilities the deviation will in one case out of three be greater than the mean error, in one case out of 20 greater than twice the mean error, while the probability that the deviation will be greater than 4 times the mean error is extremely small.

These remarks would only have a theoretical interest, if there were no correspondence between these results and the actual observations. It might be, that in reality the deviations for instance were much smaller than would be supposed according to the law of error; there is also a chance that they are greater, and till it has been proved that the law under certain circumstances holds good it has no value. By numerous experiments, for which I beg to refer to my "Theory of Statistics, 1890",* it has in fact been proved, that the limits of the casual deviations very rarely are narrower than according to the law of error, most frequently they are much wider than supposed by the theory, but very often the actual law of error can be brought to correspond to the theoretical law.

Taking now this favourable supposition that there is such a correspondence we find on returning to Mr. Manly's paper, 34 expected deaths in the group where one of the parents died from consumption against 42 as expected. The mean error being about 6 the deviation is about $1\frac{1}{3}$ times the mean error. But we very often observe much greater deviations. In one case out of 5 we may expect a deviation greater than 8, in one case out of 20 a deviation greater than 12, &c., even if there is no difference between the mortality of the group and that of the H^{M} Table. If 10 life offices investigate a certain class of applicants, each with an average of 34 deaths, one of them would, according to the theory of probabilities, find a mortality greater than 42, another a mortality smaller than 26. The former would conclude that the mortality among these applicants was several per-cent greater than ordinary, the latter that the mortality was very small. But the only true conclusion would be that the data were insufficient to allow any trustworthy statistical results.

* *Grundzuge der Theorie der Statistik*, Jena, 1890.

Returning to Mr. Manly's paper we may find by examination of *new* data, that he is right in estimating the mortality among applicants whose father or mother died from consumption between $\frac{1}{4}$ and $\frac{1}{5}$ greater than the H^M mortality. But it may be that the mortality is much greater. It is not impossible that the average mortality by further observations would be found to be even 60 per-cent or more above what is expected according to the H^M Table. If this average mortality were for instance 70 per-cent greater, the number of deaths would be 58. The mean error would be 7 to 8 and a deviation downwards like 16, or about twice the mean error, would bring the number down to 42, which is the observed number. Nor is it improbable that the mortality is smaller than according to the H^M Table. If the mortality were 10 per-cent smaller, or 31, the deviation above the average, to 42, would equally amount to twice the mean error. We thus see that the observations prove very little, the range of casual deviations being so wide that we cannot tell if the true average number be even smaller than 31 or greater than 58.

From what has been explained it follows that the alleged increased mortality among the group of applicants whose father or mother died of consumption cannot be considered as proved by Mr. Manly's data. There is some better evidence of an increased mortality in the group of persons whose mother died of childbirth. The expected number of deaths in this group being 55 and the actual number 69 we have a mean error of 7 to 8, whereas the deviation is twice the mean error. But even here the conclusion as to an intensified mortality is evidently not quite safe. Somewhat clearer evidence may be got by adding the two groups together. The number of data being then greater, the law of great numbers consequently comes better into action. We find 89 expected deaths and 111 actual ones; the mean error being $9\frac{1}{2}$ and the deviation 22, and accordingly a greater probability of an intensified mortality than in each group separately: but whether the mortality is for instance 25 per-cent above the H^M Table or 20, 30, &c., is left quite uncertain.

Mr. Manly adds up the number of deaths among persons whose sister or brother died of consumption, finding thus 128 cases against 124 as expected. The difference is very small, but nevertheless it cannot be held as proved that no difference existed between the mortality in this group and that of the H^M Table. It is not impossible that further observations will give evidence of a

mortality like that prevailing among persons whose father or mother died of phthisis. Supposing in the latter group 42 to be the exact average mortality, the number of expected deaths among the collaterals, if the mortality were the same, would be 153, with a mean error of 12 to 13. The deviation from 153 to 128, only twice the mean error, would evidently be within the scope of possibility.

Taking all the groups considered by Mr. Manly together, we find 231 expected and 255 actual deaths. The mean error is 15, and we have thus a deviation only $1\frac{1}{2}$ times the mean error. It is, therefore, not impossible that the applicants with a consumptive family history show merely the ordinary mortality.

The main objection to Mr. Manly's investigation is thus that the materials are too small. If some other life offices would treat their observations in the same way, the eventual extra risk might be calculated, but, before these data are to hand, we are ignorant as to the effect of a family predisposition on persons accepted for life insurance.

But even if this problem were solved, the question as to the mortality among persons with a consumptive family history would be far from being so. It would only be the physically sound and healthy whose mortality would be known, but behind these lucky applicants there are vast numbers of rejected. In every life office numbers will apply for insurance who are rejected on account of ill-health, family predisposition, &c., only the very healthiest being accepted. In this respect, the life offices necessarily act very cautiously; having no data as to the mortality of such persons, they prefer rejecting them altogether or charging them with a high premium, taken so to speak at random. If they had mortality tables for all such applicants, one or more for heart disease, others for consumption in different stages, &c., they might calculate extra premiums that would cover the increased risk, which would be a great benefit for the applicants. Actuarial skill has here a vast uncultivated field; having worked out the observations as to healthy persons into very minute details, that science ought to turn to these important and so long-neglected problems. That the mortality among the accepted applicants has only been 225 deaths against 231 as expected, seems to show that the medical advice of the life office is very cautious and careful. If the medical advisers were to show yet more skill and foresight, they would probably reject some more applicants, thus reducing the mortality to that of the H^M Table.

How then is this problem to be solved? In a small country like Denmark, it would not be impossible to trace the life of all the rejected applicants, genealogical history being rather flourishing in such small communities. Of course the first trial would not give quite satisfactory results, but by-and-by a series of tables might be constructed which might be used for calculating the extra premiums. In a country like England, this way of solving the problem would of course meet with many difficulties, but the fact that the register of marriages, births, and deaths in Somerset House can be kept in order seems to show that even in a large community such a problem might gradually be solved. In fact there have been collected data in England bearing on this question—namely, the materials which the late Charles Ansell, jun., has dealt with in his Family Statistics, 1874. Through the courtesy of the National Life Assurance Society I have had access to these materials, and I shall attempt in the following pages to explain the main results of an investigation of these data as to the influence of family phthisis.

Among these data I selected all the returns where a death from phthisis, consumption or tuberculosis, &c., was recorded. From the day of this death the family concerned was taken under observation.

The causes of death could not of course be considered as very exact. Very often the cause was not at all recorded, and where there are duplicates, two members of the family each sending a return, the causes of death frequently differ more or less. The popular nomenclature is of course very vague, dropsy for instance being not uncommonly assigned as the cause of death. But on the whole there is reason to believe that the general character of the selected group is not disturbed by the alleged deficiency of the data. At all events—as was explained above—it is probable that the effect of family phthisis will appear in the selected group, though perhaps with a smaller intensity than it really has.

The following table gives the chief results of a comparison between the mortality experience for families generally according to Mr. Ansell's tables, and that among the families with a consumptive history, including all cases where the father, the mother, or a sister or brother died of consumption, the first year of life having been left out of the question.

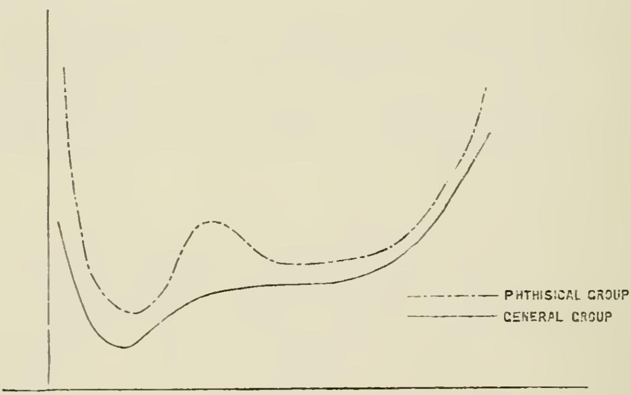
	Males					Females				
		Rate of Mortality		Number of Deaths			Rate of Mortality		Number of Deaths	
Age Years	Years of Life	According to General Table	In Consumptive Families	Actual	Expected	Years of Life	According to General Table	In Consumptive Families	Actual	Expected
1– 5	451·5	1·35	3·10	14	6·1	428·0	1·28	2·57	11	5·5
5–10	1,332·0	0·55	0·98	13	7·3	1,265·0	0·49	0·40	5	6·2
10–15	2,196·0	0·36	0·73	16	7·9	2,184·5	0·42	0·50	11	9·2
15–20	3,037·0	0·61	0·92	28	18·5	2,959·5	0·69	1·62	48	20·4
20–25	3,491·0	0·92	1·60	56	32·1	3,577·5	0·66	1·03	37	23·6
25–35	7,276·0	0·95	1·17	85	69·1	8,256·5	0·72	1·12	91	59·4
35–45	5,763·0	1·05	1·20	69	60·5	5,548·5	0·81	0·79	44	44·9
45–55	3,240·5	1·36	1·48	48	44·1	3,634·0	0·91	0·82	29	33·1
55–65	1,190·0	2·36	2·86	34	28·1	1,591·5	1·72	1·76	28	27·4
Total	27,977·0	...	...	363	274	29,445·0	...	...	304	230

There is thus a considerable difference between the mortality among persons with a consumptive family history and the general class. In the former group altogether 504 deaths were *expected*, whereas 667 deaths actually took place, the mortality being consequently about $\frac{1}{3}$ greater than according to the general table. But this abnormal mortality does not appear in all ages, the specific "phthisical" age 15–35 showing the greatest difference from the general mortality. An examination of the following abstract will show this.

	Males		Females	
Age	Number of Actual Deaths	Number of Expected Deaths	Number of Actual Deaths	Number of Expected Deaths
1–15 years	43	21·3	27	20·9
15–35 ,,	169	119·7	176	103·4
35–65 ,,	151	132·7	101	105·4
Total . .	363	273·7	304	229·7

At the age 35–65 there is apparently only a small difference between the mortality in the two groups. Taking both sexes together we have 252 actual deaths and 238 expected ones, the

difference being only 14. It may be that this is a true expression of the real sanitary state, there is also a possibility that the mortality in reality is greater, but at all events it is not probable that the mortality is proportionally less favourable than at the age 15–35 with altogether 223 expected and 345 actual deaths, the difference amounting to several times the mean error. There seems also to be a difference at the age 1–15 (70 deaths against 42 as expected, the difference being considerable compared with the mean error) but the main actuarial interest is attached to the age 15–35. Out of the 345 actual deaths during this period of life 170 were recorded as phthisical, which is much more than that of the population generally. It follows from what has been explained that the mortality table of persons with a family history of consumption has a different appearance from the common one. The former has a maximum at about 20, whereas the latter shows a constantly decreasing mortality from 1 to 15 years, and, after the age of 15, with a single slight exception, a constantly increasing mortality. The following diagram will give an approximate idea of this movement (for males), the distance between the curve and the horizontal line representing the values of the intensity of the mortality at different ages.



This peculiar character of the mortality table of persons with a consumptive family history will, in certain cases, necessitate alterations in the ordinary way of the payment of regular yearly insurance premiums. The yearly premium will commonly be constantly increasing with the age at entrance and the surrender-value of the policy will consequently always be positive. But if,

for instance, the yearly premium at 20 were higher than at 30, a person who entered at 20 might at 30 give up his insurance and apply again for insurance at a lower premium, the life office consequently suffering a loss equal to the value of the annuity of the difference of the premiums at 20 and 30. But a life office might of course avoid this difficulty by temporary premiums, ceasing for instance after 10 or 20 years.

In order to judge of the effect of the family disposition I have calculated approximate net premiums for a temporary assurance ceasing at 65 (at which age my observations stop). I have used a short method based on the ungraduated tables, and supposing the force of mortality constant during each successive period of 5 or 10 years.* The influence of a family predisposition after 65 being probably comparatively small we might perhaps be entitled to complete the life table after 65 with the H^M Table, thus being able to calculate premiums for an insurance for lifetime, but I shall not here enter upon such details.

Persons with a Family Predisposition			General Table	
Age	Single Premium	Yearly Premium	Single Premium	Yearly Premium
20	27·9	1·37	24·1	1·13
30	27·2	1·47	23·9	1·26
40	26·2	1·70	23·5	1·50
50	23·8	2·18	20·6	1·87

There is thus a marked difference between the net premiums according to the two tables, and the difference would be still greater if all the persons with a family predisposition could be left out of the general table. But at all events the premium for *an otherwise healthy* person with a family predisposition would not be so high as to be impossible for the applicant to pay, and many persons would evidently look upon it as a great benefit to be accepted at a moderate extra premium instead of being rejected, even if this extra premium for reasons which it will not be

* The force of mortality being μ_1, μ_2, &c., we have approximately, if the number of living in age x, according to the life table, is l_x:

$$l_5 = l_0\left(\frac{2-\mu_1}{2+\mu_1}\right)^5 l_{10} = l_5\left(\frac{2-\mu_2}{2+\mu_2}\right)^5 \text{ \&c.}$$

necessary to explain must be proportionately greater than these numbers show.

Returning to the mortality statistics we may ask whether the alleged influence of a family history of consumption may not often be the result of other causes. Two persons living in the same house may be attacked by the same contagion; the disease thus not arising from an inherited disposition, but being in both cases acquired.

The cause of error may however be, to a certain extent, eliminated if we date the observations from a later period, instead of taking the persons under observation at the moment when a sister or brother died of consumption we may put it off, for instance, 5 years. This will reduce the mortality a little, but there will still remain a considerable difference between the mortality in the two groups. I shall only here give an abstract of the results.

There is hardly any difference between the actual mortality and that expected according to the phthisical table (printed on page 381) during the ages 5–15 and 35-65. But between 15 and 35 there is a smaller mortality than expected, though it is still somewhat greater than according to the general table. Altogether there were 226 deaths among males and females,

Age Years	Males			Females		
	Number of Deaths			Number of Deaths		
	Actual	Expected		Actual	Expected	
		According to General Table	According to Phthisical Table		According to General Table	According to Phthisical Table
5–15	14	6·6	12·7	8	6·7	7·3
15–35	117	89·8	125·5	109	74·1	125·7
35–65	141	123·4	140·7	98	103·1	99·5
Total	272	220	279	215	184	233

whereas 251 were expected according to the phthisical table, and 164 according to the general one. In the latter case there is a great probability of an intensified mortality in the phthisical families, but also in the former case there seems to be a difference—namely, that the mortality decreases a little after the lapse of

5 years after the first death from phthisis,* leaving at all events a considerable surplus compared to the general table.

I have also followed up the cases where more than one death from phthisis has taken place in one family. Examining the members of these families from the date of the second or third death of phthisis, I find the following main results:

Two Deaths from Phthisis in the Family.

Age Years	Males: Number of Deaths: Actual	Males: Expected: According to General Table	Males: Expected: According to Phthisical Table	Females: Number of Deaths: Actual	Females: Expected: According to General Table	Females: Expected: According to Phthisical Table
1–15	9	2·4	4·5	3	1·7	2·1
15–35	54	23·3	32·4	42	17·9	30·1
35–65	46	36·1	41·2	40	34·1	31·6
Total	109	62	78	85	54	64
Three Deaths from Phthisis in the Family						
1–15	4	0·5	1·0	1	0·4	0·4
15–85	15	7·3	10·1	14	4·9	8·3
35–65	17	11·9	13·5	14	10·3	9·7
Total	36	20	25	29	6	18

The mortality is thus much greater than in families where only one death from phthisis has taken place, this being consequently an aggravating circumstance taking into consideration

* The calculation of the mean error is in this case a little more complicated than under the supposition that we can look upon the rates of mortality as definitely fixed by numerous observations. Here both the data in the group of selected persons, and in the other group of which the mortality is taken as typical, are liable to casual deviations. Let d_1 be the number of deaths during the first five years, d_2 the number of deaths after this date; let m_1 and m_2 be the years of life. The difference between the expected and actual deaths will then be $\left(\frac{d_1+d_2}{m_1+m_2}\right)m_2 - d_2 = \frac{m_1 m_2}{m_1+m_2}\left(\frac{d_1}{m_1} - \frac{d_2}{m_2}\right)$. The mean error of $\frac{d_1}{m_1}$ being approximately $\sqrt{\frac{d_1}{m_1^2}}$, we shall have as the mean error of the above expression $\frac{m_1 m_2}{m_1+m_2}\sqrt{\frac{d_1}{m_1^2}+\frac{d_2}{m_2^2}}$. In the present case we have $d_1=119$, $d_2=126$, $\frac{m_1}{m_2}=\frac{27}{73}$, and consequently the mean error $=9$. The deviation $251-226=25$ is thus nearly three times the mean error.

the health of applicants, nevertheless even the additional premiums covering the extra risk for such persons would not be unattainable for the applicants.

These observations are, as explained above, incomplete and insufficient, but I hope that this paper may at least induce others to take up the same question, and may afford some help to those who do so.

CORRESPONDENCE.

INSURANCES AGAINST ISSUE.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—Dr. Sprague sent you, along with his letter of 10 March 1887 (*J.I.A.*, xxvi, 391), a table of the amounts, &c., of issue insurances, compiled from the Board of Trade Returns for the five years 1881–85 inclusive, except in two instances; and I have now the pleasure of sending you a new table, containing similar particulars regarding these insurances, which has been compiled from the Board of Trade Returns for the following five years, 1886–90 inclusive, except in the case of the *Edinburgh* office, whose last return was made in 1892.

Dr. Sprague, in his letter referred to above, called attention to the increase in the number and amount of these insurances; and you will see, from the subjoined summary, that this progress has been scarcely less marked in the five years now under review. The average premium for this quinquennium is lower than for the previous ones, but is still fully 6 per-cent.

Summary.

Date of Table	No. of Companies	No. of Policies	Net Sums Insured	Net Premiums Received	Average Premium per-cent
			£	£	
1877	27	261	875,558	62,238	6·75
1882	41	383	1,264,166	97,495	7·51
1887	39	539	1,696,747	116,704	6·82
1894	45	710	2,117,625	135,220	6·08

I am, Sir,

Your obedient servant,

D. M. CARMENT.

26 *St. Andrew Square,*
Edinburgh,
October 1894.

Name	Date of Valuation	Number of Policies	Net Amounts, deducting Reassurances		Gross Amounts			
			Sums Insured	Premiums Received	Sums Insured	Premiums Received	Reserve	Average Premium per-cent
			£	£	£	£	£	
Eagle . . .	31 Dec. 1887	58	284,510	29,094	461,305	40,008	14,194	8·67
Guardian . . .	31 Dec. 1889	89	222,289	13,146	342,832	20,363	17,823	5·94
Equity and Law .	31 Dec. 1889	73	221,914	15,779	291,125	20,402	20,402	7·01
Law Union . .	30 Nov. 1889	68	116,075	5.454	275,075	10,085	10,085	3·67
Scottish Equitable .	1 Mar. 1888	46	106.700	6.384	113,050	6,651	6,075	5·88
North British & Merc.	31 Dec. 1885	23	104,176	6,345	117,176	7,133	6,676	6·09
Legal and General .	31 Dec. 1886	28	100,838	5,211	140,038	7,041	7,041	5·03
Law . . .	31 Dec. 1889	32	88,146	5.781	97,446	6,254	6,254	6·42
Standard . . .	15 Nov. 1885	22	81,784	5,300	187,784	11,790	4,964	6·28
Crown . . .	25 Mar. 1890	20	62,442	2,693	83,442	4,132	3,767	4·95
Edinburgh . .	31 Mar. 1892	23	54,966	2,496	54,966	2,496	2,247	4·54
Pelican . . .	31 Dec. 1885	9	54,015	3,151	72,015	3,340	3,339	4·64
National . . .	31 Dec. 1889	22	50,150	2,312	50,150	2,312	1,849	4·61
Reliance . . .	31 Dec. 1887	9	45,437	3,558	45,437	3,558	3,400	7·83
Scottish Union & Nat.	31 Dec. 1889	10	43,808	2,010	53,808	2,460	2,460	4·57
Norwich Union .	30 June 1886	8	37,582	2,200	37,582	2,200	1,100	5·85
Rock . . .	19 Aug. 1889	8	37,300	3,490	92,800	6,987	5,412	7·53
London Assurance .	31 Dec. 1885	8	30,000	1,607	30,000	1,607	1,526	5·36
Universal . . .	31 Dec. 1888	8	28,800	1,608*	30,800	1,720	2,257	5·58
Royal Exchange .	31 Dec. 1885	6	28,400	645	28,400	645	516	2·27
Commercial Union .	31 Dec. 1887	12	25,018	1,138	25,018	1,138	1,138	4·55
Clerical, Med. & Gen.	30 June 1886	5	25,000	1,363	25,000	1,363	1,363	5·45
Alliance . . .	31 Dec. 1888	8	25,000	923	25,000	923	923	3·69
Atlas . . .	31 Dec. 1889	9	24,090	1,789	26,390	2,019	2,202	7·65
Caledonian . .	31 Dec. 1885	9	23,350	983	37,850	1,292	969	3·41
Imperial . . .	31 Jan. 1886	9	21,865	767	21,865	767	767	3·51
Union . . .	30 June 1887	6	21,500	1,339	21,500	1,339	715	6·23
Northern . . .	31 Dec. 1885	8	21,050	948	21,050	948	948	4·50
Liv. & London & Globe	31 Dec. 1888	8	20,700	885	20,700	885	520	4·28
West of England .	31 Dec. 1887	10	13,750	823	13,750	823	363	5 99
British Empire .	31 Dec. 1887	5	13,500	600	13,500	600	569	4·44
General . . .	31 Dec. 1887	7	13,000	778	13,000	778	389	5·98
Mutual . . .	31 Dec. 1888	7	11,500	435	11,500	435	435	3·78
University . .	1 May 1890	5	10,300	1,083	10,300	1,083	1,083	10·51
City of Glasgow .	20 Jan. 1889	8	9,783	570	9,783	570	570	5·83
Scottish Metropolitan	31 Dec. 1888	6	7,500	356	15,000	834	300	5·56
Scottish Imperial .	31 Dec. 1885	4	7,000	403	7,000	403	341	5·76
Scottish Life . .	31 May 1886	2	6,500	140	6,500	140	97	2·15
Sovereign . . .	31 Dec. 1885	2	5,000	284	5,000	284	142	5·68
Westminster & General	31 Dec. 1886	4	4,362	213	4,362	213	171	4·89
Patriotic . . .	31 July 1889	2	4,000	480	4,000	480	100	12·00
Hand-in-Hand . .	31 Dec. 1889	1	2,000	525	2,000	525	525	26·25
Provident Clerks .	31 Dec. 1887	1	1,000	6	2,000	126	126	6·30
Marine and General .	31 Dec. 1889	1	1,000	25	1,000	25	25	2·50
Briton Medical & Gen.	31 Dec. 1889	1	225	100	†225	100	7	...†
45 Companies .	...	710	2,117,625	135,220	2,948,524	179,277	136,175	6·08

* Estimated as $\frac{288}{308} \times 1,720 = 1,608$.

† Sum insured reduced under Reconstruction Scheme.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE. April 1894.

Examination for Admission to the Class of Associate (Part I).

Examiner—Prof. S. L. Loney, M.A.

First Paper.

1. A man borrows money from a money-lender, agreeing to pay interest at the rate of one shilling per pound per month, and that at the end of every third month the interest is to be added to the capital; prove that this interest is at the rate of nearly 75 per-cent per annum, and if the sum due at the end of nine months be £114. 1*s*. $3\frac{3}{4}d.$, find the original sum that he borrowed.

2. Prove that

$$(x+y-2z)^3+(y+z-2x)^3+(z+x-2y)^3$$
$$=3(x+y-2z)(y+z-2x)(x+z-2y),$$

and simplify the expression

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}-3\sqrt{6}}.$$

3. Solve the equations:

(i) $$\frac{\sqrt{x-1}+\sqrt{x-6}}{\sqrt{x-1}-\sqrt{x-6}}-\frac{\sqrt{x-1}-\sqrt{x-6}}{\sqrt{x-1}+\sqrt{x-6}}=\frac{8}{5}\sqrt{6},$$

and (ii) $$\left.\begin{array}{l} x^4+x^2y^2+y^4=133, \\ x^2+xy+y^2=7 \end{array}\right\}.$$

4. Prove that the sum of the $(m+n)$th and the $(m-n)$th terms of an arithmetical progression is equal to twice the mth term.

Sum the series $1+11+111+1111+11111$ . . . . . to n terms.

5. Find the number of permutations of n things taken r at a time, and deduce the number of combinations.

A closed chain is to be formed of twelve links of different metals; how many different arrangements can be made?

6. A train passes two men walking beside a railway in $3\frac{1}{2}$ seconds and $3\frac{3}{5}$ seconds respectively; a second train passes the men in $4\frac{2}{5}$ and $4\frac{1}{2}$ seconds respectively. Prove that the latter train will overtake the former, and, if they be on different rails, will pass it in 36 seconds.

7. Prove that $\log_a m^n = n \log_a m$.

The population of a certain country increases at the rate of 2 per-cent every 10 years; if its population now be ten millions what will its population be at the end of a century, given

$$\log 6 = \cdot 7781513,\ \log 1\cdot 7 = \cdot 2304489,$$

and $$\log 12\cdot 18995 = 1\cdot 086002\,?$$

8. Prove that the true discount on a given sum of money for a given time and at a given rate of interest is half the harmonic mean between the sum and the simple interest on the sum for the same time and rate.

9. A cask is filled with 50 gallons of water and another with 40 gallons of brandy; x gallons are drawn from each cask, mixed, and replaced. The same operation is repeated. Find x when there are $8\frac{7}{8}$ gallons of brandy in the first cask after the second replacement.

10. Prove that the arithmetic mean of any number of positive quantities is greater than their geometric mean.

Show that the sum of the harmonic series

$$\frac{1}{a} + \frac{1}{a+b} + \frac{1}{a+2b} + \ldots \text{ to } n \text{ terms}$$

is greater than $$\frac{2n}{2a+(n-1)b}.$$

11. Find the chance that a player, taken at random from a whist party, has—

(1) all the trumps in his hand,

(2) no trumps in his hand,

and (3) no cards of any particular suit.

12. Prove that in an obtuse-angled triangle the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by either of these sides, and the part of it produced which is intercepted between the obtuse angle and the perpendicular let fall on it from the opposite angle.

In any triangle prove that the sum of the squares on the sides is equal to twice the square on half the base together with twice the square on the line joining the vertex to the middle point of the base.

Second Paper.

1. What is the essential principle of Book-keeping by Double Entry?

Illustrate it by opening the requisite accounts for any transaction between A and B.

2. Express as a decimal fraction the quantity

$$\frac{9{\cdot}\dot{6}}{3{\cdot}\dot{3}} \times \frac{14{\cdot}02\dot{3}}{{\cdot}9} \times 1\tfrac{1}{29} \times {\cdot}\dot{3} \times 1{\cdot}\dot{7}4\dot{1} \div {\cdot}\dot{0}0\dot{6} \times \frac{32}{4207}\,.$$

3. A father's age is equal to those of his three children together. In 9 years it will amount to those of the two eldest; in 3 years after that, to those of the eldest and youngest; and in 3 years after that, to those of the two youngest. Find their present ages.

4. Solve the equations

$$\text{(i)} \quad \begin{cases} x+y+z=11, \\ x^2+y^2+z^2=45, \\ yz=20, \end{cases}$$

and in positive integers the equation

$$\text{(ii)} \quad 19y-23x=7.$$

5. Expand, to four terms, the quantities $(1-x)^{-2}$ and $(1+2x)^{\frac{1}{2}}$, stating under what restrictions the expansions are respectively valid.

Prove that the two infinite series

$$1+\frac{1}{4}+\frac{1.4}{4.8}+\frac{1.4.7}{4.8.12}+\frac{1.4.7.10}{4.8.12.16}+\ldots\ldots$$

and $$1+\frac{2}{6}+\frac{2.5}{6.12}+\frac{2.5.8}{6.12.18}+\frac{2.5.8.11}{6.12.18.24}+\ldots\ldots$$

are equal to one another.

6. Sum to n terms the series

$$(1) \quad 1.2.4+2.3.5+3.4.6+\ldots\ldots$$

and $$(2) \quad \frac{2}{1.3.4}+\frac{3}{2.4.5}+\frac{4}{3.5.6}+\ldots\ldots,$$

State the rule for writing down the sum to n terms of the series whose nth term is

$$1\div\{an+b\}\{a(n+1)+b\}\{a(n+2)+b\}\ldots \text{ to } r \text{ factors.}$$

7. Find that number of six digits such that when the extreme left-hand digit is transposed to the right hand, the rest being unaltered, the number is increased threefold. Show that there are two solutions, the greater being double the other.

If the smaller solution be taken and the same process be repeated on it 2, 3, 4, and 5 times successively, then the numbers obtained are respectively 2, 6, 4, and 5 times the original number.

8. State and prove the Exponential Theorem, and write down the expansion of $\log_e(1+y)$ in powers of y.

Show that the value of $\log_e 2$ is ·693 . . . .

9. A man takes out an endowment policy for 35 years with profits. At the end of each seven years a bonus is declared, at the rate of 35 shillings per-cent per annum, on the sum insured and bonuses at that time, and is added to this sum. Prove that if he live the full time he will get back the money he has paid, together with 3 per-cent compound interest, if his premium be slightly under £2. 19*s*. per-cent on the amount of his original insurance.

Given $\log 103 = 2{\cdot}0128372$, $\log 281386 = 5{\cdot}4493020$,
$\log 11225 = 4{\cdot}0501863$, $\log 60462 = 4{\cdot}7814825$,
and $\log 29477 = 4{\cdot}4694490$.

10. The chance of a given event happening in one trial is 1 in n, where n is a large number; prove that it will be an even chance that it happens at least once in $\cdot 693\ldots \times n$ trials.

11. A bag contains any number of balls which are equally likely to be white or black; one is drawn and found to be white; prove that the chance of drawing another white ball on a second drawing, the first ball not being replaced, is $\frac{2}{3}$.

12. If a quadrilateral be inscribed in a circle prove that the sum of a pair of its opposite angles is two right angles.

If a quadrilateral can be circumscribed about a circle prove that the sum of the lengths of two opposite sides is equal to the sum of the lengths of the other pair.

Examination for Admission to the Class of Associate, (Part II).

Examiners—Messrs. R. Cross; A. W. Sunderland; Geoffrey Marks; and H. E. Nightingale.

First Paper.

1. Write down the more important formulas for obtaining the rate of interest in an annuity-certain, having given a the present value and n the term of the annuity.

2. Two loans were granted 10 years ago—(a) £20,000 at $4\frac{3}{4}$ per-cent per annum nominal, repayable by 60 equal half-yearly instalments which include both principal and interest; (b) £20,000 at $4\frac{1}{2}$ per-cent per annum nominal, repayable by 60 equal half-yearly payments of principal, interest being also paid on the balance from time to time remaining outstanding. Find in each case the amount of the payment due to-day, and show the amounts of principal and interest included in the payment. Find also the sum for which each loan may be redeemed to-day, assuming interest at $3\frac{1}{2}$ per-cent per annum nominal.

3. Interpret the following expressions, and find their values in algebraical symbols:

$$_{n-1|}q_{\overline{xy}}, \qquad {}_{|n}q_{\overline{xy}}, \qquad {}_{|n}q^{2}_{x.y}, \qquad {}_{|n}q^{\;1}_{xy}.$$

4. Of three lives, x, y, and z, find the probability that in the nth year from the present time,

(*a*) All will die.
(*b*) The last life will die.
(*c*) One and one only will die.
(*d*) One at least will die.

5. Give the single and annual premiums for an endowment assurance on the joint existence of x and y maturing n years hence.

How would you calculate such premiums in an individual case, having access only to interest tables and the Institute of Actuaries' Life Tables, when the term of the endowment is not more than 10 years?

6. State the formula for the value of a term policy effected on x for a term $n+t$ years, the duration being t years and the premium due and unpaid.

How would you conveniently arrange a table for calculating the policy values for quinquennial ages at entry and years of duration?

7. Why is it, generally speaking, impossible to construct a mortality table from a census alone, or from a mere enumeration of deaths at the various ages?

How would you proceed to form a table from the two together?

8. Explain what is meant by the term "force of mortality."

The number living at age x being represented by the function l_0a^{-x}, find an algebraic expression for the expectation of life.

9. Explain, without entering into methods of construction, the formation of the functions D_x, N_x, M_x.

Give in commutation symbols the values of π_x, ${}_n A_x$, ${}_n a_x$, and of the single premium for a deferred temporary uniformly increasing assurance on x.

10. Show how to find the fine payable now for the option of effecting at the end of n years an endowment assurance on a life now select and of age x, the annual premium for a select life aged $x+n$ to be payable on effecting the assurance.

11. Draft a form of security to be employed when loans are granted by a life office on its policies.

How should the document be executed and stamped?

12. Mention some of the most desirable investments now available for the funds of a life office, giving reasons for your selection.

Second Paper.

1. A loan of £a is granted, repayable over a period of n months by equal instalments which provide principal and interest. Find the monthly instalment and the amount of capital included in the first payment:

(1) Assuming compound interest at $\frac{i}{12}$ per month.

(2) Assuming simple interest at $\frac{i}{12}$ per month.

Find the repayments of capital for the first two months when £1,000 is advanced, repayable over seven months with compound interest at $\frac{5}{12}$ per-cent per month.

2. A bond securing an advance of 1 with interest at the rate i is to be repaid by equal annual instalments of $\frac{1}{n}$. Required the price to be paid for the bond to yield the purchaser interest at the rate j.

Give a precise verbal definition of all the symbols used in answering the question.

3. Find the probability that a life aged x will die before a life aged y or within n years after y's death.

4. A company has on its books at the beginning of a year m lives each aged x.

(1) Find the most probable number of claims which will arise.

(2) State the possible numbers of claims which may occur, and, combining them with their respective probabilities, deduce the *expected deaths*.

5. An assurance on x is payable in $n+t$ years if x be living, or at the death of x if occurring between the tth and $(n+t)$th year. If x dies in the first t years the premiums paid are returned. Find the annual premium:

(1) If payable for t years only;

(2) If payable for $n+t$ years.

6. In the case of a stationary population, free from emigration or immigration, show how to construct a mortality table from an enumeration of the number born in a year, and a register showing the deaths and ages at death for the year.

You may assume that all births occur at the beginning of the year.

7. A military power desires to maintain a standing army of a million men. Five years' service is compulsory on all males attaining the age of 20. How would you apply a table, showing the mortality amongst males, to ascertain the annual number of recruits required to maintain the army at its proper number?

8. Find a formula for the value of an endowment policy, (1) in commutation symbols, (2) in terms of the annual premiums for endowments; and give a verbal interpretation of the latter.

9. Write down in commutation symbols the formula for an office annual premium for an assurance on x deferred for n years, with a return of the premiums paid in the event of the life dying during the period of deferment.

How would you value such a policy after it had been t years in force, t being less than n?

10. Explain the principles on which conversion tables, such as Orchard's, are based.

Having given the l_x column of a mortality table, describe shortly the various steps by which a table of annual premiums for endowment assurances, payable at age 60 or previous death, could be most rapidly obtained.

11. Give in detail a brief form of absolute assignment of a life policy, stating the important points to be embodied.

State also how the document should be executed, and give particulars as to stamping.

12. Describe the ordinary routine observed in settling a claim under a life policy.

13. What are the chief points to be observed in considering the eligibility of a leasehold property as security for a loan?

Examination for Admission to the Class of Fellow (Part III, Section A).

Examiners—Messrs. T. G. C. Browne; W. J. H. Whittall; F. E. Colenso; and Ernest Woods.

First Paper.

1. A person takes a crossed cheque, bearing on its face the words "not negotiable." How are his rights as holder of the cheque restricted by these words?

A. B. steals from C. D. an uncrossed cheque made payable to the order of C. D., but not endorsed. Having forged C. D.'s signature, A. B. presents the cheque for payment and receives its value. What remedy, if any, has C. D.?

2. What is meant by a voluntary settlement? Can the validity of a voluntary settlement be affected by any, and if so what, subsequent events?

3. What amendments of the English law relating to the assignment of life policies would you suggest?

4. When moneys are receivable by trustees who cannot all attend the settlement, how should the payment be made? What modification of the general law has been made with regard to the receipt of policy moneys by trustees?

5. Enumerate the securities upon which, in the absence of express provisions to the contrary in their trust deeds, it is lawful for trustees to invest their funds.

Would it be safe for trustees to purchase stock at a price exceeding its redemption value?

6. An advance is proposed upon mortgage of reversionary interests consisting of (*a*) equitable interests arising under a settlement of real and personal estate, and (*b*) legal interests in realty. What steps would you consider it necessary for the lenders to take in order to secure priority of charge in respect of the whole of the interests to be comprised in the mortgage?

What are the advantages of a Registry of Deeds as regards (*b*)?

7. A policy upon the life of A. B. was, in the year 1879, assigned by him to his wife absolutely "for her separate use." She afterwards, by deed in which A. B. concurred, but which was not acknowledged in accordance with the provisions of the Fines and Recoveries Act, assigned the policy to C. D. Upon the death of A. B., his wife

surviving him, the question arises whether her concurrence should be required in the receipt for the policy moneys. Discuss this question.

8. The estate of a person dying in Ireland comprises a policy granted upon his life by an English office. Could you advise that office to pay the claim to the personal representatives of the assured on production of (*a*) Irish probate or (*b*) Irish letters of administration, without requiring in either case re-sealing of the grant in England? Give reasons for your answers.

9. Give an analysis of the various elements of international indebtedness which form the bases of the foreign exchanges.

10. What distinguishes the Bank of England, in its position and operations, from other banks?

When the Bank of England desires to obtain the control of the discount market, what measures does it usually take?

11. Give a brief account of the last Conversion of Consols, stating approximately the amounts of the various items at present constituting the National Debt, and any other particulars known to you in regard to each.

What are the various sinking funds at present in operation for the reduction of the National Debt?

Second Paper.

12. Set out the reasons which have been given for expecting that the closing of the Mints for the free coinage of silver in India will be of advantage to that country; and also the objections which have been urged against that step.

13. State the various objects for which the local indebtedness of this country has been created.

What resources of income, other than the rates, do the local authorities possess?

14. Illustrate the process of Finite Integration by finding the integral of

$$u_x \cdot u_{x+1} \cdot \cdot \cdot \cdot u_{x+m-1},$$

and apply your result to obtain the sum of m terms of the series

$$2.5.8+5.8.11+8.11.14+ \cdot \cdot \cdot \cdot$$

15. Show that if $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9$, be terms of a series,

$$u_5 = \frac{56(u_4+u_6)-28(u_3+u_7)+8(u_2+u_8)-(u_1+u_9)}{70}.$$

Fill up the two gaps in the following table:

u_1	·74556,	u_6	·18432,
u_2	·55938,	u_7	·13165,
u_3	·42796,	u_8	·08828,
u_4	·32788,	u_9	.
u_5	,		

16. Indicate briefly the methods of construction and the distinguishing features of the following mortality tables.

1. The Institute of Actuaries' Experience.
2. The Connecticut Mutual Experience.
3. The Government Annuitants (1883) Experience.

17. What reasons have been put forward in favour of accurately observing the rate of discontinuance amongst assured lives? Give a working formula for ascertaining such rate, and show how the *data* could be best tabulated for the purpose.

18. State what you know of any standard Sickness Tables, including the methods of their construction, the data upon which they are based, and their suitability for use.

19. Describe Makeham's law of Mortality; and the method of graduation adopted by Mr. King for the Table in the Institute Text-Book.

20. Describe fully a formula for graduation by means of columnar summation based upon the principles of Woolhouse's method.

21. Describe the method which, in your opinion, is the best for ascertaining the profit from mortality. What other methods have been suggested, and what are your objections to them?

Examination for Admission to the Class of Fellow (Part III, Section B).

Examiners—Messrs. T. G. C. Browne; W. J. H. Whittall; F. E. Colenso; and Ernest Woods.

First Paper.

1. An office doing a non-profit business charges Carlisle 3 per-cent premiums with 10 per-cent loading, and values by the English Life Table No. 3 at 4 per-cent. using gross premiums, less $12\frac{1}{2}$ per-cent. What is your opinion of this valuation?

What steps would you take to eliminate negative values? What arguments have been employed to justify the retention of such values within certain limits?

2. State the advantages of and objections to the following systems of distributing surplus:

1. In proportion to the premiums paid during the valuation period.
2. By way of a reversionary bonus at a uniform rate per-cent on the sum assured for each year's duration since the preceding division, subject to the provision that the bonus additions, though declared quinquennially, do not vest until the premiums with interest amount to the sum assured.

3. A Mutual office values by the Institute of Actuaries H^{M} and $H^{M(5)}$ Tables on the net premium basis at 3 per-cent, and divides its profits by way of a reversionary bonus at a uniform rate per-cent on the sum assured for each year's duration since the preceding division. How do fluctuations in the amount of new business affect the results of this method of division, and what alterations, if any, should you advise?

Draft a special report to the Directors, submitting your views for their approval.

4. What are the different methods of classifying endowment assurances for valuation purposes, and the relative advantages and disadvantages of each? Give in each case the formula for finding the correct reserve.

5. Is an addition to the age in all cases a satisfactory method of making a surcharge for extra mortality? If not, what alternatives can you suggest? In considering different classes of assurance, it will be sufficient to take Whole-Life Assurances, Endowment Assurances, and Temporary Assurances.

6. Life office A has just declared a large surplus after a stringent valuation, is doing a large and increasing new business at a heavy

rate of expense, and is improving its funds at an average rate of interest of 4·25 per-cent per annum. Office B is solvent, with a moderate expense ratio, but declares unsatisfactory surpluses, is doing a diminishing new business, and is earning a rate of interest of only 3·75 per-cent on its funds.

Give an outline of the points to which you would direct your attention, if you were called upon as a referee to advise both offices upon the terms of a proposed transfer of the business of office B to office A.

7. Give a form for a book, by which the premium revenue of an office can be deduced continuously from year to year and reconciled with the valuation class books.

8. How would you value the reversionary securities of an office and deal with the profits arising therefrom? What practical objection is there to bringing any portion of the profit from this source into the interest revenue of an office?

9. What, in your opinion, is the best way of valuing the marketable securities of an office in a statement of assets?

How, in your view, could the returns under the Life Assurance Companies' Act be amended so as to convey full information respecting the value of the assets?

10. Define the term debenture stock as distinguished from a debenture.

Discuss fully the suitability of American Railway Bonds as investments for English Assurance Companies, and the points to be considered before purchasing them.

11. Draft a form of letter entertaining a preliminary proposal for the purchase of a reversionary life interest in settled funds, and enumerating the various points in respect of which the Directors would require to be satisfied.

Second Paper.

12. How would you determine the average rate of interest for the purposes of the returns to the Board of Trade? Set out fully your reasons for preferring the method you would adopt to others.

13. Explain the symbols

$$ {}_np_{\overline{wxyz\ldots\ldots(m)}}^{[r]}, \qquad Q^4_{wxyz}, \qquad A^4_{\substack{wxyz\\321}}, $$

Find the value of ${}_np_{\overline{wxyz}}^{[2]}$.

✓ 14. Demonstrate the formula

$$a_x^{(m)} = a_x + \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu_x + \delta).$$

✓ 15. Show that, approximately,

$$\mathring{a}_x = a_x + \tfrac{1}{2}\bar{\mathrm{A}}_x - \tfrac{1}{12}\bar{\mathrm{A}}^{1}_{x.\overline{1|}}.$$

✓ 16. Obtain, by differentiation under the signs, the differential coefficients with respect to x alone of the following functions:

$$\int_0^\infty v^t({}_tp_x + {}_tp_y - {}_tp_{xy})(1 - {}_tp_z)\,dt.$$

$$\frac{1}{l_x . l_y . l_z}\int_0^\infty v^t . l_{x+t} . l_{y+t} . l_{z+t} . (\mu_{y+t} + \mu_{z+t})\bar{a}_{x+t}dt.$$

✓ 17. A table of mortality is constructed according to the condition:

$$\mu_x = -\log_e s - (\log_e c \log_e g)c^x.$$

Prove that $\quad \bar{\mathrm{A}}^1_{xy} = \tfrac{1}{2}c^{x-w} . \bar{\mathrm{A}}_{xy} + \log_e s(c^{x-w} - 1) . \bar{a}_{xy},$

where $\quad c^w = \tfrac{1}{2}(c^x + c^y).$

✓ 18. Write down the definite integrals representing the following benefits:

$$\bar{\mathrm{A}}^{3}_{\underset{1}{xyz}}, \qquad \bar{a}_{\underset{1}{xyz}}^{\;2}{}_{|w}, \qquad \bar{a}_{\underset{321}{wxyz}}^{\;4}{}_{|u},$$

and give some account of the processes which have been employed for obtaining approximations to their numerical values.

✓ 19. Describe the "force of mortality", and

Prove that $\mu_x = \dfrac{7(d_{x-1} + d_x) - (d_{x-2} + d_{x+1})}{12 l_x}$ very nearly.

✓ 20. Having given

$$\bar{a}_{29.45} = 12{\cdot}922$$
$$\bar{a}_{30.45} = 12{\cdot}879$$
$$\bar{a}_{31.45} = 12{\cdot}829 \quad \text{and } \bar{\mathrm{A}}^1_{30.45} = {\cdot}1454,$$

show that $\quad \mu_{30} = {\cdot}00768$ approximately.

The Candidate having handed in his answers to the foregoing questions, will be supplied with the following books:—"Institute of Actuaries' Life Tables"; and a Premium Conversion Table (Orchard or Rothery and Ryan).

21. An assurance is proposed on the joint lives of A and B, two publicans aged 30 and 35 respectively, who are both personally engaged in the sale of liquor. A is a spinster and is about to marry B. Assuming that in the case of a single life the annual extra

premium charged on account of the occupation of publican would be £1 per-cent per annum, and that a single extra premium of £1 per-cent would also be required if the life were a spinster on the point of marriage, what single and annual office rates would you quote for the joint-life assurance, taking as your basis for ordinary annual premiums the H^M 4 per-cent table, and loading by the formula

$$P = 1{\cdot}075(\pi + {\cdot}01a^{-1} + {\cdot}00225)?$$

Give reasons for the methods which you adopt.

22. A leasehold residential house, in a fashionable neighbourhood, and let on lease to a good tenant, is proposed as a security for an advance. The original lease has 65 years to run, but the borrower's interest is subject to the life interest of a lady aged 70. The gross rental of the premises being £350, and the ground rent £50, what would you consider the largest sum that it would be prudent for an office to advance? Indicate the methods by which you arrive at your result.

PROCEEDINGS OF THE INSTITUTE.—Session 1893-94.

First Ordinary Meeting, 27 *November* 1893.

The first ordinary meeting of the session 1893-94 was held at the Hall of the Institute, on the 27th day of November 1893.

The President (Mr. Aug. Hendriks) in the Chair.

The President announced that an Examiner had been appointed for Part I, and the choice of the Council had fallen on Mr. S. L. Loney, who was Third Wrangler, Cambridge, in 1882.

He also stated that a joint circular had been issued to the insurance companies, requesting them to contribute the result of their mortality experience during the thirty years which had elapsed since the last tables were formed by the Institute of Actuaries. He had every reason to anticipate that the circular would be met by the several companies in the spirit in which it was proposed.

Mr. R. P. Hardy read a paper entitled "An enquiry into the methods of representing and giving effect to the Experience of a Friendly Society: with some account of the Hearts of Oak Benefit Society, and its Experience for the years 1884-91."

The following gentlemen took part in the discussion:—Messrs. A. Hendriks, Manly, G. F. Hardy, Adler, Neison, and W. G. Bunn (a visitor).

Second Ordinary Meeting, 18 *December* 1893.

The President (Mr. Aug. Hendriks) in the Chair.

The death of George Humphreys, one of the Vice-Presidents of the Institute, was announced.

A ballot was taken for the election, as Associate, of Mr. Arthur Hunter, F.F.A. Messrs. Colquhoun and Adler acted as Scrutineers, and Mr. Hunter was declared duly elected.

A paper "On the Methods of deducing the Rate of Mortality from the Experience of Assured Lives; with some mention of a method adopted in

investigating the experience of the Clerical, Medical and General Life Assurance Society", was read by the author, Mr. W. J. H. Whittall.

The following gentlemen took part in the discussion:—Messrs. Finlaison, Wyatt, Bailey, Ryan, R. P. Hardy, G. King, Manly, and the President.

Third Ordinary Meeting, 29 *January* 1894.

The President (Mr. Aug. Hendriks) in the Chair.

A ballot was taken for the election, as Associate, of Mr. Rufus Weels Weeks. Messrs. Searle and Bumsted acted as Scrutineers, and Mr. Weeks was declared duly elected.

The following papers were read in abstract:

i. "On the Tabulation of the facts extracted from the Records of a Life Office for the purpose of investigating its Mortality Experience"—Dr. Sprague.

ii. "On a Mode of tabulating the facts, for the purpose of ascertaining the Numbers exposed to Risk, and calculating the Rate of Mortality experienced by Assurance Companies"—Mr. James Meikle.

Dr. Sprague's paper was read by the author, and Mr. Meikle's by the Honorary Secretary, Mr. Cockburn.

The following gentlemen took part in the discussion:—Messrs. G. King, R. P. Hardy, Hewat, G. F. Hardy, Whittall, Searle, Thiselton, and the President.

Fourth Ordinary Meeting, 26 *February* 1894.

Mr. T. G. C. Browne (Vice-President) in the Chair.

The Honorary Secretary reported that Mr. A. J. Finlaison had been elected Vice-President, and Mr. Ernest Woods Member of Council, to fill the vacancies caused by the death of Mr. George Humphreys.

A paper entitled "Notes on the use of Scales of Premium reduced in anticipation of Future Bonuses" was read by the author, Mr. G. F. Hardy.

The following gentlemen took part in the discussion:—Messrs. Higham, Whittall, Sorley, Nash, R. P. Hardy, Thiselton, G. King, and the Chairman (Mr. Browne).

Fifth Ordinary Meeting, 2 *April* 1894.

The President (Mr. Aug. Hendriks) in the Chair.

A ballot was taken for the election, as Associate, of Mr. Alfred Ernest Sprague, B.Sc., M.A., F.F.A.

Messrs. Cross and Faulks acted as Scrutineers, and Mr. A. E. Sprague was declared duly elected.

A paper entitled "Reversionary Interests contingent on Survival of the Reversioner", was read by the author, Mr. S. G. Warner.

The following gentlemen took part in the discussion:—Messrs. Browne, Mackenzie, Faulks, Bumsted, Bailey, Hemming, G. F. Hardy, and the President.

The President announced that arrangements had been made for an Institute dinner, at the Holborn Restaurant, on Wednesday, 18 April.

Sixth Ordinary Meeting, 30 *April* 1894.

The President (Mr. AUG. HENDRIKS) in the Chair.

A paper "On the application of Makeham's Modification of Gompertz's Expression for the Law of Mortality to the Practical Calculation of Survivorship Benefits", was read by the author, Mr. F. E. Colenso.

The following gentlemen took part in the discussion:—Messrs. Sunderland, G. F. Hardy, G. King, Hall, R. P. Hardy, Byers, and the President.

The Forty-Seventh Annual General Meeting, 9 *June* 1894.

The President (Mr. AUG. HENDRIKS) in the Chair.

The proceedings at the Annual General Meeting will be found on page 409.

REPORT, 1893-94.

"The Council have pleasure in reporting to the members upon the progress of the Institute during the session of 1893-94, the forty-sixth year of its existence.

"The increase in the number of members was 60, as compared with 29 in the preceding year. Since the date of the Charter the number of members has been as follows:

1884–85	434	1889–90	601
1885–86	441	1890–91	620
1886–87	484	1891–92	645
1887–88	521	1892–93	674
1888–89	563	1893–94	734

"The following schedule (p. 404) shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

"The Council have, with great regret, to report the loss by death during the year of six Fellows, namely: Mr. A. F. M. Gamble, Mr. R. W. Griffin, Mr. G. Humphreys, Mr. R. J. Lodge, Mr. Joseph Mills, and Mr. H. J. Rothery; also of Mr. W. S. B. Woolhouse, an Honorary Member.

"Mr. Humphreys had served the Institute in many capacities, and at the time of his death was one of the Vice-Presidents.

"Mr. Rothery was a member of the Council, and also Honorary Editor of the *Journal*.

"Of Mr. Woolhouse it is hardly too much to say that perhaps he had no equal among the non-academical mathematicians of his time. The improvements in the *Nautical Almanac*, in the office of which some of his early years were passed, were devised by him. The *Journal of the Institute* contains many of his contributions, and the graduation of the tables of mortality, both of the 17 Offices' Experience and those collected by the Institute, was his work. He had served on the Council of the Institute, and had filled the office of Vice-President.

"The vacancies on the Council caused by the deaths of Mr. Rothery and Mr. Humphreys have been filled by the election of Mr. T. E. Young and Mr. Ernest Woods.

"Mr. A. J. Finlaison was elected a Vice-President in the place of Mr. Humphreys, and the Council are pleased to be able to announce that Mr. G. H. Ryan has undertaken the duties of Honorary Editor of the *Journal* in succession to Mr. Rothery.

Schedule of Membership, 31 *March* 1894.

	Honorary Members	Fellows	Associates	Students	Corres-ponding Members	Total
i. Number of Members in each class on 31 March 1893 .	2	171	196	294	11	674
ii. Withdrawals by						
(1) Death . . .	1	6	...	1	...	47
(2) Resignation . .	...	2	2	18	...	
(3) Default in payment of Subscriptions . .	...	...	1	16	...	
	1	163	193	259	11	627
iii. Additions to Membership						
(1) By Election . .	...	...	2	...	...	107
(2) By Order of Council	...	...	...	103	...	
(3) By Re-instatement	...	...	1	1	...	
	1	163	196	363	11	734
iv. Transfers						
(1) By Examination:						
from Associates	...	...	3	...	...	...
to Fellows .	...	3	...	...	...	...
	1	166	193	363	11	734
(2) By Examination:						
from Students	...	...	...	3	...	...
to Fellows .	...	3	...	...	...	...
	1	169	193	360	11	734
(3) By Examination:						
from Students	...	...	...	15	...	...
to Associates .	...	...	15	...	...	...
v. Number of Members in each Class on 31 March 1894 .	1	169	208	345	11	734

"The Accounts for the year will be seen to be satisfactory, the total amount of funds now being £5,135. 6*s*. 2*d*., showing an increase in the year of £582. 9*s*. 11*d*.

"The Income and Expenditure Account and Balance Sheet are given herewith (p. 408).

"The Annual Subscriptions, together with admission and other fees, amounted to £1,473. 3*s*. 0*d*., showing an increase of £162. 15*s*. 0*d*. over those of the previous year.

"The total Income for the year was £1,844. 10*s*. 10*d*., and the total Expenditure £1,262. 0*s*. 11*d*.

"The stock in hand of the Institute publications at date is as follows:

No. of Copies	Description of Work
269	*Text-Book*, Part I.
504	,, ,, II.
108	Mortality Experience Tables.
6	Mortality Experience.
597	Logarithm Cards.
410	Messenger Prize Essay (Friendly Societies).
512	Index to 10 Vols.
119	,, to 20 ,,
9,311	Parts of *Journal*.

"The following papers were submitted at the sessional meetings of the Institute, namely:

"27 *November* 1893—'An enquiry into the methods of representing and giving effect to the experience of a Friendly Society: with some account of the Hearts of Oak Benefit Society and its experience for the years 1884-91,' Mr. Ralph P. Hardy.

"18 *December* 1893—'On the Methods of deducing the Rate of Mortality from the Experience of Assured Lives; with some mention of a method adopted in investigating the experience of the Clerical, Medical & General Life Assurance Society,' Mr. W. J. H. Whittall.

"29 *January* 1894—Abstracts of the following papers:

'On the Tabulation of the facts extracted from the Records of a Life Office for the purpose of investigating its Mortality Experience,' Dr. Sprague.

'On a Mode of tabulating the facts, for the purpose of ascertaining the Numbers exposed to Risk, and calculating the Rate of Mortality experienced by Assurance Companies,' Mr. James Meikle.

"26 *February* 1894—'Notes on the use of Scales of Premium reduced in anticipation of future Bonuses,' Mr. G. F. Hardy.

"2 *April* 1894—'Reversionary Interests contingent on Survival of the Reversioner,' Mr. S. G. Warner.

"30 *April* 1894—'On the application of Makeham's Modification of Gompertz's Expression for the Law of Mortality to the Practical Calculation of Survivorship Benefits,' Mr. F. E. Colenso.

"For the Examinations held in the United Kingdom on 20, 21, 23 and 24 April last, 125 candidates presented themselves, namely:

64 for Part	I.		
41 ,, ,,	II.		
5 ,, ,,	III,	Section	A.
3 ,, ,,	III,	,,	B.
12 ,, ,,	III,	Sections	A and B.

"Of these the following numbers were successful:

31 in Part	I.		
25 ,, ,,	II.		
10 ,, ,,	III,	Section	A.
12 ,, ,,	III,	,,	B.

"The following are the successful candidates, the names in each class being arranged alphabetically.

PART I.

Class I:

T. L. Coates.
A. Morland.

Class II:

W. T. Butterfield.
F. Gibberd.
T. Hartley.
M. Rees.
W. Stott.
T. E. Streeter.
E. B. Wilkinson.
O. W. Williams.
A. T. Winter.

Class III:

D. H. Aaron.
S. J. Askew.
H. H. Austin.
W. T. Featherstonehaugh.
A. W. Findlay.
A. C. Freeman.
T. G. Haward.
W. H. Hodgson.
E. E. Jackson.
J. E. S. Kemp.
G. W. Mead.
A. C. Nash.
A. H. Raisin.
V. E. Ridewood.
E. Ryley.
E. R. Searls.
Hy. Weatherill.
A. W. Woolfe.
F. A. Williams.
W. A. Workman.

PART II.

Class I:

A. Barrand.
W. A. Sim.
C. T. Weeden.

Class II:

J. M. Allen.
S. J. H. W. Allin.
E. H. Brown.
E. J. Bull.
K. W. Elder.
S. Jackson.
W. A. King.
A. Levine.
A. Lowndes.
G. E. May.
C. E. Reeve.
H. N. Sheppard.
G. Watt.

Class III:

R. W. Barton.
R. C. Hawkin.
N. C. M. Home.
N. M. Johannessen.
D. Legg.
F. Marchbank.
N. Miller.
C. R. Ray.
C. F. Trenerry.

PART III.—SECTION A.

Class I:

*R. Todhunter.

Class II:

*W. H. Aldcroft.
*J. Burn.
*A. E. Sprague.
*J. Taylor.
J. Watson.

Class III:

*W. A. Hutcheson.	*J. McDonald.
*A. M. Laughton.	J. F. Moran.

PART III.—SECTION B.

Class I:

H. A. Thomson.	*R. Todhunter.

Class II:

A. D. Besant.	*W. A. Hutcheson.
*J. Burn.	*J. McDonald.

*A. E. Sprague.

Class III:

*W. H. Aldcroft.	R. H. Fellows.
A. H. Clarke.	*A. M. Laughton.

*J. Taylor.

Those marked (*) passed in both sections.

"In the Colonies the Examination entries numbered 46, as under:

For Part I, 29.
,, II, 13.
,, III, Section A, 2.
,, III, Sections A and B, 2.

"The results of the Colonial Examinations will be duly announced.*

"The Council have given much attention during the year to the proposed New Mortality Experience investigation. Sixty-three Companies have agreed to contribute their Experience of Assured Lives, and many of these, together with certain other Offices transacting Annuity business, have promised to contribute their Experience of Annuity Nominees.

"The Institute and the Faculty, acting in conjunction, have recently issued to all the contributing Companies a circular for their guidance, with specimens of the cards to be used, and the distribution of the cards is now in progress.

"The Commissioners for the Reduction of the National Debt, and their Actuary, Mr. A. J. Finlaison, C.B., have kindly placed at the disposal of the Institute Tables relating to Joint Life Annuities calculated in 1884 upon the observations of the Mortality of Government Annuitants, and the Council propose, without unnecessary delay, to publish these tables. The volume, they hope, will be useful to the profession and to the public.

"The new Catalogue of the Library is now almost ready for publication, and within a very short period a copy will be sent to each member of the Institute.

"The proposed French edition of the *Text-Book* is making good progress, nearly two-thirds of Part II being now in print and stereotyped. It is hoped that by the end of 1894, Part II will be ready for issue to the public.

"Negotiations were entered into with the Imperial Institute, and have resulted in the President of the Institute of Actuaries becoming, *ex officio*, a member of the Governing Body of the Imperial Institute.

* See p. 412.

Dr. Income and Expenditure for Year ending 31 March 1894. Cr.

1893-94.		£ s. d.	£ s. d.
Amount of Funds, 31 March 1893—			
Messenger Legacy Fund		297 8 1	
Brown Prize Fund		211 16 9	
Library Fund		298 5 0	
General Fund		3,745 6 5	4,552 16 3
Annual Subscriptions—			
Town Fellows	£211 1 0		
Country ,,	65 2 0		
Fellows admitted since 1884:			
(1) By Examination	144 18 0		
(2) By Ballot	31 10 0	452 11 0	
Town Associates	186 18 0		
Country ,,	76 13 0		
Associates admitted since 1884:			
(1) By Examination	60 18 0		
(2) By Ballot	29 8 0	353 17 0	
Students		347 11 0	1,153 19 0
One Annual Subscription compounded for			37 16 0
Entrance Fees—			
Associates		4 4 0	
Students		106 1 0	110 5 0
Examination Fees			171 3 0
Sales of Publications—			
Journal		108 12 6	
Text-Book, Part I.		38 6 9	
,, ,, II.		69 16 0	
Log. Cards		1 11 10	
Mortality Experience		16 4 6	
Life Tables		14 5 0	
Hardy's Friendly Societies		0 5 4	
		249 1 11	
Less Cost of Binding		21 11 3	227 10 8
Received for Use of Hall			15 15 0
Dividends and Interest			128 2 2
			£6,397 7 1

1893-94.		£ s. d.	£ s. d.
Journal—			
Cost of Nos. 169, 170, 171, and 172		338 13 7	
Honorarium to Assistant Editor	£15 15 0		
Clerical Assistance	8 8 0	24 3 0	362 16 7
Library—			
Binding and Purchases		5 13 5	
Assistance *re* Catalogue		28 10 0	34 3 5
General Expenditure—			
Rent		260 0 0	
Salaries (including Income Tax)		233 2 6	
Lecturers		78 15 0	
Examination Charges		41 9 10	
Meetings		32 6 6	
House Expenses		28 0 4	
Corporation Duty		5 0 9	
Fire Insurance		11 8 0	
Stationery and Printing		138 6 6	
Postage and Telegrams		30 12 5	
Sundries		5 19 1	865 0 11
Funds 31 March 1894—			
Messenger Legacy Fund		303 1 1	
Brown Prize Fund		219 12 1	
Library Fund		298 5 0	
General Fund		4,314 8 0	5,135 6 2
			£6,397 7 1

Examined and found correct, 18 *April* 1894.

J. W. Miller,
R. M. Moore,
W. R. Makeham, } *Auditors.*

Balance Sheet, 31 March 1894.

LIABILITIES.	£ s. d.	£ s. d.
Messenger Legacy Fund (£211. 1*s.* 10*d.* Consols), cost	203 17 8	
Accumulated Dividends	99 3 5	303 1 1
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock), cost	200 0 0	
Accumulated Dividends	19 12 1	219 12 1
Library Fund		298 5 0
General Fund		4,314 8 0
		£5,135 6 2

ASSETS.	£ s. d.
Consols (£2,000), cost	1,886 10 0
Metropolitan Railway 4 per-cent Debenture Stock (£1,050), cost	1,185 11 3
Great Eastern Railway 4 per-cent Debenture Stock (£800), cost	1,031 10 3
On Deposit at the London and Westminster Bank	500 0 0
On Current Account at the London and Westminster Bank	500 4 8
Arrears of Subscriptions	31 10 0
	£5,135 6 2

Examined and found correct, 18 *April* 1894.

J. W. Miller,
R. M. Moore,
W. R. Makeham, } *Auditors.*

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The President (Mr. A. Hendriks), in proposing the adoption of the report, said the Institute continued in the same satisfactory condition that it had been in now for a long series of years. In every way it not only held its own, but had made sensible progress—in fact, taking the report and adverting to the increase in the number of members, one finds that in the decennial period, 1884 to 1894, the numbers had increased from 434 to 734—say, exactly 300 members in excess of the number 10 years ago. This was a very satisfactory increase, and the schedule attached to the report showed that the increase had been considerably over that of the preceding year. Following upon that statement, he had to submit to the meeting the names of those who had departed from this scene. He need hardly advert to them one by one, but would refer more particularly to one of the younger and more prominent members, Mr. Rothery, who was taken from them in the prime of life, and in the zenith of his usefulness to this society. They had, also, lost a veteran in Mr. Woolhouse, whose contributions, not only to their particular branch of science, but to mathematics in general, were of world-wide interest and importance. Again, they had to deplore the departure of their cordial and genial friend, Mr. Humphreys. The vacancy caused by these deaths had all been filled up with, he thought, great advantage to the Institute—that is, the best choice possible had been made to replace those who were so usefully employed here previously. The election of Mr. Finlaison as a Vice-President was very satisfactory to every member of the Institute, and they also acclaimed the choice made of Mr. Ryan when he undertook the duties of honorary editor of the *Journal* in succession to Mr. Rothery. After referring in detail to the accounts, he alluded to the progress that had been made in regard to the proposed new mortality experience investigation. That investigation commenced when he took office, but no novelty could be claimed for the suggestion he had made in his address, as his two immediate predecessors had already referred to the same subject, and it was only the pressure of other matters which led to its postponement during their term of office. The number of companies which would contribute was 63. That meant a very large number of lives exposed to risk, and a very great task to be undertaken by the Institute, and they were only now upon the threshold of the actual work pertaining to such an enquiry. A considerable amount of time must necessarily elapse before the results could be arrived at, and there were two reasons for that. One was that the enquiry would extend over a vast number of observations, which would involve an immense amount of work. But the second reason, even greater, was that such an enquiry, once commenced, could not be brought to a termination in a short time. All who would be engaged in the enquiry were also engaged in their usual avocations, and had heavy duties to perform officially, and these extra duties would have of necessity to be undertaken by very busy men. They had, however, already sent out a circular and the cards, and the duty which commenced now would be to fill up these cards, and in due time to return them to the Institute for examination and manipulation. Then they had to congratulate themselves and to thank Mr. Finlaison for placing at their disposal the results of his calculations relating to joint lives under the table published by the Government. That table was admitted to be at present a sound test by which companies might enquire into their annuity obligations, and it was a very great assistance to them to be placed in possession of the tables relating to joint lives which had not hitherto been at their disposal. Then a heavy duty had been undertaken by one of the committees in preparing a catalogue to the works in the library. The proof-sheets were before him, and were of a very voluminous character. The work when completed would be a great boon to all who used the library of the Institute. It would be in the knowledge of most that the excellent *Text-Book*, for which they were so much

indebted to a previous President (Mr. Sutton) and to Mr. George King, had been translated into French for the use of the French-speaking community. The work had been undertaken by a Belgian gentleman, M. Bégault, and two of his colleagues, and it had now made very considerable progress. The last paragraph of the report dealt with the election of the President, *ex officio*, as one of the governing body of the Imperial Institute. It was felt that as a very large number of other institutions of a cognate character had associated themselves with the Imperial Institute, the Institute of Actuaries ought to be there represented. This was admitted by the Imperial Institute, and resulted in the election of himself to occupy that position in their deliberations. Subsequent to printing the report, it had been decided to again offer the Messenger prize. The matter had occupied the attention of a committee, which had arrived at a decision as to the subject-matter of the proposed prize essay, which, shortly, was this: "On the books and forms to be used in scheduling the particulars of risks of a life assurance company under its assurance and annuity contracts for periodical or interim valuations, and for the distribution of surplus." The exact terms of the prize had not yet been arrived at, but would be announced in due course. The subject was a very practical one, which he hoped would lead to some excellent papers. The last point to which he would refer was the measure now before Parliament called the Finance Bill. Amongst other things it dealt with the transfer of shares, funds, and securities, and laid down certain regulations for companies; and it also, in clause 8, sub-section (*b*), more particularly referred to policies of insurance, as follows: "Where any moneys become payable in the United Kingdom on the death of a deceased person under a policy of insurance or other contract, whether by way of a capital sum or of an annuity, the same shall not be capable of being transferred, disposed of, or paid, nor shall any person be able to give a good discharge for the same unless it is certified by the Commissioners that there is no claim for estate duty thereon, and the Commissioners shall, on application in the prescribed form, if satisfied that there is no claim, give such certificate." Upon this Bill being announced there was a meeting of a kindred society—the Life Offices' Association. It seemed that there were very grave objections on the part of the insurance companies to this sub-section, and, as President of this Institute, he had been invited to associate himself with that body in representing to the Chancellor of the Exchequer the views of the insurance companies. A letter jointly addressed to the Chancellor of the Exchequer resulted in the right hon. gentleman kindly giving them an interview, which was of a satisfactory character, resulting as it did in the promise to withdraw that clause. They had felt that this provision would cause considerable delay in the payment of claims, and that it would lead more particularly to misapprehension in the mind of the public, in the sense that they might suppose, however erroneously, that the insurance companies, instead of one and all being not only desirous but most willing to meet their engagements immediately they occurred, were making needless delay. They had explained this to the Chancellor of the Exchequer, and it was in consequence of that representation, and also of the manner in which they had assured him that in every other way upon the Bill becoming law it would be their earnest endeavour to assist the Government, that he, after due consideration and consultation with the appointed officers of the Inland Revenue, had come to the decision that the clause objected to should be expunged. He (Mr. Hendriks) then formally moved "That the report and accounts be received and adopted."

Mr. T. G. C. Browne, in seconding the motion, referred to the library fund. That fund was a monument to the effusive generosity of the members of the body corporate. It was raised some five years ago, and as yet he

believed not a single shilling of it had been spent. And how it was to be expended on the library appeared an almost insoluble problem. He would suggest that a prize, say of 25 guineas, should be offered to the ingenious gentleman who could find out how the *corpus* of the fund could be satisfactorily expended upon the library. He also referred to the course the Council had adopted in appointing a professional examiner. Their choice had fallen upon a gentleman who had had great experience in examinations, and was a high Cambridge wrangler. The result so far had been quite satisfactory. The difficulties of having amateur examiners would no doubt have been enhanced this year, in consequence of the great increase in the number of candidates. To examine 64 papers certainly required a considerable amount of experience, and involved an immense amount of labour.

The motion was unanimously agreed to.

Messrs. Terry and Strachan having been appointed Scrutineers, a ballot was taken for the election of President, Vice-Presidents, Council, and Officers for the ensuing year. The Scrutineers reported that the following gentlemen, recommended by the Council, had been unanimously elected:

President.

Alexander John Finlaison, C.B.

Vice-Presidents.

Thomas G. C. Browne.	Charles Daniel Higham.
Ralph Price Hardy.	George King.

Council.

*Alfred Barton Adlard.	Henry William Manly.
Arthur Hutcheson Bailey.	Benjamin Newbatt.
Thomas G. C. Browne.	Gerald H. Ryan.
Arthur Francis Burridge.	Frederick Schooling.
*James Chisholm.	*Louis Michael Simon.
Henry Cockburn.	James Sorley.
*Francis E. Colenso, M.A.	Thomas Bond Sprague, M.A.
Stanley Day.	A. W. Sunderland, M.A.
Alex. John Finlaison, C.B.	William Sutton, M.A.
George Francis Hardy.	John Bell Tennant.
Ralph Price Hardy.	Robert Charles Tucker.
Augustus Hendriks.	*Samuel Geo. Warner.
Charles Daniel Higham.	Ernest Woods.
George King.	Frank Bertrand Wyatt.
Alex. George Mackenzie.	Thomas Emley Young.

* New Members of Council.

Treasurer.

James Chisholm.

Honorary Secretaries.

Henry Cockburn.	Frank Bertrand Wyatt.

Mr. A. J. Finlaison returned thanks on behalf of the Council, the Vice-Presidents and Officers. He also thanked the meeting for having elected him to the foremost position in their profession. His endeavours would be to maintain the high place the Institute had attained during the administration of his talented predecessors.

Mr. George R. Jellicoe proposed that Mr. J. W. Miller, Mr. R. M. Moore and Mr. E. H. Holt, be elected Auditors for the ensuing year.

Mr. H. A. Thomson seconded the motion, which was carried.

Mr. John Coles proposed a vote of thanks to the President, Vice-Presidents, Council, Officers and Examiners for their services during the past year.

Mr. H. C. Thiselton seconded the motion, which was cordially agreed to.

The President briefly acknowledged the vote on his own behalf, and on behalf of the Vice-Presidents, Council, Treasurer and other Officers.

On the motion of Mr. J. Sorley, seconded by Mr. H. E. Nightingale, a vote of thanks was given to Messrs. Miller, Makeham and Moore for their services as Auditors during the past year.

COLONIAL EXAMINATIONS.

Examinations were held on 20 and 21 April 1894, at Sydney, Melbourne, Wellington, Montreal, and Toronto, with the following results:

Part I.

Twenty-nine Candidates sent in their names, of whom twenty-one presented themselves, and fourteen passed as follows:

Class I:

None.

Class II:

Sanderson, Wm. (Toronto).
Wood, A. B. (Montreal).

Class III:

Brough, F. (Toronto).
Eedy, A. M. (Sydney).
Farrell, J. (Sydney).
Harris, F. I. (Sydney).
Hindmarsh, J. (Melbourne).
Jobson, A. (Melbourne).
Martin, S. G. (Wellington).
Norris, C. A. (Melbourne).
Owen, E. T. (Melbourne).
O'Reilly, A. J. (Montreal).
Shlager J. (Melbourne).
Wylie, S. B. (Toronto).

Part II.

Thirteen Candidates sent in their names, of whom eleven presented themselves, and seven passed as follows:

Class I:

Henderson, R. (Montreal).

Class II:

Elliott, C. A. (Sydney).

Class III:

Adams, C. E. (Wellington).
Fraser, T. J. (Melbourne).
Hardcastle, E. E. (Wellington).
Hollingworth, A. C. (Sydney).
Johnston, F. H. (Montreal).

Part III (Section A).

Three gentlemen sent in their names, of whom two presented themselves and passed as under:

Class II:

Blackadar, A. K. (Montreal).

Class III:

Muter, Percy (Wellington).

Part III (Section B).

Of the two gentlemen who sent in their names, one presented himself and passed, namely:

Class II:—Blackadar, A. K. (Montreal).

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Opening Address by the President,

ALEXANDER JOHN FINLAISON, Esq., C.B.

[Delivered 26 November 1894.]

THE most important financial event since my predecessor addressed you from this chair has been the stoppage, in June 1893, of the free coinage of silver in British India, with a view to the substitution of gold as the medium to be used for the satisfaction of every public or private debt in that country. This action of the Government has affected the enterprises of commerce, the profits of trade, and the command which a coin of the smallest denomination has in India over the necessaries of life.

From the year 1835 until 1893 the standard prescribed for the lawful discharge of every description of debt in India was a definite weight of silver, certified to be such by the stamp of the Government, freely impressed at the mints upon all bullion deposited for that purpose. The Government of India altered the standard for the discharge of debts in June 1893, to a definite number of coins called rupees, the number of which is restricted to such an extent that a factitious value has been given to them, equivalent to about one-sixth more than the worth of the silver contained in the coins, that is to say, this is the result of the action of the Government according to a statement made in the House of Lords, on the 29th of June 1894, by Lord Lansdowne,

who was the Governor-General of India when the mints were closed to the public for the free coinage of silver.

One of the elementary functions of the Government of every country is to prescribe the standards of measurement to be used in the various ordinary transactions of life. Some of these standards—such as those used for the measurement of length, surface, or bulk—can be fixed with a high degree of precision, but the discovery of a standard which will accurately measure the value of commodities at any given time, as well as at other dates, agreed upon for the payment of a supposed equivalent to the original value, has hitherto eluded the wit of man. But, as it is the duty of a Government to prescribe the best standard attainable for the measurement of value, so it is also the duty of a Government to change a standard adopted at a former date, if it has been discovered to have become unstable and unfair as between debtor and creditor, that is to say, when a more trustworthy and less variable standard can be obtained and substituted for the relinquished measure of value.

The Government of a country within whose boundaries neither silver nor gold is produced in any considerable quantity will probably be a better judge of the merits of these rival metals as a standard for the measurement of value, than a country in which there are mines of either metal.

Germany is a country which does not contain any considerable mines either of silver or gold, and Germany, in 1873, deliberately changed the standard for the measurement of value within its boundaries from silver to gold.

France is a country which produces neither silver nor gold to any considerable extent, and France, since 1878, has been endeavouring to substitute gold for silver as the standard measure of value. In 1878 the mints of France were closed to the free coinage of silver, while they are declared to be open to the free coinage of gold. Silver five-franc pieces remain a legal tender for any amount in France, and are still, for all practical purposes, a token of one-fourth part of a gold twenty-franc piece. No gold is, however, voluntarily brought to the French mint, and the gold coins in the country are in process of being hoarded. The Bank of France is in possession of large amounts of gold coin, but it will not part with any considerable sum in gold in exchange for its notes, which are convertible into either silver or gold, at the option of the Bank. The Bank of France virtually retains the gold it obtains, and parts with all the silver it can get rid of.

Silver five-franc pieces, it is needless to say, however, are worth no more than silver bullion outside the Latin Union.

The decision to close the mints of India to the free coinage of silver has, however, been made on account of the grave difficulties with which the Government is confronted through the heavy fall in the value of silver as compared with the value of gold, and not on account of the example or experience of other countries, nor upon the basis of conclusions derived from reasoning upon first principles.

Much time has been consumed in discussion as to whether the fall in the value of silver has not been more apparent than real, that is to say, whether there has not been an increase in the value of gold. It is very difficult to discover with any great degree of accuracy the amount of the annual production of the silver mines of the world, but the most careful estimates show that from 1876 to 1885 it was, on the average, about 2,500 tons a year; from 1886 to 1890, about 3,500 tons a year; in 1891, about 4,500 tons; and, in 1892, nearly 5,000 tons. It is almost certain that, during the above-mentioned period, the demand for silver for use in arts or manufactures has not increased to an extent corresponding to the increase in the supply of the metal.

The proof that there has been a real fall in the value of silver has, however, been demonstrated to the Government of India in an unmistakable manner; for, notwithstanding the fall in the rate of exchange, and in the price at which rupee bills could be sold in London, the imports of silver into India continued to increase while the mints were open to the free coinage of that metal.

The proposals of the Government of India for stopping the free coinage of silver with a view to the introduction of a gold standard were not accepted by the Secretary of State until they had been referred to a committee that was presided over by Lord Herschell. This committee, besides being asked to advise whether it would be expedient to allow the proposals to be carried into effect, were invited to make suggestions for the modification of the proposed measures for the introduction of a gold standard into India, in case they were of opinion that there was no sufficient ground for overruling the proposals of the Government of India.

The suggestion for stopping the free coinage of silver in India was understood by the committee to be supplemented by a proposal, that the mints should be opened for the free coinage

of gold at an indefinite future date, but not until the effect of the limitation of the number of coined rupees on their value in gold had been discovered.

The committee reported that, in view of the serious evils with which the Government of India may at any time be confronted, if matters remained unchanged, they could not advise that the proposals should be overruled. The committee further stated, however, that they considered the following modifications of the proposals advisable, namely, that "The closing of the mints "against the free coinage of silver should be accompanied by an "announcement that, though closed to the public, they will "be used by the Government for the coinage of rupees in exchange "for gold at a ratio to be then fixed, say 1*s*. 4*d*. per rupee; and that, "at the Government treasuries, gold will be received in satisfaction "of public dues, at the same ratio."

If the above recommendation means that gold coins should be given for gold bullion taken to the mints, then it was proposed that, if anyone should take as much pure gold as is contained in a sovereign to an Indian mint, it should be stamped into a coin that should be equivalent to 15 rupees: that is to say, into gold coins that should be accepted at the Government treasuries in satisfaction of public dues at that rate. The recommendation of the committee does not contain a proposal that gold coins should be made legal tender in India; and appeared to be otherwise so vague to two distinguished members of the committee itself, as to require the following explanatory note, namely, "We "are anxious to state more fully and explicitly than is done in "the report we have signed, what is the full effect of the imme-"diate step which we have agreed in recommending. The step "recommended is, that the Indian Government should be "empowered to close the Indian mints against the free coinage "of silver, until the rupee rises in value, so as to stand at a given "ratio with the sovereign, such ratio to be a little above the ratio "which has recently been current, say 1*s*. 4*d*.; and that the "Government should then be required to give rupees at that "ratio for all gold brought to their mints. The immediate effect "of this step will be to alter the Indian measure of value. As "long as the Indian mint is open, the measure of value is the "market value of the weight of silver contained in the rupee; "but, as soon as the mint is closed, we can no longer be sure "that this will be the case. Further, so soon as the rupee has "risen to the given ratio, the fraction of an English gold

"sovereign, represented by 1*s.* 4*d.*, will become the measure of "value."

It is questionable whether the explanatory note just quoted is more explicit than the original recommendation it is stated to elucidate. It is not clear whether the rupees to be coined in exchange for all gold brought to the mints are proposed to be of gold or silver. If the coins to be given in exchange for gold are proposed to be silver coins, what have the mints to do with the transaction? Gold bullion cannot be stamped into silver coins. It would be a mere matter of purchase and sale of one commodity for another. If the coins proposed to be given in exchange for gold bullion are to be gold coins, the pure gold in the coins should be equivalent to the amount of pure gold contained in the bullion deposited at the mint, less any defined charge that may be prescribed for mintage. A necessary preliminary to the voluntary deposit of gold bullion at a mint for coinage, is an explicit declaration of the ratio at which the gold coins to be received in exchange for the bullion would be legal tender, in comparison with the ordinary current coinage of the country. No declaration that gold coins should be made legal tender in India was explicitly recommended by the committee, nor has any such declaration been made by the Government of India.

The closure of the Indian mints to the free coinage of silver was, however, carried into effect on 26 June 1893, by "The Indian Coinage and Paper Currency Act, 1893", which repealed the whole of that portion of "The Indian Coinage Act, 1870", which related to the free coinage of silver for the public, the Government of India retaining the power to coin silver rupees on its own account.

The Government do not, however, disclose the method by which they propose to obtain silver bullion in order to coin it into rupees. There seems to be an idea that the stoppage of the free coinage of silver has already conferred a value on a coined rupee, in excess of the value as bullion of the silver contained in it, and that a continuation of the restriction will further enhance the value of the coins to an extent that gold will voluntarily be taken to the Indian treasuries and mints, in the shape either of bullion or full-weight sovereigns, in order to purchase silver rupees at the rate of 15 silver rupees in exchange for one sovereign. Is it proposed that, if gold should be obtained at this rate, it should be employed to purchase silver bullion at a cheaper rate, to be coined for the benefit of the Government of India into rupees artificially increased in value by restricting their numbers?

On the same day that the stoppage of the free coinage of silver took effect, notices were issued by the Government of India that notes, payable in silver rupees, would be given in exchange for sovereigns, at the rate of 15 rupees for a sovereign; that sovereigns would be received in all the treasuries of British India in payment of sums due to the Government at the rate of a sovereign for 15 rupees; and that, subject to a charge of one-fourth per mille, gold bullion would be received at the Indian mints in exchange for "Government rupees", at the rate of 15 rupees for the amount of pure gold in a sovereign.

The present relative market prices of gold and silver bullion would seem to have to be materially altered before any considerable amount of either sovereigns or of gold bullion is taken for the voluntary purchase of silver rupees on the above-mentioned conditions, for at the current price of silver, about as much silver bullion as is contained in $20\frac{1}{2}$ rupees can be obtained in London for a sovereign.

Apart from the general interest which the important financial experiment of the Government of India will have for the Members of the Institute, it will have a particular attraction from the persistent advocacy which an apparently similar scheme received from a distinguished Fellow of the Institute. Colonel J. T. Smith, who was elected on the 23rd of February 1863, and remained a Fellow of the Institute until his death in 1883, was formerly Master of the Mints of Madras and Calcutta. In the year 1876—that is to say, about three years after the substitution of gold for silver as the standard measure of value in Germany, but two years before the closure of the mints of France to the free coinage of silver—Colonel Smith submitted a proposal to the Government of India suggesting that "the coinage of silver on " behalf of private individuals, and advances upon silver bullion, " should be suspended; that part of Act 23 of 1870, which makes " it incumbent on the Government to receive and coin it, being " repealed, the Government retaining in their own hands the " power of replenishing the silver currency whenever they may " deem it expedient." Colonel Smith goes on to propose that gold bullion should be received by the Government of India for the purpose of being struck into coins which should be a legal tender at a rate which had reference to the price of silver at the date of his recommendation.

The proposal made by Colonel Smith in 1876 will be seen to

correspond very closely to the recommendation of Lord Herschell's committee, and to be founded on a similar idea, namely, that the stoppage of the free coinage of silver would gradually enhance the value of the stock of silver rupees already coined until they attained a value corresponding to the ratio to gold promulgated in a decree of the Government at the time when the mints were closed to silver.

It is now open to the Government, in continuation of the action taken last year, to declare at any time the ratio at which gold, accepted by them for coinage, should be legal tender for the payment of ordinary debts, but this ratio has not yet been disclosed. The Government has, as yet, done no more than state the price at which they will sell legal tender silver rupees for sovereigns, or gold bullion. Now, until it is less costly for a debtor to discharge his debts by buying rupees at the rate of 15 for a sovereign than by any other means, there is little prospect of gold being voluntarily taken to the Indian treasuries.

The silver currency of India at the present time is said to have been described by a great authority as a token currency of unparalleled magnitude. There is some doubt, however, whether the amount of silver coined into five-franc pieces is not as great as the amount coined into rupees. The essential difference between the two silver coins is that, while a silver five-franc piece is a token of the one-fourth part of a gold twenty-franc piece, a silver rupee is a token of an unknown and unstable standard, although it is asserted to be worth more than the value of the silver contained in it.

The familiarity of the French people with silver five-franc pieces, the system on which business is conducted in France, the knowledge that a large amount of gold is stored in the Bank of France, and the continuation in circulation of a certain number of scattered gold pieces which are exchanged for silver coins at their nominal worth, maintains the token silver coinage of France at its nominal value, in a corresponding manner to the way in which a paper currency is sustained, so long as it is not too excessive for the habits of the people among whom it circulates, notwithstanding the fact that more of it is in circulation than could be suddenly exchanged for the metal it is assumed to represent. The issue of a proportionately small amount of a paper currency, beyond the quantity which is readily absorbed by the people among whom it circulates, would, however, rapidly cause the whole to fall to a discount out of all proportion to the surplus amount issued.

Similarly, a change in the business habits of the people of France, such as a growth of the system of banking, would quickly cause the redundant silver currency to fall in value, and the necessity for the purchase of any unusually large quantity of commodities from a country in which five-franc pieces were not a legal tender, would be likely to have the same effect.

The very considerable increase in the annual production of the silver mines of the world has already been alluded to, and it has been shown that, according to the best estimates that can be made, the production in the year 1892 was nearly 5,000 tons: that is to say, about twice as much as the average annual production between 1876 and 1885. The increase which has already taken place in the amount of silver raised from the mines is not, however, the sole cause of the fall which has taken place in its market value; neither has the fall been entirely caused by a diminution in the demand for the metal. The true cause of the fall in price has been the diminished cost of the production of silver, by the discovery of new and fertile mines, by improvements in the methods of separating it from inferior metals, and by a reduction in the cost of transport.

Gold is not more valuable than silver because of its greater brilliancy, durability, or ductility, but because a far greater outlay is required to produce a quantity of gold than is required to produce an equivalent bulk of silver. If new and abundant gold mines are discovered, or improved methods of working the old mines are invented, the value of gold will diminish. A restriction on the free coinage of gold might in such a case temporarily put an artificial value on the coins of a country using gold as a standard of value; but the laws of the most powerful Government cannot eventually conquer the laws of Nature, and, one way or another, the prices of other commodities would tend towards an agreement with the natural relative price of the standard, which is variable in a similar manner, although, if well chosen, not to a similar extent, as the commodities, the value of which it measures from time to time.

The production of either gold or silver is not subjected to any monopoly or restraint, and there are no conceivable limits to the extent to which the supply of either may be increased. The competition of the producers compels them to sell their bullion for the price that can be obtained for it, until they are obliged to cease working the mines which produce less than the ordinary rate of profit on the capital employed in them.

A question that will, perhaps, have more attraction for the members of the Institute of Actuaries than the fall in the value of silver and the grave difficulties with which the Government of India is, in consequence, confronted, is an enquiry, whether the increased production of the silver mines has had a material effect on the reduction of the rate of interest obtainable for money, which has taken place in recent years in so marked a manner as to be beyond dispute.

The diminution in the rate of interest which began to take place towards the end of the 16th century and continued throughout the 17th century, was, while it was taking place, generally attributed to the additions made to the amount of gold circulating in Europe after the discovery of America. A similar reduction in the rate of interest was observed in 1852 and 1853, after the gold discoveries which took place about that time in California and Australia; and, again, at the present time, the largely increased production of silver which has recently occurred, has been accompanied by a reduction in the rate of interest for money. Not only was the discovery of fertile mines ordinarily accepted as a self-evident reason for a fall in the rate of interest, but such a master of logic as John Locke accepted the proposition as conclusive.

Adam Smith, in the 4th Chapter of the 2nd Book of *The Wealth of Nations*, ascribes to Locke, to Law, to Montesquieu, "as well as many other writers", an idea "that the increase of the "quantity of gold and silver, in consequence of the discovery of the "Spanish West Indies, was the real cause of the lowering of "the rate of interest through the greater part of Europe. Those "metals, they say, having become of less value themselves, "the use of any particular portion of them necessarily became of "less value too, and consequently the price which could be paid "for it. This notion, which at first sight seems so plausible, has "been so fully exposed by Mr. Hume, that it is perhaps "unnecessary to say anything more about it."

The precise form in which Locke put his proposition in his essay entitled "Considerations of the Consequences of the Lowering of Interest and raising the value of money", in a letter sent to a Member of Parliament, 1691, was that: "The "natural value of money, as it is apt to yield such a yearly "income by interest, depends on the whole quantity of the then "passing money of the Kingdom, in proportion to the whole trade "of the Kingdom."

But, as Adam Smith appeals to Hume, to Hume let us go. Hume in his famous essay on interest says: "Lowness of interest "is generally ascribed to plenty of money. But money, however "plentiful, has no other effect than to raise the price of labour and "commodities.

"Low interest proceeds from: A small demand for borrowing, "great riches to supply that demand, and small profits arising "from commerce. These circumstances are all connected together, "and proceed from the increase of industry and commerce, not of "gold and silver."

Great or little riches to supply the demand for borrowed money, Hume goes on to show, depend on "the habits and way of "living of a people, not on the quantity of gold or silver. In "order to have, in any state, a great number of lenders, it is not "sufficient nor requisite that there be great abundance of the "precious metals. It is only requisite that the property or "command of that quantity which is in the state, whether great "or small, should be gathered in particular hands, so as to form "considerable sums. This begets a number of lenders, and sinks "the rate of usury; and this, I shall venture to affirm, depends "not on the quantity of specie, but on particular manners and "customs, which make the specie gather into separate sums or "masses of considerable value.

"In some nations, after a sudden acquisition of money or of "the precious metals by foreign conquest, the interest has fallen, "not only among them, but in all the neighbouring states.

"In the conquering country, it is natural to imagine that this "new acquisition of money will fall into few hands and be "gathered into large sums which seek a secure revenue, either by "the purchase of land, or by interest, and consequently the same "effect follows, for a little time, as if there had been a great "accession of industry and commerce. The increase of lenders "above the borrowers sinks the interest; and so much the faster, "if those who have acquired those large sums find no industry or "commerce in the state, and no method of employing their money "but by lending it at interest. But, after this new mass of money "has been digested, and has circulated through the whole state, "affairs will soon return to their former situation, while the "landlords and new money-holders, living idly, squander above "their income, and the former daily contract debts, and the latter "encroach on their stock until its final extinction. The whole

"money may still be in the state, and make itself felt by the "increase of prices; but not being now collected into any large "masses or stocks, the disproportion between the borrowers and "lenders is the same as formerly, and consequently the high "interest returns."

Adam Smith was, therefore, hardly justified, in view of the above-mentioned passages in David Hume's essay on interest, in saying that he had so fully exposed the notion that the increase of the quantity of gold and silver, in consequence of the discovery of the Spanish West Indies, was the real cause of the lowering of the rate of interest in the 17th century, that it was, perhaps, unnecessary to say anything more about it.

Hume no doubt demonstrated that, ultimately, when the "new mass of gold and silver had been digested and had "circulated throughout the whole state, the former high rate of "interest would return." But, in the progress to this thorough digestion and circulation, the augmentation of new wealth in the hands of comparatively few people would have, according to Hume, a material effect in lowering the rate of interest.

There is every reason to believe that the concentration of great riches in the hands of a comparatively small number of persons, through the recent great enhancement of the production of silver, has occasioned a reduction of the rate of interest obtainable on those classes of securities, for which the silver favourites of fortune have been competing with other possessors of wealth.

There has, however, been another and probably more potent influence at work in this country, namely, the perfect organization with which the savings of all classes of the people are collected together, and formed into considerable sums for the purpose of being invested at interest.

The organization of Joint Stock and Private Banks, whose vast funds are no doubt in great measure employed in advances for the temporary purposes of commerce, but who to a large extent compete for more permanent investments; the British Assurance Societies, whose funds exceed £200,000,000 sterling; the not inconsiderable funds of Friendly and Building Societies; the £130,000,000 sterling in the custody of the National Debt Commissioners, which has been collected throughout Great Britain and Ireland by means of Savings Banks, not only from the great towns but also from every remote village and hamlet in the land.

These are the organizations which, with others of kindred nature, and with the addition of the large possessions of comparatively few wealthy people, gather together considerable sums in particular hands and occasion that disproportion between lenders and borrowers which, as shown by Hume, is the true cause of reductions in the rate of interest.

The money collected through Savings Banks, and gathered together in the hands of the National Debt Commissioners, has been the means not only in an indirect manner of aiding in the reduction of the rate of interest by the purchase and absorption of public securities from the market as they have been presented for sale, but this particular money was also the principal instrument which enabled the Chancellor of the Exchequer in 1888 to effect the conversion of £565,000,000 of 3 per-cent stock into a $2\frac{1}{2}$ per-cent stock with a terminable annuity of a $\frac{1}{4}$ per-cent which expires in 1903, attached to it. This great operation could only be effected by an offer to pay in cash the full amount of the debt owing to those stockholders who might refuse to accept the new stock, at the lower rate of interest, tendered to them in exchange for their 3 per-cent stock, as an alternative for the money due to them. Comparatively few stockholders did want their money, for the holders of other eligible securities would not part with their property, except on such terms as were not sufficient to tempt any considerable number of holders of 3 per-cents to claim their money from the Government. Many claims were made, however, and although much less money was required than could have been produced, sufficient of the Savings Banks' funds was employed to show that their aid was indispensable in carrying out the great conversion.

As a matter of fact, at the present time a sum of about £13,000,000 sterling of the Savings Banks' funds, which was utilized for the purposes of the conversion, is still due from the nation to the Savings Banks.

Another cause of the fall in the rate of interest, besides the ever-increasing sums which are being collected together in a form which renders them disposable for investment, is the reduction in the amount of capital necessary to carry on foreign commerce, owing to the very great improvements which have taken place in late years in marine steam-engines and the construction of ships. The greatly-increased speed and certainty with which produce is carried by sea from one part of the world to another, causes less

capital to be locked up than used to be the case, in the shape of commodities passing to market: that is to say, less in comparison to the volume of trade which is carried on. The opening of the Suez Canal, 25 years ago, had a great effect in this way, but the influence of the Suez Canal in quickening the transit of goods is as nothing in comparison with the improvement in the speed and capacity of shipping.

While the accumulation of money into large heaps seeking for investment has been progressing, many of the more apparent channels of investment have become restricted. Although high-class securities have greatly risen in price, there appears to be great reluctance on the part of the better-informed and larger holders of securities to realize the profit they could now obtain, over the price at which they invested, for the way is not clear even to them, to trustworthy securities that would yield a better rate of interest. The debts of local government boards are certainly expanding on all sides, but not faster than the assistance offered to them. Comparatively obscure towns seem to be able readily to borrow money at a mere fraction above 3 per-cent per annum. The National Debt, which at one time appeared to be so vast that all the money that offered could be readily invested in it, is fast becoming restricted in extent: that is to say, as far as regards the amounts freely coming for sale on the market.

The reductions which have been made in the amount of the funded debt of the country in recent years are matters of current knowledge, but the increase of the amount of the funded debt held by Government departments, although an account has been periodically published in recent years, does not appear to be so generally known.

The total amount of Consols and $2\frac{1}{2}$ per-cents, that is to say, the total of the only form of debt practically in the market for general investment was, indeed, still about £561,000,000 on the 31st of March 1894, but of this amount, no less a sum than about £116,000,000 was held by public departments, and the amount so held is increasing at a rapid rate.

The stock held by public departments has, for all practical purposes, been removed from the market, and the balance of about £445,000,000, held by the general public, is to such a large extent in the hands of trustees, that it is only liberated for sale at long intervals of time. It is not improbable that the

large and increasing demands of Government departments are making such considerable encroachments upon the amount of stock periodically brought forward for sale as to leave but a comparatively small margin for other purchasers, even at the present extremely high prices.

These high prices, and the difficulty that ordinary persons have in obtaining a remunerative rate of interest upon trustworthy security, have recently had an effect upon the market for life annuities in a direction which has been noticed on former occasions, when ordinary first-class securities have been scarce. The amount of Government Stock, directly or indirectly, converted into immediate life annuities in the undermentioned financial years, which end on 31 March, has been as follows:

1864	£611,561
1874	640,972
1884	882,635
1894	1,354,913

The published accounts of British life assurance companies afford an additional illustration of the increase which has taken place in recent years in the demand for life annuities. A summary of the revenue accounts of ordinary British companies from the Blue Book issued in 1894 shows that the "consideration for annuities" received by all the companies was £1,359,476. Considerably more than two millions and a half sterling of securities may therefore be said to have been appropriated in 1893 in consequence of the purchase of life annuities in that year. There is every reason to suppose that the amount of purchases has increased in the current year.

Life annuities, as the Members of the Institute of Actuaries are aware, have for very many years formed an appreciable amount of the National Debt. They are, in fact, one of the oldest forms of debt resting upon parliamentary security. It has been the practice of every English Government, from a period of immemorial antiquity, to contract debts, but, until the year 1692, money was borrowed either upon the personal security of the king or upon the security of taxes which had been granted to the king. The so-called loan might, indeed, sometimes be properly described as the sale of the produce of a tax which had been granted to the king, as the collection of the tax might be confided to the person who had made an advance of money on the security of its produce. In the year 1692, however, Parliament imposed excise duties upon

beer, which were directed to continue for the term of 99 years, and thus to form a fund which was to be appropriated, in the first place, to the payment of life annuities to the nominees of capitalists who, it was hoped, would advance £1,000,000 on the security of the fund, coupled with the inducement of a very high rate of annuity. The Government were unable to obtain the whole of the £1,000,000 they attempted to borrow, but a sum of about £881,500 was raised, the security being apparently more distrusted than the rate of annuity was liked.

This transaction was the origin of what for many years was termed the "funding system", that is to say, the system of borrowing money upon the security of a fund to be formed from the produce of taxes to be collected from the people throughout a long period of time. Nowadays but small thought seems to be given to the form of security upon which the National Debt rests, but, by a perversion of language, the debt itself is termed the "Funds", as if it formed the security for its own repayment.

Life annuities continued to be granted in batches in this way for a hundred years until the year 1789, according to the wants of the Government and the capacity of the public for this form of security. The demand for life annuities did not, however, take the fitful shape that the wants of the Government assumed, and an attempt to borrow any considerable sum within a short period of time in the form of life annuities usually resulted in a failure to obtain the whole of the capital required. A regular and steady demand for life annuities secured upon Government funds seemed, however, always to exist, and in the year 1808 a form of machinery was set up which has virtually continued to work ever since, by which other forms of debt might be converted into life annuities.

Under the Provisions of the Acts of Parliament which regulate this form of commutation of the National Debt £60,000,000 capital of perpetual annuities have been converted into life annuities, from 1808 to the 31st of March 1894, at which date £1,150,000 a year of life annuities were chargeable on the Consolidated Fund. These annuities were estimated to be then of equivalent capital value to £11,200,000 Consols.

The National Debt is proverbially one of the most uninteresting subjects to discuss, but before I finally leave it I should wish with your permission to call attention to an erroneous idea which seems to be prevalent as to the remote origin of the debt which still

presses upon the country. Except the debts due to the Banks of England and Ireland, and with regard to these, only from a technical point of view are they more ancient, the whole of the National Debt, as it now exists, dates from the present century: that is to say, the whole of the present debt of about £669,000,000, with the comparatively small exceptions mentioned above, has been borrowed since the year 1800 for the purpose of actual expenditure, the loans which may have been contracted for sinking fund purposes being left out of consideration.

On an Investigation of the Mortality and Marriage Experience of the Widows' Funds of the Scottish Banks. By Archibald Hewat, F.F.A., F.I.A., F.S.S., *Secretary, and* James Chatham, F.F.A., F.I.A., *one of the Assistant-Actuaries, of The Edinburgh Life Assurance Company.*

[Read before the Institute, 17 December 1894.]

Prefatory Note.

Having on several occasions been employed to investigate and report upon the Widows' Funds of certain Banks in Scotland, and having found no available tables specially suited to work of that kind, Mr. Hewat had, like other actuaries, to fall back upon existing tables which have been considered more or less appropriate and presumably safe enough for the purpose in view. Not being quite satisfied that these tables are sufficiently reliable for use in the actuarial investigations of these increasingly important Funds, and holding the opinion that the actuary, like other skilled workmen, should never allow his work to suffer for want of a proper instrument or tool, he resolved to emulate the intelligent and enterprising workman in other professions—the surgeon and the engineer, for example—and manufacture an instrument specially adapted to the particular class of work in hand.

He resolved upon this the more readily as he knew where to find the raw material, of reliable quality and in sufficient quantity, for his purpose. He accordingly communicated his idea to the Managers of the Banks, and through them to the Committees of Management of the respective Funds, who entered heartily into the project, and, in the most courteous manner, offered to supply

all the particulars which might be required, so far as their records could disclose.

As a Fellow of the Institute, the motto of which corporation reminds us that "every man is a debtor to his profession", and desiring, in a humble way and to some small extent, to discharge that debt by an endeavour "to be a help thereunto", it occurred to Mr. Hewat that the results of the investigation might be usefully submitted to the Institute, he having received the cordial assent of the Banks to his request for permission to make this investigation and its results the subject of the following Paper.

It will be readily understood that the investigation contemplated was too much for one actuary to wholly undertake himself—especially one whose time and energies are fully occupied with ordinary official duties. He accordingly asked Mr. Chatham, also a Fellow of the Institute, to assist him in the details of the work. As his valued services deserve more than a mere passing acknowledgment, Mr. Hewat asked the Council to allow him to submit the following as a joint paper.

There are ten Banking Companies in Scotland. Of these, five have their Head Offices in Edinburgh, two in Glasgow, two in Aberdeen, and one in Inverness. The Banks whose head-quarters are in Edinburgh have each a Widows' Fund, the Members of which are the employees at their Head Offices and Branches throughout Scotland, as well as at their London Offices. These five Scottish metropolitan Banks date respectively from the years 1695, 1727, 1746, 1810, and 1825, and are the oldest existing Banks in Scotland. Their total assets amount to £84,250,000—being more than 70 per-cent of the assets of all the Scottish Banks.

The Widows' Funds in connection with these Banks were commenced in the years 1808, 1820, 1821, 1851, and 1866. They have already, up to the close of their last year's Accounts (1894), paid to widows and orphans of employees a sum not far short of a quarter-of-a-million sterling. The amount paid last year exceeded £15,500.

At the close of the last Accounts, the total membership of these Funds was 2,281, and the Annuitants numbered 283. The number of Beneficiaries is thus equal to nearly 12½ per-cent of the number of the present Contributors. The Funds, taken as a whole, amounted to a sum not far short of half-a-million sterling. They are maintained by the annual contributions of

the Members; marriage and equalizing "taxes"; interest on the investments and on the bank balances; in most cases by an annual donation, or honorarium, voted by the Directors of the Bank; and sundry other items. In some cases, there is also the periodical surplus on the Guarantee Funds maintained by the employees against defalcations. Last year the contributions (including "taxes") yielded nearly £10,000, and the interest a little over £21,500.

Membership is, as a rule, practically compulsory on all in the service of the Banks. In the few exceptional cases where an option may be exercised, the employees almost invariably join the Fund, the Benefits being so great in proportion to the Contributions required. In one Fund, membership was at one time not compulsory until marriage, but the advantages of joining before marriage are such that most of the employees find it in their interest to join the Fund while they are yet bachelors. Membership is not usually open to those entering the service above the age of 50; but as few, if any, enter the service at such a comparatively advanced age, the restriction is little more than nominal. In some cases a minimum age for entry is fixed—generally 20—but in most cases the limit is a given minimum salary.

The Members' Contributions to the Funds vary considerably, not only among the different Banks, but also according to the varying status of the Members from time to time. These are, generally, a fixed annual contribution payable (half-yearly or quarterly) by all, irrespective of status, certain sums known as marriage and equalizing "taxes" being paid by Members on marriage. Only in a very few exceptional cases is medical evidence of good health required before admission to membership. The risk of "selection" against the Fund is comparatively small, because most of the Members enter when they are young bachelors, the Banks do not knowingly take into their service weak and delicate men, membership is practically compulsory, the unhealthy do not usually marry, and the scale of salaries does not encourage early marriages.

Members who leave the service may, in most cases, continue their connection with their Fund, the annual contribution being increased, generally by about 25 per-cent or more; a further addition being made in the event of their leaving the United Kingdom, going abroad to an unhealthy climate, or entering the Army or Navy.

The rate of Annuity has steadily increased with the increasing prosperity of the Funds. In one case, where the original amount was £25, increased to £30 in 1839, it has steadily increased, by seven stages, until it is now £72. In another, the Annuity contemplated was £50, which, increased to £60 in 1858, has been further increased, by three stages, until it reached £90 in 1867, at which amount it now remains; but that class was closed in 1875, when a new class was commenced, the Annuity in connection with which is £65. In another, the Annuity commencing at £20, is now £45, having been increased to that sum by five stages; and in the most recent of the Funds the Annuity has been increased, by two stages, from £30 to £40, its present amount. In one of the Funds the Annuity during the first half-century was uniform, and after increasing by successive stages from £40 to £100 a new scheme was adopted, under which the Annuities are divided into three classes; namely, £60, £40, and £20, which have since been increased, by three stages, to £84, £56, and £28 respectively.

The Annuity lapses, in most cases, by the re-marriage of the widow; but if there are orphan children the Annuity is, in most cases, continued to them until the youngest reaches majority or some minor age.

The foregoing particulars are given in order to indicate the general character of the statistics, or data, which form the basis of the investigation, the results of which are now submitted. A special value attaches to the results brought out in this investigation in that there is an almost total absence of the disturbing effects of *selection* and of *secession*, or of the influx of new Members in larger proportions in any one part of the year than in another.

Careful consideration was given to the framing of a suitable card by means of which the desired particulars were to be obtained. A specimen of the card is here submitted:

S. B. W. F.

Initials of Member....................

MEMBER.	
Date of—	
Birth, . . .	...... 18..............................
Entry, . .	.............18 ....
1*st Marriage*, .	...........18........ ....
2*nd Marriage*, .	......18........ ....
Exit (............)	..........18........ ....
FIRST WIFE.	
Date of—	
Birth, . . .	.... 18 ....
Death (*or re-marge.*)	.... 18 ....
SECOND WIFE.	
Date of—	
Birth, . .	....18.........
Death (*or re-marge.*)	..........................18........
ORPHANS.	
Date of—	
First Payment. .	18.........
Last ,, .	..........................18.........

A supply of cards was sent to the Clerks or Treasurers of the respective Widows' Funds. A private mark was placed on them, so that, while the information supplied was kept private, a means was provided of knowing to which Bank to apply for any explanations or corrections that might be found necessary as the investigation proceeded. Along with the cards was sent a Memorandum of Instructions for filling them up, from which the following are extracts:

"The Statistics to be supplied on the accompanying cards are "those relating to all the Members who have been connected with "the Fund *since it was commenced* (so far as the Registers show) "as well as those at present on the Roll.

"*Initials.*—Write the initials in the usual way, those of the "Christian name (or names) first and the Surname last.

"*Dates.*—Insert the day, month, and year of the events, "whenever practicable. If the first or second wife has re-married, "write the date in *red* ink. If the Member has married *more than* "*twice,* write the other particulars on the *back* of the card, making "a reference to them on the front of it underneath the words "'2nd Marriage.'

"*Entry.*—Insert the date when the Member *entered the Fund* "(not the service of the Bank).

"*Exit.*—Insert the date when the Member *left the Fund* (not "the service of the Bank). In the blank space after 'Exit' write "*D* if the Member has *died*; *W* if he has *withdrawn*; and a "horizontal stroke (—) if he was still on the Fund at "————— 1893 [the close of the observations].

"*Orphans.**—Children coming on the Fund through the "re-marriage of their mother, are to be treated as orphans.

"*No Marking* is to be made in the last column on the card."

As the cards came in from the Banks they were carefully examined; and after rejecting the comparatively few which were useless on account of the date of birth not being given or from some other cause, the number available was found to be 3,806. The whole of those cards were not complete in every respect. For instance, a Member may have married twice without the card containing full particulars of the first wife, though giving full particulars of the second. Such a card was not rejected as wholly useless, the information on it being used as far as it was of value. An attempt was made to deal with those cards only which contained *full* particulars of the Member's history in connection with the Fund; but it was found that, if that course were persisted in, a considerable portion of valuable material would be lost at stages where, from various causes, the number of facts was otherwise very small. The attempt was therefore abandoned.

As an official in one of these Banks sometimes transfers his services to another of them, and still retains his connection with the Widows' Fund of the former, while becoming a Member of the Fund in the latter, the cards were arranged alphabetically, when 22 such cases were discovered, thus reducing the number to 3,784. As the scheme in the case of one of the Banks was at one time not compulsory until marriage, 37 cards on the lives of bachelors in that Bank were rejected, thus still further reducing the number to 3,747. The remainder of the

* The statistics of these were insufficient to be of any use in this investigation.

cards from that Bank, relating to the lives of 208 married men, were dealt with only as from the date when marriage did eventually take place.

Of the 3,747 mentioned above, 2,979 were bachelors at entry. The following were the numbers existing at the close of the observations, and the number withdrawing, marrying and dying during the period in this class:

	Bachelors at entry		
Existing . . .	1,136 =	38·1	per-cent
Withdrawing . .	706 =	23·7	,,
Marrying . . .	932 =	31·3	,,
Dying . . .	205 =	6·9	,,
	2,979 =	100	,,

Of the 932 bachelors who married, the particulars of 15 were found to be incomplete, reducing the number to 917. The difference between the 3,747 and the 2,979 consisted of 759 who were married at entry, and 9 widowers, but the information relating to 42 of the former was found to be incomplete; thus leaving 717, which, added to the 917, gave 1,634 married men. The following are the same particulars relating to them:

	Bachelors at entry who married			Married Men at entry		
Existing . .	558 =	60·9	per-cent	357 =	49·8	per-cent
Withdrawing .	65 =	7·1	,,	48 =	6·7	,,
Widowed . .	119 =	13·0	,,	122 =	17·0	,,
Dying . .	175 =	19·0	,,	190 =	26·5	,,
	917 =	100	,,	717 =	100	,,

The number of *bachelors* at entry who married and became *widowers* is, as above, 119; and the number of married men at entry who became widowers, 122, together 241; and as there were 9 who entered as widowers, this gave 250 widowers. The following are the same particulars relating to them:

	Widowers from Bachelors and Married Men			Widowers at entry		
Existing . .	63 =	26·1	per-cent	2 =	22·2	per-cent
Withdrawing .	22 =	9·1	,,		...	
Marrying . .	85 =	35·3	,,	6 =	66·7	,,
Dying . . .	71 =	29·5	,,	1 =	11·1	,,
	241 =	100	,,	9 =	100	,,

The number of widowers who re-married is 91 $(=85+6)$. Two of these were rejected on account of the information being

incomplete. It was, however, found possible to use 32 with more or less incomplete particulars, so far as early history was concerned, making 121 in all. The following are the same particulars relating to this class:

	Widowers who re-married		
Existing . . .	61 =	50·4	per-cent
Withdrawing . .	6 =	5·0	,,
Widowed . . .	13 =	10·7	,,
Died	41 =	33·9	,,
	121 =	100	,,

Turning now to the statistics relating to the wives of the Members, there were 917 married to those who entered the Fund as bachelors, and 717 to those who had entered as married men, making 1,634 in all; but as the information in 2 and 4 of the cards in these classes respectively was defective, they were rejected, reducing the number to 1,628. The following are the particulars relating to them:

	Wives of Bachelors at entry			Wives of Married Men at entry		
Existing . .	556 =	60·8	per-cent	357 =	50·0	per-cent
Withdrawing .	65 =	7·1	,,	48 =	6·8	,,
Widowed . .	175 =	19·1	,,	189 =	26·5	,,
Dying . .	119 =	13·0	,,	119 =	16·7	,,
	915 =	100	,,	713 =	100	,,

There were 121 who became second wives of Members. The following are the particulars of them:

	Wives of Widowers who re-married		
Existing . . .	61 =	50·4	per-cent
Withdrawing . .	6 =	5·0	,,
Widowed . . .	41 =	33·9	,,
Died	13 =	10·7	,,
	121 =	100·0	,,

The widows of those who entered as bachelors numbered 175, and of those who entered as married men, 190; together 365. As the information on one card was incomplete, this number was reduced to 364. The following are the particulars of them:

	Widows of Bachelors and of Married Men at entry		
Existing . . .	206 =	56·7	per-cent
Re-marrying . .	15 =	4·1	,,
Dying	143 =	39·2	,,
	364 =	100	,,

The next point to be considered was how the particulars contained in the cards could best be tabulated. After weighing the advantages and disadvantages of the various known methods, it was decided to tabulate them according to the nearest age at entry into the various conditions through which a Member of the Fund may pass; thus, in the case of a bachelor, his nearest age at entry into membership, marriage, widowerhood, &c.; and to tabulate those passing out of observation in the year of membership, marriage, widowerhood, &c., in which the event took place—*i.e.*, the years were reckoned from the date of entry into membership, marriage, widowerhood, &c. It will be seen that this is analogous to what is known as the policy-year method, that having been thought the most suitable to employ in an investigation like the one now under consideration.

In dealing with "experiences" of this kind, where it is necessary to deduce marriage and mortality rates, it was considered simpler, as well as more accurate, to employ central marriage and central death rates. In order to arrive at the number at risk at the middle of the year, the marriages and withdrawals were investigated, and it was found, as was to be expected in the circumstances peculiar to these Funds, that these were practically uniformly spread over the year of membership, and the convenient supposition was made that they were exposed to risk, on the average, for six months in the year in which the event took place. The usual method in which to treat the *existing* in a case of this kind, is to take them to the anniversary in the year of membership *preceding* the close of the observations; but a considerable body of facts would have been lost if this had been done, and as the numbers under observation were few, it was considered desirable to take them to the anniversary *succeeding* the close of the observations. As indicated in the Memorandum of Instructions, a fixed date (1892-93) was taken for the termination of the observations in each Fund, the date varying with the close of the respective financial years; and in this case also they would be exposed to risk on the average for six months. This method of tabulation enabled the number at risk at the middle of the year† to be ascertained very readily, as the following table,

† $$\begin{aligned} E_{x+\frac{1}{2}} &= \Sigma(n_{x-1}-f_{x-1})+n_x \\ &\quad -\tfrac{1}{2}[e_x+w_x+(bm)_x+(bd)_x] \\ &= \Sigma(n_{x-1}-f_{x-1})+n_x-\tfrac{1}{2}f_x, \end{aligned}$$

where $f_x=e_x+w_x+(bm)_x+(bd)_x$.

for those who entered the Fund as bachelors at age 20, will show:

BACHELORS.—*Age at Entry* 20.

Year of Membership	Existing	Withdrawn	Married	Died	Total	Entered on year	Number at risk $=(7)-\frac{1}{2}(6)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	22	23	3	3	51	368	342·5
2	16	26	...	4	46	317	294
3	25	13	3	1	42	271	250
4	5	9	4	...	18	229	220
5	6	5	5	1	17	211	202·5
6	6	2	6	1	15	194	186·5
7	9	6	10	2	27	179	165·5
8	1	11	10	2	24	152	140
9	4	4	9	1	18	128	119
10	3	1	6	...	10	110	105
*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*
	141	114	92	21	368	2,909	2,725

This will be at once recognized as being analogous to what is known as the *exact-duration* method. After carefully considering the whole matter, it was adopted as being the most appropriate in the circumstances. Its only rival is that which is known as the *nearest-duration* method; but, in an investigation like the one under consideration, that latter method would have been unsuitable, involving, as it would have done, various tabulations of the materials. It would seem as if the *exact-duration* method was applicable to nearly all cases, is more accurate, and, in investigations like the present, much simpler.

Although the cards were kept separate for each age at entry into the various classes, no attempt has been made in the present instance to deduce *select* rates, but mixed rates only; and the results were therefore summarized according to age attained. The class of Widowers was a small one, and as it was thought that the mortality in that class would not materially differ from that of Husbands, the two classes were amalgamated, and the term Married Men has been used to designate the combined class. In the following table the years of life for each of the classes are given, also the number of deaths:

	Years of Life	Number of Deaths
Bachelors . . .	22,423·5	205
Married Men . .	25,219·5	478
Wives	23,537·5	251
Widows . . .	4,466·0	143
	75,646·5	1,077

Although the observations extend over long periods, yet the numbers in the various classes into which they have been divided are comparatively small, with the result that there are considerable irregularities. The graphic method of adjustment is known to be specially applicable to a case of this kind, and it was accordingly adopted. The central death and central marriage rates were calculated, grouped, and adjusted in the manner explained by Dr. Sprague in the *Journal* (xxvi, 77, *et seq.*). It has been thought well to give the whole of the figures, in order that it may be seen to what extent the adjusted results represent the original observations. This is done in the appended tables:

F, (*a*), (*b*), (*c*) and (*d*); G, (*a*) and (*b*); H, (*a*) and (*b*); J, (*a*) and (*b*); and K, (*a*) and (*b*); to which has been added a Marriage and Mortality Table for Bachelors and Married Men (L).

In the following tables (A and B, pp. 439, 440) the adjusted central death and central marriage rates are brought together in order that they may be readily compared.

Although the death-rates were obtained independently of each other, they yet corroborate each other to a remarkable extent. It will be observed that the rate for bachelors attains a first maximum at age 28, which is somewhat later than usual, and falls again to a last minimum at age 34. In the case of married men, the first maximum occurs a year later, and the last minimum a year earlier. It will also be observed that the rate for married men is lower than that for bachelors up to age 33, which is what might have been anticipated, as there is a natural selection in marriage which tends to lower the rate of mortality; but from 34 to 45 the rate is higher than that for bachelors, which is contrary to what has been previously generally observed (*J.I.A.*, xxii, 233–243). Without wishing to lay too much stress on the observations, keeping in view the numbers at risk, it seems to indicate that the cares of married life tell considerably upon the husbands, and that after a few years subsequent to marriage they fare worse than their bachelor friends (during these dozen years from age 34 to age 45). It is possible, however, that the feature referred to may be due, in some measure, to resignations and dismissals, on account of

ill-health, being more numerous among the bachelors than among the married men. Afterwards, however, they experience on the whole a more favourable rate of mortality.

The rate of mortality experienced by wives is very high at the outset (age 20)—about double that of bachelors or married men—but falls rapidly at first and afterwards more gradually to a minimum at age 42, when the rate is almost the same as that of bachelors at age 37, and married men at age 36. The high rate at the earlier ages may be accounted for by the risks attending the child-bearing period. The rate at age 42 (which is the same as at age 43) is about one-third less than that for

TABLE A.—*Adjusted Central* DEATH-*rates*.

Age	Bachelors	Married Men	Wives	Widows	Age	Bachelors	Married Men	Wives	Widows
14	·0058	...	...	...	55	·0200	·0177	·0122	·0180
15	·0059	...	...	...	56	·0214	·0189	·0132	·0188
16	·0060	...	...	...	57	·0230	·0204	·0143	·0196
17	·0061	...	...	...	58	·0248	·0223	·0155	·0204
18	·0061	...	...	...	59	·0270	·0246	·0168	·0212
19	·0062	...	...	...	60	·0294	·0275	·0182	·0220
20	·0062	·0061	·0131	...	61	·0320	·0309	·0197	·0229
21	·0063	·0061	·0127	·0110	62	·0348	·0347	·0213	·0239
22	·0063	·0062	·0123	·0110	63	·0378	·0389	·0231	·0250
23	·0064	·0062	·0118	·0111	64	·0411	·0435	·0252	·0262
24	·0066	·0063	·0113	·0111	65	·0444	·0478	·0276	·0277
25	·0069	·0065	·0108	·0112	66	·0476	·0519	·0304	·0296
26	·0074	·0067	·0103	·0112	67	·0508	·0558	·0336	·0320
27	·0081	·0069	·0099	·0113	68	·0540	·0594	·0373	·0370
28	·0086	·0071	·0096	·0113	69	·0572	·0627	·0415	·0420
29	·0081	·0074	·0093	·0114	70	·0603	·0657	·0463	·0470
30	·0077	·0072	·0090	·0114	71	·0634	·0687	·0517	·0520
31	·0073	·0070	·0087	·0115	72	·0664	·0717	·0578	·0570
32	·0070	·0068	·0085	·0115	73	·0694	·0747	·0646	·0620
33	·0068	·0067	·0083	·0116	74	·0724	·0777	·0722	·0670
34	·0067	·0069	·0082	·0117	75	·0760	·0810	·0806	·0720
35	·0068	·0073	·0081	·0118	76	·0810	·0850	·0899	·0770
36	·0071	·0079	·0080	·0119	77	·0880	·0900	·1001	·0830
37	·0076	·0087	·0079	·0120	78	·0980	·0970	·1113	·0890
38	·0082	·0095	·0079	·0121	79	·1110	·1070	·1236	·0960
39	·0089	·0101	·0079	·0122	80	·1270	·1210	·1371	·1030
40	·0096	·0107	·0078	·0124	81	·1470	·1300	·1519	·1100
41	·0104	·0112	·0078	·0126	82	·1720	·1550	·1671	·1180
42	·0111	·0117	·0077	·0128	83	·2030	·1850	·1838	·1270
43	·0118	·0122	·0077	·0130	84	·2410	·2210	·2021	·1400
44	·0124	·0127	·0078	·0132	85	·2870	·2640	·2221	·1600
45	·0130	·0131	·0078	·0134	86	·3420	·3150	·2439	·1800
46	·0136	·0134	·0079	·0136	87	...	·3750	...	·2100
47	·0142	·0136	·0081	·0138	88	...	·4460	...	·2500
48	·0148	·0138	·0083	·0140	89	...	·5300	...	·3000
49	·0154	·0141	·0085	·0143	90	...	·7000	...	·3600
50	·0159	·0144	·0088	·0147	91	...	1·0000	...	·4300
51	·0164	·0148	·0092	·0142	92	...	...	...	·5200
52	·0170	·0153	·0098	·0158	93	...	...	...	·6500
53	·0178	·0159	·0105	·0165	94	...	...	...	·8000
54	·0188	·0167	·0113	·0172	95	...	...	...	1·0000

TABLE B.—*Adjusted Central* MARRIAGE-*rates.*

Age	Bachelors	Widows	Age	Bachelors	Widows	Age	Bachelors	Widows
20	·0025	...	42	·0458	·0050	64	·0088	...
21	·0085	·0001	43	·0428	·0035	65	·0084	...
22	·0150	·0015	44	·0399	·0025	66	·0080	...
23	·0220	·0032	45	·0371	·0018	67	·0076	...
24	·0300	·0051	46	·0344	·0014	68	·0072	...
25	·0390	·0072	47	·0318	·0012	69	·0068	...
26	·0470	·0095	48	·0293	·0011	70	·0064	...
27	·0540	·0120	49	·0269	·0010	71	·0060	...
28	·0600	·0145	50	·0246	·0009	72	·0056	...
39	·0650	·0170	51	·0224	·0008	73	·0052	...
30	·0690	·0195	52	·0203	·0007	74	·0048	...
31	·0720	·0220	53	·0183	·0006	75	·0044	...
32	·0740	·0245	54	·0165	·0005	76	·0040	...
33	·0755	·0270	55	·0149	·0004	77	·0036	...
34	·0725	·0270	56	·0135	·0003	78	·0032	...
35	·0693	·0230	57	·0123	·0002	79	·0028	...
36	·0658	·0195	58	·0114	·0002	80	·0024	...
37	·0623	·0165	59	·0108	·0001	81	·0020	...
38	·0588	·0140	60	·0104	·0001	82	·0016	...
39	·0554	·0115	61	·0100	...	83	·0012	...
40	·0521	·0090	62	·0096	...	84	·0008	...
41	·0489	·0070	63	·0092	...	85	·0004	...

bachelors or married men, and this superiority is maintained almost throughout the whole of life. The rate for widows starts (at age 21) lower than that for wives; but, instead of diminishing slightly, it increases throughout the entire length of the table, the result being that the rate for widows, after the first few ages, is very much higher than that for wives. This may be due, in some measure, to the strain of bringing up a family single-handed, and to the greater struggles they have for existence as compared with their married sisters. There is a slight change for the better, however, after age 65.

The results of an investigation like the present are more interesting when looked at relatively to results brought out in similar investigations in other classes. So far as the marriage experience is concerned a ready means of comparison is found by reference to the *Journal* (vol. xxi, p. 420) where Dr. Sprague, in one of his valuable and lucid contributions, supplies, from various sources, a Table showing the probability of marrying in a year among bachelors—the classes being Schoolmasters,* Clergymen,† Advocates,‡ and Peerage-Families. Dr. Sprague's Table is here reproduced, with a column added for Scottish Bankers:

* The old Burgh and Parochial Schoolmasters in Scotland.
† Ministers of the Church of Scotland.
‡ Members of the Faculty of Advocates [or "Barristers"], Edinburgh.

rrying in a

ue … ge ies	Hewat Scottish Bankers	Age
	...	15
	...	16
	...	17
	...	18
	...	19
	·0025	20
	·0084	21
	·0148	22
	·0217	23
	·0295	24
	·0381	25
	·0458	26
	·0524	27
	·0580	28
	·0627	29
	·0665	30
	·0693	31
	·0711	32
	·0725	33
	·0697	34
	·0668	35
	·0635	36
	·0602	37
	·0569	38
	·0537	39
	·0505	40
	·0475	41
	·0445	42
	·0417	43
	·0389	44
	·0362	45
	·0336	46
	·0311	47
	·0287	48
	·0263	49
	·0241	50
	·0220	51
	·0199	52
	·0180	53
	·0162	54
	·0146	55
	·0133	56
	·0121	57
	·0112	58
	·0106	59
	·0102	60
	·0098	61
	·0094	62
	·0090	63
	·0086	64
	·0082	65
	·0078	66

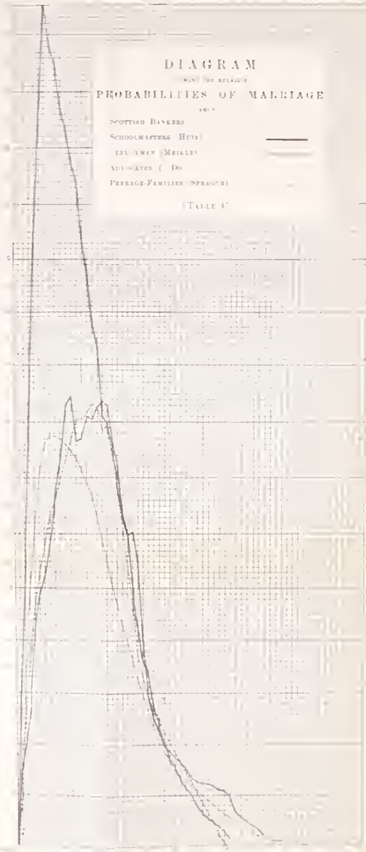


DIAGRAM
PROBABILITIES OF MARRIAGE
Scottish Bankers
Schoolmasters (Huie)
(Meikle)
(Do
Peerage-Families (Sprague)
(Table C

TABLE C.—BACHELORS.—*Probability of Marrying in a Year, $(bmq)_x$.*

Age	HUIE		MEIKLE			SPRAGUE	HEWAT	Age
	School-masters	Clergymen	Clergymen	Advocates	Peerage Families	Peerage Families	Scottish Bankers	
15	...	...	...	...	...	·0001	...	15
16	...	...	...	...	...	·0004	...	16
17	...	...	...	...	...	·0009	...	17
18	...	...	...	...	...	·0020	...	18
19	...	...	...	...	...	·0050	...	19
20	·01625	·00125	...	·01792	·01024	·0100	·0025	20
21	·01904	·00242	·05123	·02530	·03199	·0300	·0084	21
22	·02701	·00407	·08379	·03709	·04403	·0500	·0148	22
23	·04275	·01132	·11492	·04691	·05612	·0600	·0217	23
24	·04880	·01567	·13676	·05333	·06500	·0700	·0295	24
25	·05270	·02435	·15634	·05680	·07251	·0760	·0381	25
26	·05772	·03849	·14831	·06322	·07676	·0775	·0458	26
27	·06420	·04891	·14531	·06586	·07786	·0773	·0524	27
28	·07277	·05521	·13751	·06955	·07743	·0770	·0580	28
29	·08312	·06328	·13356	·07219	·07595	·0760	·0627	29
30	·08472	·06625	·12735	·07682	·07350	·0750	·0665	30
31	·07624	·07006	·12326	·08053	·07213	·0735	·0693	31
32	·07691	·07171	·11379	·08053	·07147	·0715	·0711	32
33	·08092	·07288	·10738	·08158	·06891	·0692	·0725	33
34	·08068	·07379	·09784	·08308	·06741	·0660	·0697	34
35	·08270	·07736	·09337	·08255	·06447	·0623	·0668	35
36	·08358	·07800	·08117	·07826	·06041	·0585	·0635	36
37	·08322	·07551	·07948	·07923	·05620	·0547	·0602	37
38	·07714	·07792	·07473	·07507	·05383	·0509	·0569	38
39	·07370	·08019	·07001	·06800	·04875	·0472	·0537	39
40	·06347	·07967	·06077	·06138	·04546	·0435	·0505	40
41	·05956	·06805	·05927	·05446	·04102	·0399	·0475	41
42	·06024	·05834	·05100	·04571	·03664	·0363	·0445	42
43	·05473	·04879	·04291	·03700	·03220	·0328	·0417	43
44	·03999	·04596	·03591	·03254	·02918	·0294	·0389	44
45	·02829	·04391	·03453	·02911	·02531	·0261	·0362	45
46	·02480	·04026	·02827	·02685	·02194	·0232	·0336	46
47	·02262	·03923	·02384	·02465	·01842	·0208	·0311	47
48	·01947	·03996	·02095	·02251	·01555	·0188	·0287	48
49	·01784	·04159	·01834	·02043	·01379	·0171	·0263	49
50	·01724	·04168	·01573	·01841	·01100	·0158	·0241	50
51	·01657	·03987	·01340	·01613	·00821	·0146	·0220	51
52	·01585	·03437	·01143	·01417	·00573	·0134	·0199	52
53	·01420	·03102	·01070	·01257	·00356	·0122	·0180	53
54	·01287	·02643	·00819	·01067	·00176	·0111	·0162	54
55	·01235	·02380	·00688	·00881	·00015	·0100	·0146	55
56	·01226	·02216	·00575	·00695	...	·0090	·0133	56
57	·01168	·02172	·00478	·00489	...	·0081	·0121	57
58	·01106	·01881	·00395	·00223	...	·0073	·0112	58
59	·01094	·01612	·00324	·00037	...	·0065	·0106	59
60	·00922	·01528	·00263	...	...	·0057	·0102	60
61	·00680	·01495	...	...	...	·0051	·0098	61
62	·00591	·01341	...	...	...	·0046	·0094	62
63	·00493	·01228	...	...	...	·0040	·0090	63
64	·00386	·01034	...	...	...	·0035	·0086	64
65	·00337	·00884	...	...	...	·0030	·0082	65
66	·00212	·00716	...	...	...	·0026	·0078	66

TABLE C.—BACHELORS.—*Probability of Marrying in a Year,* $(bmq)_x$—(continued).

Age	Huie		Meikle			Sprague	Hewat	Age
	School-masters	Clergymen	Clergymen	Advocates	Peerage Families	Peerage Families	Scottish Bankers	
67	...	·00602	...	...	...	·0023	·0074	67
68	...	·00555	...	...	...	·0021	·0070	68
69	...	·00418	...	...	...	·0019	·0066	69
70	...	...	...	...	...	·0018	·0062	70
71	...	...	...	...	...	·0016	·0058	71
72	...	...	...	...	...	·0015	·0054	72
73	...	...	...	...	...	·0013	·0050	73
74	...	...	...	...	...	·0012	·0046	74
75	...	...	...	...	...	·0010	·0042	75
76	...	...	...	...	...	·0009	·0038	76
77	...	...	...	...	...	·0007	·0034	77
78	...	...	...	...	...	·0006	·0031	78
79	...	...	...	...	...	·0004	·0027	79
80	...	...	...	...	...	·0003	·0023	80
81	...	...	...	...	...	·0001	·0019	81
82	...	...	...	...	...	...	·0015	82
83	...	...	...	...	...	...	·0011	83
84	...	...	...	...	...	...	·0007	84
85	...	...	...	...	...	...	·0004	85
86	...	...	...	...	...	...	·0001	86

From this Comparative Table it will be observed that the probability of bachelors, among Scottish Bankers, marrying in a year, $\left(\frac{(bm)_x}{(bl)_x}\right)$, is

Lower than in Mr. Huie's *Schoolmasters* ... up to age 44
„ „ „ Mr. Meikle's *Clergymen* ... „ „ „ 43
„ „ „ „ „ *Advocates* ... „ „ „ 42
„ „ „ „ „ *Peerage-Families* „ „ „ 32
„ „ „ Dr. Sprague's *Peerage-Families* „ „ „ 32

and

Higher than in Mr. Huie's *Clergymen* ... „ „ „ 29

From this it appears that the rate of marriage among Scottish Bankers is, on the whole, lower than what has been found to obtain among the Schoolmasters, Clergymen, Advocates, and Peerage-Families, whose experience has been investigated, except among the Clergymen up to age 29, if Mr. Huie's results are accepted. But Dr. Sprague has, in the article already referred to, made observations upon these results which cause some hesitation in accepting them as being sufficiently reliable.

The probability of marriage is

Greatest among	Meikle's	*Clergymen*	... at age	25
,, ,,	Sprague's	*Peerage-Families*	,, ,,	26
,, ,,	Meikle's	*Peerage-Families*	,, ,,	27
,, ,,	Huie's	*Schoolmasters*	.. ,, ,,	30
,, ,,	SCOTTISH	BANKERS	... ,, ,,	33
,, ,,	Meikle's	*Advocates*	... ,, ,,	34
,, ,,	Huie's	*Clergymen*	... ,, ,,	39

Taking the average of the rates from age 25 to age 55, the probability of marriage among Scottish Bankers comes nearest to that of Dr. Sprague's peerage rates.

They may be placed thus in the order of probability—the highest being first in order and the lowest being last—Meikle's Clergymen; Huie's Clergymen; Huie's Schoolmasters; Meikle's Advocates; SCOTTISH BANKERS; Sprague's Peerage; and Meikle's Peerage.

It must be remembered that Schoolmasters* and Clergymen, unlike Bank employees, generally enter at once upon the full emoluments of their office, with few and small (if any) occasional increments, and usually have official residences and other inducements to form a home and seek the companionship of a wife, which is all the more necessary in the more or less remote villages and rural parts where many in these professions discharge the duties of their respective offices. Also that Advocates are not infrequently in circumstances, apart from their profession, to marry earlier than they might otherwise do if they were wholly dependent upon professional income. The employees of Banks are, however, differently situated, being for the most part mainly dependent upon their gradually increasing salaries, and, in their earlier years, generally finding their comfort in lodgings, or in residing with their parents or other relatives.

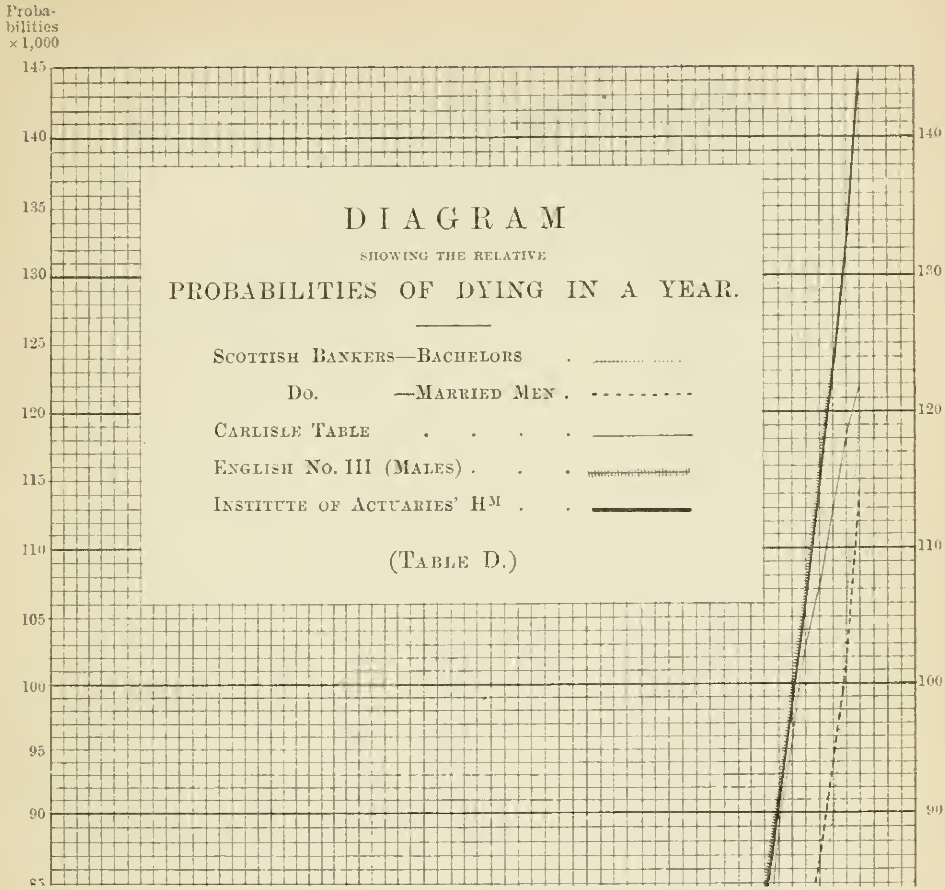
It must also be remembered that our Scottish Bankers' experience has been investigated down to the year 1892-93; while the experience of the other classes, included in the foregoing Comparative Table, referred to more or less remote periods; the Peerage-Families so far back as 1855; the Schoolmasters to 1861; the Clergymen, Mr. Huie's, to 1863, and Mr. Meikle's to 1875; and the Advocates to 1877. It is well known that in more recent times marriage is longer delayed among the salaried and professional classes than formerly, owing largely to the higher style of living now assumed to be desirable.

* The old Burgh and Parochial Schoolmasters.

The next Table (D) shows the probability of dying in a year, (q_x), among Scottish Bankers as compared with that of well-known standard tables.

Table D.—Males.—*Probability of Dying in a Year*, (q_x).

Age	Carlisle	English No. 3 Males	Institute of Actuaries' H^M	Scottish Bankers		Age
				Bachelors	Married Men	
15	·00619	·00517	·00287	·00588	...	15
16	·00671	·00563	·00325	·00598	...	16
17	·00691	·00620	·00388	·00608	...	17
18	·00696	·00686	·00479	·00608	...	18
19	·00701	·00757	·00575	·00618	...	19
20	·00706	·00828	·00633	·00618	·00608	20
21	·00695	·00846	·00673	·00628	·00608	21
22	·00699	·00864	·00684	·00628	·00618	22
23	·00704	·00883	·00676	·00638	·00618	23
24	·00709	·00899	·00664	·00658	·00628	24
25	·00731	·00917	·00663	·00688	·00648	25
26	·00737	·00933	·00669	·00737	·00668	26
27	·00777	·00951	·00690	·00807	·00688	27
28	·00870	·00969	·00717	·00856	·00707	28
29	·00983	·00988	·00743	·00807	·00737	29
30	·01010	·01008	·00772	·00767	·00717	30
31	·01021	·01029	·00792	·00727	·00697	31
32	·01013	·01051	·00811	·00698	·00678	32
33	·01005	·01074	·00829	·00678	·00668	33
34	·01015	·01099	·00850	·00668	·00688	34
35	·01026	·01126	·00877	·00678	·00727	35
36	·01055	·01156	·00911	·00707	·00787	36
37	·01086	·01188	·00946	·00757	·00866	37
38	·01117	·01222	·00978	·00817	·00945	38
39	·01188	·01258	·01008	·00886	·01005	39
40	·01300	·01297	·01031	·00955	·01064	40
41	·01378	·01340	·01049	·01035	·01114	41
42	·01437	·01386	·01073	·01104	·01163	42
43	·01458	·01433	·01113	·01173	·01213	43
44	·01480	·01486	·01156	·01232	·01262	44
45	·01481	·01542	·01219	·01292	·01301	45
46	·01482	·01601	·01294	·01351	·01331	46
47	·01460	·01667	·01370	·01410	·01351	47
48	·01393	·01735	·01444	·01469	·01370	48
49	·01368	·01807	·01522	·01528	·01400	49
50	·01342	·01884	·01595	·01578	·01430	50
51	·01429	·02022	·01667	·01627	·01469	51
52	·01520	·02123	·01755	·01686	·01518	52
53	·01615	·02227	·01860	·01764	·01578	53
54	·01690	·02337	·01973	·01863	·01656	54
55	·01792	·02454	·02103	·01980	·01754	55
56	·01900	·02584	·02245	·02117	·01872	56
57	·02090	·02725	·02399	·02274	·02019	57
58	·02421	·02882	·02563	·02450	·02205	58
59	·02827	·03058	·02754	·02664	·02430	59
60	·03349	·03252	·02968	·02897	·02713	60
61	·03579	·03468	·03204	·03150	·03043	61
62	·03741	·03708	·03464	·03421	·03411	62
63	·03825	·03974	·03749	·03710	·03816	63
64	·03977	·04267	·04041	·04027	·04258	64
65	·04109	·04591	·04343	·04344	·04668	65


Probabilities × 1,000
145
140
135
130
125
120
115
110
105
100
95
90
DIAGRAM
SHOWING THE RELATIVE
PROBABILITIES OF DYING IN A YEAR.
Scottish Bankers—Bachelors
Do. —Married Men
Carlisle Table
English No. III (Males)
Institute of Actuaries' H^M
(Table D.)
140
130
120
110
100
90

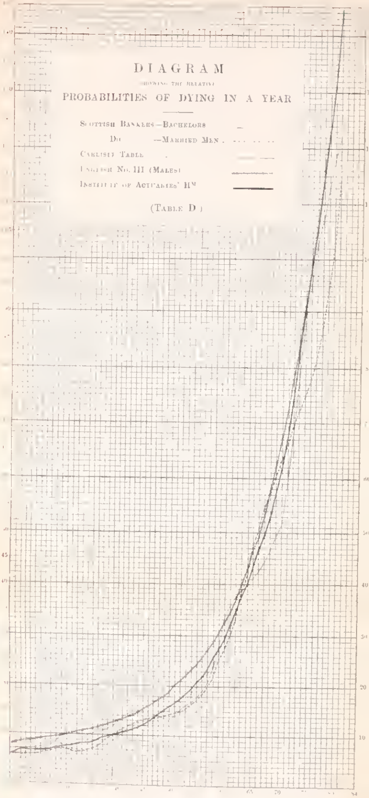


DIAGRAM
PROBABILITIES OF DYING IN A YEAR
Scottish Bankers—Bachelors
Do —Married Men
Carlisle Table
English No. III (Males)
Institute of Actuaries' HM
(Table D)

TABLE D.—MALES.—*Probability of Dying in a Year,* (q_x)—(continued).

Age	Carlisle	English No. 3 Males	Institute of Actuaries' H^M	Scottish Bankers		Age
				Bachelors	Married Men	
66	·04250	·04946	·04657	·04649	·05059	66
67	·04439	·05337	·04989	·04954	·05429	67
68	·04645	·05763	·05323	·05258	·05769	68
69	·04911	·06227	·05734	·05561	·06079	69
70	·05165	·06728	·06219	·05854	·06361	70
71	·05885	·07270	·06805	·06145	·06642	71
72	·06813	·07855	·07494	·06427	·06922	72
73	·07812	·08483	·08286	·06707	·07201	73
74	·09017	·09157	·09120	·06987	·07480	74
75	·09552	·09876	·09836	·07322	·07785	75
76	·10297	·10641	·10637	·07785	·08153	76
77	·10743	·11454	·11469	·08429	·08612	77
78	·10882	·12316	·12321	·09342	·09251	78
79	·11841	·13226	·13306	·10516	·10157	79
80	·12172	·14184	·14465	·11942	·11410	80
81	·13381	·15193	·15804	·13694	·12206	81
82	·14069	·16251	·17135	·15838	·14385	82
83	·15088	·17356	·18585	·18430	·16934	83
84	·15879	·18511	·19888	·21508	·19901	84
85	·17528	·19714	·20989	·25098	·23321	85
86	·19346	·20965	·21966	·29206	·27214	86
87	·21622	·22263	·23123	...	·31579	87
88	·21983	·23604	·23930	...	·36467	88
89	·21547	·24992	·25320	...	·41898	89
90	·26056	·26420	·27945	...	·51853	90
91	·28571	·27889	·31274	...	·66544	91
...	* * *	* * *	* * *			...
97	·22222	·37448	1·00000			97
...	* * *	* * *				...
103	·66666	·47863				103
...	* * *	* * *				...
108	...	·56712				108

From this Comparative Table it will be observed that (omitting ages under 20):

(1) The probability of dying in a year among *bachelors* is

Lower than that indicated by the Carlisle Table up to age 47 (except at age 27), and again between ages 59 to 63 and 72 to 80.

" than the English Life Table No. 3 (Males) up to age 82.

" than the Actuaries' "Healthy Males" Table, at all ages, except ages 25 to 29, 42 to 49, and 84 to 86.

(2) The probability of dying in a year among *married men* is

Lower than that indicated by the Carlisle Table up to age 63 (except at ages 49 to 51); and from ages 73 to 81.

" than the English Life Table No. 3 up to age 83, except at ages 65 to 68.

" than the Actuaries' "Healthy Males" Table up to age 62 (except at ages 40 to 46); and from ages 71 to 83.

It will also be observed that the Mortality Experience of the Scottish Bankers, as exhibited in this Table D, corresponds in a remarkable manner with that of the *Healthy Males* of the Institute of Actuaries' Tables. At ages above 43 the close resemblance to "assured-life" experience is still more marked when compared with the $H^{M(5)}$ Table.

The following Table (E, p. 447) shows the probability of dying in a year among the wives and widows of the Scottish Bankers as compared with that of well-known standard tables.

From this Comparative Table it will be observed that:

(1) The probability of dying in a year among the *Wives* of Scottish Bankers is *lower* than that indicated by the *Healthy Females* of the Institute of Actuaries' Tables, at all ages between 24 and 80; after which age it is a little higher: and

(2) The probability of dying in a year among the *Widows corresponds closely* with that of the H^F Table at nearly all ages up to age 58; after which it becomes *increasingly lower,* until the age of 89, beyond which the observations are too few to rely upon.

The close resemblance between the experience of the Bankers' Widows and that of the "Healthy Females" at ages under 60 is striking. After that age, the probabilities of the former become increasingly lower, on to the end of the Table.

As the rate of mortality among Government Annuitants is not given at ages under 50, a comparison cannot be made at the younger ages. It will be observed that the probability of dying in a year among the Scottish Bankers' *Widows* (after age 49) is *higher* than that of Government Annuitants (females) up to age 62, after which it is lower up to the age of 89, except at ages

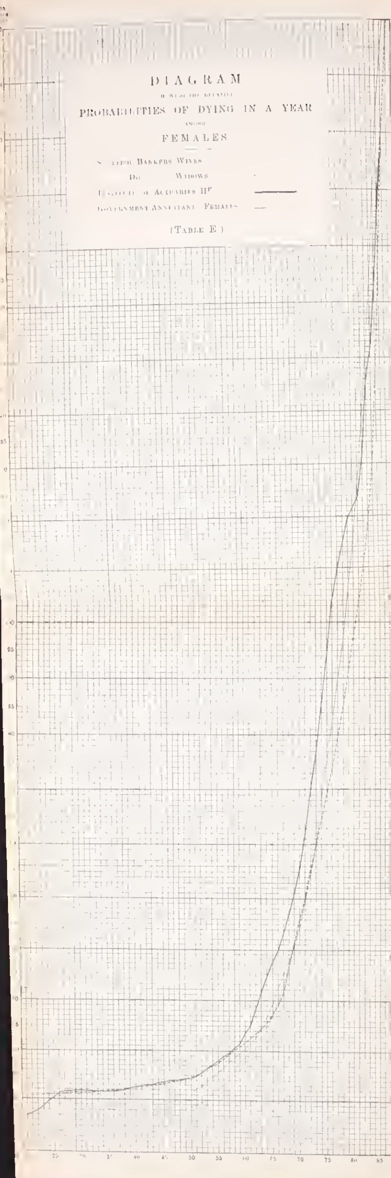


DIAGRAM
PROBABILITIES OF DYING IN A YEAR
FEMALES
Bankers Wives
Do. Widows
Actuaries H^F
Females
(Table E)
50
55
70
75
85

TABLE E.—FEMALES.—*Probability of Dying in a Year*, (q_x).

Age	Institute of Actuaries' H^F	Government Annuitants (Females)	Scottish Bankers Wives	Scottish Bankers Widows	Age	Institute of Actuaries' H^F	Government Annuitants (Females)	Scottish Bankers Wives	Scottish Bankers Widows
20	·00688	...	·01302	...	60	·02368	·02050	·01804	·02176
21	·00729	...	·01262	·01094	61	·02587	·02166	·01951	·02264
22	·00792	...	·01223	·01094	62	·02846	·02283	·02108	·02362
23	·00882	...	·01173	·01104	63	·03137	·02471	·02284	·02469
24	·00984	...	·01124	·01104	64	·03451	·02695	·02489	·02586
25	·01081	...	·01074	·01114	65	·03738	·02944	·02723	·02732
26	·01138	...	·01025	·01114	66	·04018	·03212	·02995	·02917
27	·01182	Not available until Age 49	·00985	·01124	67	·04340	·03489	·03305	·03150
28	·01201		·00955	·01124	68	·04698	·03753	·03662	·03633
29	·01195		·00926	·01134	69	·05099	·04086	·04066	·04114
30	·01185		·00896	·01134	70	·05564	·04457	·04525	·04592
31	·01171		·00866	·01143	71	·06195	·04931	·05040	·05068
32	·01157		·00846	·01143	72	·06917	·05492	·05618	·05542
33	·01166		·00827	·01153	73	·07708	·06154	·06258	·06014
34	·01172		·00817	·01163	74	·08576	·06825	·06969	·06483
35	·01176		·00807	·01173	75	·09485	·07548	·07748	·06950
36	·01190		·00797	·01183	76	·10405	·08239	·08603	·07415
37	·01204		·00787	·01193	77	·10976	·08994	·09533	·07969
38	·01219		·00787	·01203	78	·11505	·09743	·10543	·08521
39	·01234		·00787	·01213	79	·11934	·10618	·11641	·09160
40	·01255		·00777	·01232	80	·12179	·11534	·12831	·09796
41	·01275		·00777	·01252	81	·12358	·12651	·14118	·10427
42	·01294		·00767	·01272	82	·13170	·13743	·15422	·11142
43	·01315		·00767	·01292	83	·14526	·15043	·16833	·11942
44	·01338	...	·00777	·01311	84	·16280	·16376	·18355	·13084
45	·01362	...	·00777	·01331	85	·18564	·17983	·19990	·14815
46	·01376	...	·00787	·01351	86	·21214	·19391	·21739	·16514
47	·01388	...	·00807	·01371	87	·23696	·21294	...	·19005
48	·01401	...	·00827	·01390	88	·25822	·23112	...	·22222
49	·01416	...	·00846	·01420	89	·27255	·25082	...	·26087
50	·01445	·01123	·00876	·01459	90	·27504	·26632	...	·30508
51	·01496	·01211	·00916	·01509	91	·26901	·29102	...	·35391
52	·01562	·01273	·00975	·01568	92	·23200	·30731	...	·41270
53	·01641	·01379	·01045	·01637	93	·17969	·32689	...	·49056
54	·01739	·01495	·01124	·01705	94	·18571	·34211	...	·57143
55	·01827	·01585	·01213	·01784	95	·20858	·37818	...	·66667
56	·01907	·01668	·01311	·01863	96	·25616	·40058	...	...
57	·01989	·01791	·01420	·01941	97	·33775	...	...	...
58	·02074	·01882	·01538	·02019	98	·50000	...	...	...
59	·02180	·01953	·01666	·02098	99	1·00000	...	...	...

69 to 72. Among the *Wives*, on the other hand, the probability is higher at all ages above 69.

What has now been communicated by no means exhausts the results of the investigation. Many other interesting, though minor, points are being looked into. Two of special interest in the wider field of life assurance are here given, by way of example.

The *first* indicates marriage as being practically equivalent to the medical selection of lives. This is shown by the following table:

Husbands			
Year of Marriage	At Risk	Expected Deaths	Actual Deaths
1	824	3·96	4
2	773	5·07 } 10·55	7 } 11
3	726·5	5·48	4
4	688·5	5·67 } 11·37	4 } 11
5	650	5·70	7
	3,662	25·88	26

The *second* indicates the special risk of death during the first year of marriage. This is shown by the following table:

Wives			
Year of Marriage	At Risk	Expected Deaths	Actual Deaths
1	562	2·22	9
2	520·5	3·07 } 9·8	5 } 10
3	485·5	3·35	5
4	456·5	3·38	...
5	430	3·41	2
6	398	3·31	2
7	369	3·15	3
8	344·5	3·09	3
9	322	2·96	1
10	301	2·88	3
	4,189	30·82	33

This result would seem to indicate a net extra premium for the first year of £1. 4*s*. 2*d*., found thus:

$$\frac{9-2{\cdot}22}{562}=\frac{6{\cdot}78}{562}=1{\cdot}2064=\text{£}1.\ 4s.\ 2d.\ \text{per £}100.$$

The cases are, however, too few to be relied upon as a suitable basis on which to calculate the required "Extra."

Monetary Tables, based on the results of this investigation, are now being framed for use in connection with the periodical valuations of these funds, and for other purposes.

TABLE F(*a*).—BACHELORS.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping.*

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
14	1	...	...				
15	5	...	...				
16	32	...	...				
17	129·5	...	...	14–20	2123	13	·0061
18	387	2	·0051				
19	664·5	5	·0075				
20	904	6	·0066				
21	1069	9	·0084				
22	1170	5	·0043	21–24	4804	30	·0063
23	1237	6	·0049				
24	1328	10	·0075				
25	1342·5	9	·0067	25, 26	2635	19	·0072
26	1292·5	10	·0077				
27	1231·5	11	·0089				
28	1123	7	·0062	27–30	4317	38	·0088
29	1023·5	15	·0147				
30	939	5	·0053				
31	852	7	·0082				
32	765	2	·0026				
33	685	5	·0073	31–35	3441	22	·0064
34	602·5	3	·0050				
35	536·5	5	·0093				
36	481	2	·0042				
37	433·5	3	·0069	36–39	1659·5	13	·0078
38	392	2	·0051				
39	353	6	·0170				
40	319	2	·0063				
41	285	3	·0105	40–42	864	9	·0104
42	260	4	·0154				
43	232	2	·0086				
44	204	4	·0196				
45	187·5	3	·0160	43–48	1116·5	15	·0134
46	176	3	·0170				
47	165	2	·0121				
48	152	1	·0066				
49	139	2	·0144				
50	126	2	·0159				
51	110·5	...	...				
52	98·5	1	·0102	49–55	717·5	12	·0167
53	88·5	1	·0113				
54	81	1	·0123				
55	74	5	·0676				
56	71	1	·0141				
57	67	1	·0149	56–59	259·5	6	·0231
58	63·5	1	·0157				
59	58	3	·0517				
60	51·5	2	·0388				
61	48·5	...	...				
62	44·5	...	...	60–65	250·5	9	·0359
63	42·5	2	·0471				
64	35	3	·0857				
65	28·5	2	·0702				
66	26	1	·0385				
67	23·5	2	·0851				
68	22	1	·0455	66–70	111·5	6	·0538
69	20·5	1	·0488				

TABLE F(*a*).—BACHELORS.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping*—(continued).

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
70	19·5	1	·0513				
71	17·5	3	·1714				
72	14·5	1	·0690				
73	12·5	...	...				
74	11·5	1	·0870	71–77	83	6	·0723
75	11	...	...				
76	9	1	·1111				
77	7	...	...				
78	7	...	...				
79	7	...	...				
80	6	2	·3333				
81	5	...	...				
82	5	...	...	78–86	41·5	7	·1687
83	4·5	1	·2222				
84	3·5	1	·2857				
85	2·5	1	·4000				
86	1	2	2·0000				
	22423·5	205			22423·5	205	

TABLE F(*b*).—BACHELORS.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences.*

Age (1)	Adjusted central Death-rate (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)—(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
14	·0058	1	·01	...	+ ·01	+ ·01	14
15	·0059	1	·03	...	+ ·03	+ ·04	15
16	·0060	1	·19	...	+ ·19	+ ·23	16
17	·0061	0	·79	...	+ ·79	+1·02	17
18	·0061	1	2·36	2	+ ·36	+1·38	18
19	·0062	0	4·12	5	− ·88	+ ·50	19
20	·0062	1	5·60	6	− ·40	+ ·10	20
21	·0063	0	6·73	9	−2·27	−2·17	21
22	·0063	1	7·37	5	+2·37	+ ·20	22
23	·0064	2	7·92	6	+1·92	+2·12	23
24	·0066	3	8·76	10	−1·24	+ ·88	24
25	·0069	5	9·26	9	+ ·26	+1·14	25
26	·0074	7	9·56	10	− ·44	+ ·70	26
27	·0081	5	9·98	11	−1·02	− ·32	27
28	·0086	−5	9·66	7	+2·66	+2·34	28
29	·0081	−4	8·29	15	−6·71	−4·37	29
30	·0077	−4	7·23	5	+2·23	−2·14	30
31	·0073	−3	6·22	7	− ·78	−2·92	31
32	·0070	−2	5·36	2	+3·36	+ ·44	32
33	·0068	−1	4·66	5	− ·34	+ ·10	33
34	·0067	1	4·04	3	+1·04	+1·14	34
35	·0068	3	3·65	5	−1·35	− ·21	35
36	·0071	5	3·42	2	+1·42	+1·21	36

TABLE F (*b*).—BACHELORS.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences*—(continued).

Age	Adjusted central Death-rate	$\Delta \times 10^4$	Expected Deaths	Actual Deaths	Difference (4)—(5)	Accumulated Difference Σ(6)	Age
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
37	·0076	6	3·29	3	+ ·29	+1·50	37
38	·0082	7	3·21	2	+1·21	+2·71	38
39	·0089	7	3·14	6	−2·86	− ·15	39
40	·0096	8	3·06	2	+1·06	+ ·91	40
41	·0104	7	2·96	3	− ·04	+ ·87	41
42	·0111	7	2·89	4	−1·11	− ·24	42
43	·0118	6	2·74	2	+ ·74	+ ·50	43
44	·0124	6	2·53	4	−1·47	− ·97	44
45	·0130	6	2·44	3	− ·56	−1·53	45
46	·0136	6	2·39	3	− ·61	−2·14	46
47	·0142	6	2·34	2	+ ·34	−1·80	47
48	·0148	6	2·25	1	+1·25	− ·55	48
49	·0154	5	2·14	2	+ ·14	− ·41	49
50	·0159	5	2·00	2	·00	− ·41	50
51	·0164	6	1·81	...	+1·81	+1·40	51
52	·0170	8	1·67	1	+ ·67	+2·07	52
53	·0178	10	1·58	1	+ ·58	+2·65	53
54	·0188	12	1·52	1	+ ·52	+3·17	54
55	·0200	14	1·48	5	−3·52	− ·35	55
56	·0214	16	1·52	1	+ ·52	+ ·17	56
57	·0230	18	1·54	1	+ ·54	+ ·71	57
58	·0248	22	1·57	1	+ ·57	+1·28	58
59	·0270	24	1·57	3	−1·43	− ·15	59
60	·0294	26	1·51	2	− ·49	− ·64	60
61	·0320	28	1·55	...	+1·55	+ ·91	61
62	·0348	30	1·55	...	+1·55	+2·46	62
63	·0378	33	1·61	2	− ·39	+2·07	63
64	·0411	33	1·44	3	−1·56	+ ·51	64
65	·0444	32	1·27	2	− ·73	− ·22	65
66	·0476	32	1·24	1	+ ·24	+ ·02	66
67	·0508	32	1·19	2	− ·81	− ·79	67
68	·0540	32	1·19	1	+ ·19	− ·60	68
69	·0572	31	1·17	1	+ ·17	− ·43	69
70	·0603	31	1·18	1	+ ·18	− ·25	70
71	·0634	30	1·11	3	−1·89	−2·14	71
72	·0664	30	·96	1	− ·04	−2·18	72
73	·0694	30	·87	...	+ ·87	−1·31	73
74	·0724	36	·83	1	− ·17	−1·48	74
75	·0760	50	·84	...	+ ·84	− ·64	75
76	·0810	70	·73	1	− ·27	− ·91	76
77	·0880	100	·62	...	+ ·62	− ·29	77
78	·0980	130	·69	...	+ ·69	+ ·40	78
79	·1110	160	·78	...	+ ·78	+1·18	79
80	·1270	200	·76	2	−1·24	− ·06	80
81	·1470	250	·74	...	+ ·74	+ ·68	81
82	·1720	310	·86	...	+ ·86	+1·54	82
83	·2030	380	·91	1	− ·09	+1·45	83
84	·2410	460	·84	1	− ·16	+1·29	84
85	·2870	550	·72	1	− ·28	+1·01	85
86	·3420		·34	2	−1·66	− ·65	86
			204·35	205			

TABLE F(*c*).—BACHELORS.—*Number at Risk, Number Marrying, Unadjusted Central Marriage-rate; and Grouping.*

Age	Number at risk	Number marrying	Unadjusted central Marriage-rate	Age	Number at risk	Number marrying	Unadjusted central Marriage-rate
14	1	...	...				
15	5	...	...				
16	32	...	...				
17	129·5	...	...				
18	387	...	...				
19	664·5	...	...				
20	904	5	·0055	20–21	1973	6	·0030
21	1069	1	·0009				
22	1170	19	·0162	22	1170	19	·0162
23	1237	25	·0202	23	1237	25	·0202
24	1328	39	·0294	24	1328	39	·0294
25	1342·5	50	·0372	25	1342·5	50	·0372
26	1292·5	58	·0449	26	1292·5	58	·0449
27	1231·5	72	·0585	27	1231·5	72	·0585
28	1123	88	·0784				
29	1023·5	63	·0616	28–30	3085·5	196	·0635
30	939	45	·0479				
31	852	70	·0822				
32	765	49	·0641	31–34	2904·5	223	·0768
33	685	53	·0774				
34	602·5	51	·0846				
35	536·5	35	·0652				
36	481	30	·0624	35–38	1843	121	·0657
37	433·5	24	·0554				
38	392	32	·0816				
39	353	17	·0482				
40	319	18	·0564	39–42	1217	59	·0485
41	285	12	·0421				
42	260	12	·0462				
43	232	12	·0517				
44	204	6	·0294				
45	187·5	7	·0373	43–48	1116·5	42	·0376
46	176	9	·0511				
47	165	6	·0363				
48	152	2	·0132				
49	139	4	·0288				
50	126	3	·0238				
51	110·5	3	·0271	49–53	562·5	13	·0231
52	98·5	3	·0305				
53	88·5	...	...				
54	81	1	·0123				
55	74	1	·0135	54–57	293	4	·0137
56	71	...	...				
57	67	2	·0299				
58	63·5	...	...				
59	58	...	...				
60	51·5	...	...				
61	48·5	...	...				
62	44·5	1	·0225	58–66	398	4	·0100
63	42·5	1	·0235				
64	35	1	·0286				

TABLE F(*c*).—BACHELORS.—*Number at Risk, Number Marrying, Unadjusted Central Marriage-rate; and Grouping*—(continued).

Age	Number at risk	Number marrying	Unadjusted central Marriage-rate	Age	Number at risk	Number marrying	Unadjusted central Marriage-rate
65	28·5	...	...				
66	26	1	·0385				
67	23·5	...	...				
68	22	...	...				
69	20·5	...	...				
70	19·5	...	...				
71	17·5	...	...				
72	14·5	...	...				
73	12·5	1	·0800				
74	11·5	...	...				
75	11	...	...				
76	9	...	...	67–86	210	1	·0048
77	7	...	...				
78	7	...	...				
79	7	...	...				
80	6	...	...				
81	5	...	...				
82	5	...	...				
83	4·5	...	...				
84	3·5	...	...				
85	2·5	...	...				
86	1	...	...				
	22423·5	932			22423·5	932	

TABLE F(*d*).—BACHELORS.—*Adjusted Central Marriage-rate, Expected and Actual Marriages; with Differences.*

Age (1)	Adjusted central Marriage-rate (2)	$\Delta \times 10^4$ (3)	Expected Marriages (4)	Actual Marriages (5)	Difference (4)—(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
20	·0025	60	2·26	5	− 2·74	− 2·74	20
21	·0085	65	9·09	1	+ 8·09	+ 5·35	21
22	·0150	70	17·55	19	− 1·45	+ 3·90	22
23	·0220	80	27·21	25	+ 2·21	+ 6·11	23
24	·0300	90	39·84	39	+ ·84	+ 6·95	24
25	·0390	80	52·36	50	+ 2·36	+ 9·31	25
26	·0470	70	60·75	58	+ 2·75	+12·06	26
27	·0540	60	66·50	72	− 5·50	+ 6·56	27
28	·0600	50	67·38	88	−20·62	−14·06	28
29	·0650	40	66·53	63	+ 3·53	−10·53	29
30	·0690	30	64·79	45	+19·79	+ 9·26	30
31	·0720	20	61·34	70	− 8·66	+ ·60	31
32	·0740	15	56·61	49	+ 7·61	+ 8·21	32
33	·0755	−30	51·72	53	− 1·28	+ 6·93	33
34	·0725	−32	43·68	51	− 7·32	− ·39	34
35	·0693	−35	37·18	35	+ 2·18	+ 1·79	35
36	·0658	−35	31·65	30	+ 1·65	+ 3·44	36

TABLE F(*d*).—BACHELORS.—*Adjusted Central Marriage-rate, Expected and Actual Marriages; with Differences*—(continued).

Age (1)	Adjusted central Marriage-rate (2)	$\Delta \times 10^4$ (3)	Expected Marriages (4)	Actual Marriages (5)	Difference (4)—(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
37	·0623	−35	27·01	24	+ 3·01	+ 6·45	37
38	·0588	−34	23·05	32	− 8·95	− 2·50	38
39	·0554	−33	19·56	17	+ 2·56	+ ·06	39
40	·0521	−32	16·62	18	− 1·38	− 1·32	40
41	·0489	−31	13·94	12	+ 1·94	+ ·62	41
42	·0458	−30	11·91	12	− ·09	+ ·53	42
43	·0428	−29	9·93	12	− 2·07	− 1·54	43
44	·0399	−28	8·14	6	+ 2·14	+ ·60	44
45	·0371	−27	6·96	7	− ·04	+ ·56	45
46	·0344	−26	6·05	9	− 2·95	− 2·39	46
47	·0318	−25	5·25	6	− ·75	− 3·14	47
48	·0293	−24	4·45	2	+ 2·45	− ·69	48
49	·0269	−23	3·74	4	− ·26	− ·95	49
50	·0246	−22	3·10	3	+ ·10	− ·85	50
51	·0224	−21	2·48	3	− ·52	− 1·37	51
52	·0203	−20	2·00	3	− 1·00	− 2·37	52
53	·0183	−18	1·62	...	+ 1·62	− ·75	53
54	·0165	−16	1·34	1	+ ·34	− ·41	54
55	·0149	−14	1·10	1	+ ·10	− ·31	55
56	·0135	−12	·96	...	+ ·96	+ ·65	56
57	·0123	− 9	·82	2	− 1·18	− ·53	57
58	·0114	− 6	·72	..	+ ·72	+ ·19	58
59	·0108	− 4	·63	...	+ ·63	+ ·82	59
60	·0104	− 4	·54	...	+ ·54	+ 1·36	60
61	·0100	− 4	·49	...	+ ·49	+ 1·85	61
62	·0096	− 4	·43	1	− ·57	+ 1·28	62
63	·0092	− 4	·39	1	− ·61	+ ·67	63
64	·0088	− 4	·31	1	− ·69	− ·02	64
65	·0084	− 4	·24	...	+ ·24	+ ·22	65
66	·0080	− 4	·21	1	− ·79	− ·57	66
67	·0076	− 4	·18	...	+ ·18	− ·39	67
68	·0072	− 4	·16	...	+ ·16	− ·23	68
69	·0068	− 4	·14	...	+ ·14	− ·09	69
70	·0064	− 4	·12	...	+ ·12	+ ·03	70
71	·0060	− 4	·11	...	+ ·11	+ ·14	71
72	·0056	− 4	·08	...	+ ·08	+ ·22	72
73	·0052	− 4	·07	1	− ·93	− ·71	73
74	·0048	− 4	·06	...	+ ·06	− ·65	74
75	·0044	− 4	·05	...	+ ·05	− ·60	75
76	·0040	− 4	·04	...	+ ·04	− ·56	76
77	·0036	− 4	·03	...	+ ·03	− ·53	77
78	·0032	− 4	·02	...	+ ·02	− ·51	78
79	·0028	− 4	·02	...	+ ·02	− ·49	79
80	·0024	− 4	·01	...	+ ·01	− ·48	80
81	·0020	− 4	·01	...	+ ·01	− ·47	81
82	·0016	− 4	·01	...	+ ·01	− ·46	82
83	·0012	− 4	·01	...	+ ·01	− ·45	83
84	·0008	− 4	...	...	...	...	84
85	·0004	− 4	...	...	...	..	85
86	·0000		...	...	...	...	86
			931·55	932			

TABLE G(*a*).—MARRIED MEN.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping.*

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
20	2	...	...				
21	7	...	...				
22	19	...	...				
23	42·5	...	...	20–27	811	5	·0062
24	87	...	...				
25	150	1	·0067				
26	212	...	...				
27	291·5	4	·0137				
28	383	4	·0104				
29	470·5	1	·0021	28–31	1980·5	15	·0076
30	529	7	·0132				
31	598	3	·0050				
32	660·5	2	·0030	32, 33	1371·5	9	·0066
33	711	7	·0098				
34	747·5	5	·0067	34	747·5	5	·0067
35	781	6	·0077				
36	801·5	8	·0100				
37	805	8	·0099	35–40	4814·5	44	·0091
38	811	6	·0074				
39	811·5	7	·0086				
40	804·5	9	·0112				
41	790	4	·0051				
42	783·5	11	·0141				
43	778	11	·0141				
44	770	14	·0182	41–47	5273	68	·0129
45	745	14	·0188				
46	717·5	8	·0111				
47	689	6	·0087				
48	669	11	·0164				
49	649·5	8	·0123	48–51	2531·5	36	·0142
50	623·5	8	·0128				
51	589·5	9	·0153				
52	562	10	·0178				
53	527·5	7	·0133				
54	498·5	5	·0100	52–56	2531·5	40	·0158
55	482	10	·0207				
56	461·5	8	·0173				
57	440·5	10	·0227				
58	411	10	·0243	57–60	1608·5	37	·0230
59	391	9	·0230				
60	366	8	·0219				
61	348·5	12	·0344				
62	329·5	14	·0425	61–63	988	34	·0344
63	310	8	·0258				
64	284·5	14	·0492	64, 65	548·5	25	·0456
65	264	11	·0417				
66	243·5	19	·0780				
67	222·5	8	·0360				
68	203·5	15	·0737	66–71	1164·5	72	·0618
69	180	10	·0556				
70	168	10	·0595				
71	147	10	·0680				
72	127	11	·0866				
73	114	3	·0263				

TABLE G(*a*).—MARRIED MEN.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping*—(continued).

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
74	100	8	·0800	72–77	572·5	43	·0752
75	88·5	3	·0339				
76	78·5	8	·1019				
77	64·5	10	·1550				
78	55·5	2	·0360				
79	50	9	·1800				
80	40·5	3	·0741	78–83	226	30	·1327
81	32·5	8	·2462				
82	25·5	4	·1569				
83	22	4	·1818				
84	15·5	5	·3226				
85	11	1	·0909				
86	7·5	4	·5333				
87	5	...	...	84–91	50·5	15	·2990
88	5	...	...				
89	4	2	·5000				
90	2	2	1·0000				
91	·5	1	2·0000				
	25219·5	478			25219·5	478	

TABLE G(*b*).—MARRIED MEN.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences.*

Age (1)	Adjusted central Death-rate (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)—(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
20	·0061	0	·01	...	+ ·01	+ ·01	20
21	·0061	1	·04	...	+ ·04	+ ·05	21
22	·0062	0	·12	...	+ ·12	+ ·17	22
23	·0062	1	·26	...	+ ·26	+ ·43	23
24	·0063	2	·55	...	+ ·55	+ ·98	24
25	·0065	2	·98	1	− ·02	+ ·96	25
26	·0067	2	1·42	...	+1·42	+2·38	26
27	·0069	2	2·01	4	−1·99	+ ·39	27
28	·0071	3	2·72	4	−1·28	− ·89	28
29	·0074	−2	3·48	1	+2·48	+1·59	29
30	·0072	−2	3·81	7	−3·19	−1·60	30
31	·0070	−2	4·19	3	+1·19	− ·41	31
32	·0068	−1	4·49	2	+2·49	+2·08	32
33	·0067	2	4·76	7	−2·24	− ·16	33
34	·0069	4	5·16	5	+ ·16	·00	34
35	·0073	6	5·70	6	− ·30	− ·30	35
36	·0079	8	6·33	8	−1·67	−1·97	36
37	·0087	8	7·00	8	−1·00	−2·97	37
38	·0095	6	7·70	6	+1·70	−1·27	38
39	·0101	6	8·20	7	+1·20	− ·07	39
40	·0107	5	8·61	9	− ·39	− ·46	40
41	·0112	5	8·85	4	+4·85	+4·39	41

TABLE G(*b*).—MARRIED MEN.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences*—(continued).

Age (1)	Adjusted central Death-rate (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)—(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
42	·0117	5	9·17	11	−1·83	+2·56	42
43	·0122	5	9·49	11	−1·51	+1·05	43
44	·0127	4	9·78	14	−4·22	−3·17	44
45	·0131	3	9·76	14	−4·24	−7·41	45
46	·0134	2	9·61	8	+1·61	−5·80	46
47	·0136	2	9·37	6	+3·37	−2·43	47
48	·0138	3	9·23	11	−1·77	−4·20	48
49	·0141	3	9·16	8	+1·16	−3·04	49
50	·0144	4	8·98	8	+ ·98	−2·06	50
51	·0148	5	8·72	9	− ·28	−2·34	51
52	·0153	6	8·60	10	−1·40	−3·74	52
53	·0159	8	8·39	7	+1·39	−2·35	53
54	·0167	10	8·32	5	+3·32	+ ·97	54
55	·0177	12	8·53	10	−1·47	− ·50	55
56	·0189	15	8·72	8	+ ·72	+ ·22	56
57	·0204	19	8·99	10	−1·01	− ·79	57
58	·0223	23	9·17	10	− ·83	−1·62	58
59	·0246	29	9·62	9	+ ·62	−1·00	59
60	·0275	34	10·07	8	+2·07	+1·07	60
61	·0309	38	10·77	12	−1·23	− ·16	61
62	·0347	42	11·43	14	−2·57	−2·73	62
63	·0389	46	12·06	8	+4·06	+1·33	63
64	·0435	43	12·38	14	−1·62	− ·29	64
65	·0478	41	12·62	11	+1·62	+1·33	65
66	·0519	39	12·64	19	−6·36	−5·03	66
67	·0558	36	12·42	8	+4·42	− ·61	67
68	·0594	33	12·09	15	−2·91	−3·52	68
69	·0627	30	11·29	10	+1·29	−2·23	69
70	·0657	30	11·04	10	+1·04	−1·19	70
71	·0687	30	10·10	10	+ ·10	−1·09	71
72	·0717	30	9·11	11	−1·89	−2·98	72
73	·0747	30	8·52	3	+5·52	+2·54	73
74	·0777	33	7·77	8	− ·23	+2·31	74
75	·0810	40	7·17	3	+4·17	+6·48	75
76	·0850	50	6·67	8	−1·33	+5·15	76
77	·0900	70	5·81	10	−4·19	+ ·96	77
78	·0970	100	5·38	2	+3·38	+4·34	78
79	·1070	140	5·35	9	−3·65	+ ·69	79
80	·1210	190	4·90	3	+1·90	+2·59	80
81	·1300	250	4·23	8	−3·77	−1·18	81
82	·1550	300	3·95	4	− ·05	−1·23	82
83	·1850	360	4·07	4	+ ·07	−1·16	83
84	·2210	430	3·43	5	−1·57	−2·73	84
85	·2640	510	2·90	1	+1·90	− ·83	85
86	·3150	600	2·36	4	−1·64	−2·47	86
87	·3750	710	1·88	...	+1·88	− ·59	87
88	·4460	840	2·23	...	+2·23	+1·64	88
89	·5300	1700	2·12	2	+ ·12	+1·76	89
90	·7000	3000	1·40	2	− ·60	+1·16	90
91	1·0000		·50	1	− ·50	+ ·66	91
			478·66	478			

Table H(*a*).—Wives.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping.*

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
17	2	...	...				
18	15	...	...				
19	41·5	...	...				
20	93	2	·0215	17–23	840	11	·0131
21	147·5	2	·0136				
22	222	4	·0180				
23	319	3	·0094				
24	398·5	4	·0100				
25	491	5	·0102				
26	602·5	5	·0083	24–28	2912	30	·0103
27	679	5	·0074				
28	741	11	·0148				
29	793	9	·0113				
30	816	6	·0074				
31	828·5	2	·0024	29–34	5032·5	43	·0085
32	847	11	·0130				
33	871·5	6	·0069				
34	876·5	9	·0103				
35	855	9	·0105				
36	829·5	8	·0096				
37	786	9	·0115				
38	766·5	6	·0078	35–41	5379·5	44	·0082
39	746·5	5	·0067				
40	714·5	4	·0056				
41	681·5	3	·0044				
42	655	4	·0061				
43	641	5	·0078				
44	616·5	6	·0097	42–47	3570	27	·0076
45	586	4	·0068				
46	552·5	5	·0090				
47	519	3	·0058				
48	484·5	7	·0144				
49	459	6	·0131				
50	435	4	·0092				
51	410·5	3	·0073	48–54	2885	26	·0090
52	388	1	·0026				
53	365·5	2	·0055				
54	342·5	3	·0088				
55	317·5	7	·0220				
56	293	2	·0068				
57	273·5	3	·0110	55–60	1556·5	23	·0148
58	250·5	3	·0120				
59	222	6	·0270				
60	200	2	·0100				
61	182·5	6	·0328				
62	164·5	6	·0365				
63	147	1	·0068				
64	133·5	3	·0225	61–67	945·5	23	·0243
65	119	4	·0336				
66	105	...	...				
67	94	3	·0319				
68	81·5	6	·0736				
69	65·5	4	·0611				
70	54·5	1	·0183				

Table H(*a*).—Wives.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping*—(continued).

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
71	45·5	4	·0879	68–75	363·5	19	·0523
72	37	3	·0811				
73	30·5	...	...				
74	26·5	...	...				
75	22·5	1	·0444				
76	16	2	·1250				
77	10·5	2	·1905				
78	7·5	...	...				
79	5	...	...				
80	3·5	1	·2857				
81	3	...	...	76–86	53	5	·0943
82	2·5	...	...				
83	2	...	...				
84	1·5	...	...				
85	1	...	...				
86	·5	...	...				
	23537·5	251			23537·5	251	

Table H(*b*).—Wives.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences.*

Age (1)	Adjusted central Death-rate (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)–(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
20	·0131	−4	1·22	2	− ·78	− ·78	20
21	·0127	−4	1·87	2	− ·13	− ·91	21
22	·0123	−5	2·73	4	−1·27	−2·18	22
23	·0118	−5	3·76	3	+ ·76	−1·42	23
24	·0113	−5	4·50	4	+ ·50	− ·92	24
25	·0108	−5	5·30	5	+ ·30	− ·62	25
26	·0103	−4	6·21	5	+1·21	+ ·59	26
27	·0099	−3	6·72	5	+1·72	+2·31	27
28	·0096	−3	7·11	11	−3·89	−1·58	28
29	·0093	−3	7·37	9	−1·63	−3·21	29
30	·0090	−3	7·34	6	+1·34	−1·87	30
31	·0087	−2	7·21	2	+5·21	+3·34	31
32	·0085	−2	7·20	11	−3·80	− ·46	32
33	·0083	−1	7·23	6	+1·23	+ ·77	33
34	·0082	−1	7·19	9	−1·81	−1·04	34
35	·0081	−1	6·93	9	−2·07	−3·11	35
36	·0080	−1	6·64	8	−1·36	−4·47	36
37	·0079	0	6·21	9	−2·79	−7·26	37
38	·0079	0	6·06	6	+ ·06	−7·20	38
39	·0079	−1	5·90	5	+ ·90	−6·30	39
40	·0078	0	5·57	4	+1·57	−4·73	40

Table H(*b*).—Wives.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences*—(continued).

Age (1)	Adjusted central Death-rate (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)—(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
41	·0078	− 1	5·32	3	+2·32	−2·41	41
42	·0077	0	5·04	4	+1·04	−1·37	42
43	·0077	1	4·94	5	− ·06	−1·43	43
44	·0078	0	4·81	6	−1·19	−2·62	44
45	·0078	1	4·57	4	+ ·57	−2·05	45
46	·0079	2	4·36	5	− ·64	−2·69	46
47	·0081	2	4·20	3	+1·20	−1·49	47
48	·0083	2	4·02	7	−2·98	−4·47	48
49	·0085	3	3·90	6	−2·10	−6·57	49
50	·0088	4	3·83	4	− ·17	−6·74	50
51	·0092	6	3·78	3	+ ·78	−5·96	51
52	·0098	7	3·80	1	+2·80	−3·16	52
53	·0105	8	3·84	2	+1·84	−1·32	53
54	·0113	9	3·87	3	+ ·87	− ·45	54
55	·0122	10	3·87	7	−3·13	−3·58	55
56	·0132	11	3·87	2	+1·87	−1·71	56
57	·0143	12	3·91	3	+ ·91	− ·80	57
58	·0155	13	3·88	3	+ ·88	+ ·08	58
59	·0168	14	3·73	6	−2·27	−2·19	59
60	·0182	15	3·64	2	+1·64	− ·55	60
61	·0197	16	3·60	6	−2·40	−2·95	61
62	·0213	18	3·50	6	−2·50	−5·45	62
63	·0231	21	3·40	1	+2·40	−3·05	63
64	·0252	24	3·37	3	+ ·37	−2·68	64
65	·0276	28	3·28	4	− ·72	−3·40	65
66	·0304	32	3·19	...	+3·19	− ·21	66
67	·0336	37	3·16	3	+ ·16	− ·05	67
68	·0373	42	3·04	6	−2·96	−3·01	68
69	·0415	48	2·72	4	−1·28	−4·29	69
70	·0463	54	2·52	1	+1·52	−2·77	70
71	·0517	61	2·35	4	−1·65	−4·42	71
72	·0578	68	2·14	3	− ·86	−5·28	72
73	·0646	76	1·97	...	+1·97	−3·31	73
74	·0722	84	1·91	...	+1·91	−1·40	74
75	·0806	93	1·81	1	+ ·81	− ·59	75
76	·0899	102	1·44	2	− ·56	−1·15	76
77	·1001	112	1·05	2	− ·95	−2·10	77
78	·1113	123	·83	...	+ ·83	−1·27	78
79	·1236	135	·62	...	+ ·62	− ·65	79
80	·1371	148	·48	1	− ·52	−1·17	80
81	·1519	152	·45	...	+ ·45	− ·72	81
82	·1671	167	·42	...	+ ·42	− ·30	82
83	·1838	183	·37	...	+ ·37	+ ·07	83
84	·2021	200	·30	...	+ ·30	+ ·37	84
85	·2221	218	·22	...	+ ·22	+ ·59	85
86	·2439		·12	...	+ ·12	+ ·71	86
			251·71	251			

TABLE J(*a*).—WIDOWS.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping.*

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
21	1	...	...				
22	2		...				
23	4·5	1	·2222				
24	6	..	...				
25	10	..	..				
26	12·5	..	.				
27	19	..	...				
28	24·5	..	...				
29	27	1	·0370				
30	35	..	...				
31	42·5	2	·0471				
32	44	...	..	21–44	1051	12	·0114
33	45	...	...				
34	45	2	·0444				
35	43	..	...				
36	60·5	...	...				
37	64	..	...				
38	71·5	..	...				
39	74·5	1	·0134				
40	78	...	...				
41	84	1	·0119				
42	86	2	·0233				
43	85	2	·0235				
44	86·5	...	...				
45	99	1	·0101				
46	103·5	2	·0193				
47	106·5	3	·0282				
48	106·5	1	·0094				
49	103·5	3	·0290				
50	105	1	·0095	45–55	1119	16	·0143
51	99	1	·0101				
52	94·5	...	..				
53	98	1	·0102				
54	103	1	·0097				
55	100·5	2	·0199				
56	103	3	·0291				
57	103·5	2	·0193				
58	105·5	3	·0284				
59	107	...	..	56–63	841	18	·0214
60	107·5	3	·0279				
61	105·5	3	·0284				
62	104	2	·0192				
63	105	2	·0190				
64	100	4	·0400				
65	101	3	·0297				
66	99	2	·0202	64–68	502	14	·0279
67	103·5	2	·0193				
68	98·5	3	·0305				
69	94·5	...	...				
70	90·5	8	·0884				
71	79	6	·0759				
72	72·5	6	·0828				
73	68·5	2	·0292	69–78	688·5	44	·0639
74	65	5	·0769				
75	58·5	6	·1026				
76	56·5	4	·0708				

TABLE J(*a*).—WIDOWS.—*Number at Risk, Number Dying, Unadjusted Central Death-rate; and Grouping*—(continued).

Age	Number at risk	Number dying	Unadjusted central Death-rate	Age	Number at risk	Number dying	Unadjusted central Death-rate
77	53	3	·0566				
78	50·5	4	·0792				
79	45·5	5	·1099				
80	41	3	·0732				
81	37	1	·0270				
82	34·5	3	·0870				
83	27·5	7	·2545				
84	19·5	5	·2564	79–88	250·5	32	·1277
85	15	3	·2000				
86	11·5	3	·2609				
87	10	...	...				
88	9	2	·2222				
89	6	4	·6667				
90	3	1	·3333				
91	1·5	1	·6667				
92	1	...	...	89–95	14	7	·5000
93	1	...	...				
94	1	...	...				
95	·5	1	2·0000				
	4466	143			4466	143	

TABLE J(*b*). — WIDOWS. — *Adjusted Central Death-rate, Expected and Actual Deaths; with Differences.*

Age (1)	Adjusted central Death-rate (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)−(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
21	·0110	0	·01	...	+ ·01	+ ·01	21
22	·0110	1	·02	...	+ ·02	+ ·03	22
23	·0111	0	·05	1	− ·95	− ·92	23
24	·0111	1	·07	...	+ ·07	− ·85	24
25	·0112	0	·11	...	+ ·11	− ·74	25
26	·0112	1	·14	...	+ ·14	− ·60	26
27	·0113	0	·21	...	+ ·21	− ·39	27
28	·0113	1	·28	...	+ ·28	− ·11	28
29	·0114	0	·31	1	− ·69	− ·80	29
30	·0114	1	·40	...	+ ·40	− ·40	30
31	·0115	0	·49	2	−1·51	−1·91	31
32	·0115	1	·51	...	+ ·51	−1·40	32
33	·0116	1	·52	..	+ ·52	− ·88	33
34	·0117	1	·53	2	−1·47	−2·35	34
35	·0118	1	·51	...	+ ·51	−1·84	35
36	·0119	1	·72	...	+ ·72	−1·12	36
37	·0120	1	·77	...	+ ·77	− ·35	37
38	·0121	1	·87	...	+ ·87	+ ·52	38
39	·0122	2	·91	1	− ·09	+ ·43	39
40	·0124	2	·97	...	+ ·97	+1·40	40
41	·0126	2	1·06	1	+ ·06	+1·46	41
42	·0128	2	1·10	2	− ·90	+ ·56	42
43	·0130	2	1·11	2	− ·89	− ·33	43
44	·0132	2	1·14	..	+1·14	+ ·81	44

TABLE J(*b*).—WIDOWS.—*Adjusted Central Death-rate, Expected and Actual Deaths; with Differences*—(continued).

Age	Adjusted central Death-rate	$\Delta \times 10^4$	Expected Deaths	Actual Deaths	Difference (4)—(5)	Accumulated Difference $\Sigma(6)$	Age
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
45	·0134	2	1·33	1	+ ·33	+1·14	45
46	·0136	2	1·41	2	− ·59	+ ·55	46
47	·0138	2	1·47	3	−1·53	− ·98	47
48	·0140	3	1·49	1	+ ·49	− ·49	48
49	·0143	4	1·48	3	−1·52	−2·01	49
50	·0147	5	1·54	1	+ ·54	−1·47	50
51	·0152	6	1·50	1	+ ·50	− ·97	51
52	·0158	7	1·49	...	+1·49	+ ·52	52
53	·0165	7	1·62	1	+ ·62	+1·14	53
54	·0172	8	1·77	1	+ ·77	+1·91	54
55	·0180	8	1·81	2	− ·19	+1·72	55
56	·0188	8	1·94	3	−1·06	+ ·66	56
57	·0196	8	2·03	2	+ ·03	+ ·69	57
58	·0204	8	2·15	3	− ·85	− ·16	58
59	·0212	8	2·27	...	+2·27	+2·11	59
60	·0220	9	2·37	3	− ·63	+1·48	60
61	·0229	10	2·42	3	− ·58	+ ·90	61
62	·0239	11	2·49	2	+ ·49	+1·39	62
63	·0250	12	2·63	2	+ ·63	+2·02	63
64	·0262	15	2·62	4	−1·38	+ ·64	64
65	·0277	19	2·80	3	− ·20	+ ·44	65
66	·0296	24	2·93	2	+ ·93	+1·37	66
67	·0320	50	3·31	2	+1·31	+2·68	67
68	·0370	50	3·64	3	+ ·64	+3·32	68
69	·0420	50	3·97	...	+3·97	+7·29	69
70	·0470	50	4·25	8	−3·75	+3·54	70
71	·0520	50	4·11	6	−1·89	+1·65	71
72	·0570	50	4·13	6	−1·87	− ·22	72
73	·0620	50	4·25	2	+2·25	+2·03	73
74	·0670	50	4·36	5	− ·64	+1·39	74
75	·0720	50	4·21	6	−1·79	− ·40	75
76	·0770	60	4·35	4	+ ·35	− ·05	76
77	·0830	60	4·40	3	+1·40	+1·35	77
78	·0890	70	4·49	4	+ ·49	+1·84	78
79	·0960	70	4·37	5	− ·63	+1·21	79
80	·1030	70	4·22	3	+1·22	+2·43	80
81	·1100	80	4·07	1	+3·07	+5·50	81
82	·1180	90	4·07	3	+1·07	+6·57	82
83	·1270	130	3·49	7	−3·51	+3·06	83
84	·1400	200	2·73	5	−2·27	+ ·79	84
85	·1600	200	2·40	3	− ·60	+ ·19	85
86	·1800	300	2·07	3	− ·93	− ·74	86
87	·2100	400	2·10	...	+2·10	+1·36	87
88	·2500	500	2·25	2	+ ·25	+1·61	88
89	·3000	600	1·80	4	−2·20	− ·59	89
90	·3600	700	1·08	1	+ ·08	− ·51	90
91	·4300	900	·65	1	− ·35	− ·86	91
92	·5200	1300	·52	...	+ ·52	− ·34	92
93	·6500	1500	·65	...	+ ·65	+ ·31	93
94	·8000	2000	·80	...	+ ·80	+1·11	94
95	1·0000		·50	1	− ·50	+ ·61	95
			143·61	143			

TABLE K(*a*).—WIDOWS.—*Number at Risk, Number Marrying, Unadjusted Central Marriage-rate; and Grouping.*

Age	At risk	Number Marrying	Unadjusted central Marriage-rate	Age	At risk	Number Marrying	Unadjusted central Marriage-rate
21	1	...	...				
22	2	...	...				
23	4·5	...	...				
24	6	...	...				
25	10	...	...				
26	12·5	1	·0800	21–31	184	2	·0109
27	19	...	...				
28	24·5	1	·0408				
29	27	...	...				
30	35	...	...				
31	42·5	...	...				
32	44	2	·0455				
33	45	1	·0222	32–35	177	5	·0282
34	45	2	·0444				
35	43	...	...				
36	60·5	...	...				
37	64	1	·0156				
38	71·5	...	...				
39	74·5	2	·0268	36–43	603·5	7	·0116
40	78	2	·0256				
41	84	...	...				
42	86	2	·0233				
43	85	...	...				
44	86·5	...	...				
45	99	...	...				
46	103·5	...	...				
47	106·5	...	...				
48	106·5	...	...				
49	103·5	...	...				
50	105	...	...				
51	99	...	...				
52	94·5	...	...	44–60	1732	1	·0006
53	98	...	...				
54	103	...	...				
55	100·5	...	...				
56	103	1	·0097				
57	103·5	...	...				
58	105·5	...	...				
59	107	...	...				
60	107·5	...	...				
	2696·5	15			2696·5	15	

TABLE K(b).—WIDOWS.—*Adjusted Central Marriage-rate, Expected and Actual Marriages; with Differences.*

Age (1)	Adjusted central Marriage-rate (2)	$\Delta \times 10^4$ (3)	Expected Marriages (4)	Actual Marriages (5)	Difference (4)−(5) (6)	Accumulated Difference Σ(6) (7)	Age (8)
21	·0001	14	·00	...	·00	·00	21
22	·0015	17	·00	...	·00	·00	22
23	·0032	19	·01	...	+ ·01	+ ·01	23
24	·0051	21	·03	...	+ ·03	+ ·04	24
25	·0072	23	·07	...	+ ·07	+ ·11	25
26	·0095	25	·12	1	− ·88	− ·77	26
27	·0120	25	·23	...	+ ·23	− ·54	27
28	·0145	25	·36	1	− ·64	−1·18	28
29	·0170	25	·46	...	+ ·46	− ·72	29
30	·0195	25	·68	...	+ ·68	− ·04	30
31	·0220	25	·94	...	+ ·94	+ ·90	31
32	·0245	25	1·08	2	− ·92	− ·02	32
33	·0270	0	1·22	1	+ ·22	+ ·20	33
34	·0270	−40	1·22	2	− ·78	− ·58	34
35	·0230	−35	·99	...	+ ·99	+ ·41	35
36	·0195	−30	1·18	...	+1·18	+1·59	36
37	·0165	−25	1·06	1	+ ·06	+1·65	37
38	·0140	−25	1·00	...	+1·00	+2·65	38
39	·0115	−25	·86	2	−1·14	+1·51	39
40	·0090	−20	·70	2	−1·30	+ ·21	40
41	·0070	−20	·59	...	+ ·59	+ ·80	41
42	·0050	−15	·43	2	−1·57	− ·77	42
43	·0035	−10	·30	...	+ ·30	− ·47	43
44	·0025	− 7	·22	...	+ ·22	− ·25	44
45	·0018	− 4	·18	...	+ ·18	− ·07	45
46	·0014	− 2	·14	...	+ ·14	+ ·07	46
47	·0012	− 1	·13	...	+ ·13	+ ·20	47
48	·0011	− 1	·12	...	+ ·12	+ ·32	48
49	·0010	− 1	·10	...	+ ·10	+ ·42	49
50	·0009	− 1	·09	...	+ ·09	+ ·51	50
51	·0008	− 1	·08	...	+ ·08	+ ·59	51
52	·0007	− 1	·07	...	+ ·07	+ ·66	52
53	·0006	− 1	·06	...	+ ·06	+ ·72	53
54	·0005	− 1	·05	...	+ ·05	+ ·77	54
55	·0004	− 1	·04	...	+ ·04	+ ·81	55
56	·0003	− 1	·03	1	− ·97	− ·16	56
57	·0002	0	·02	...	+ ·02	− ·14	57
58	·0002	− 1	·02	...	+ ·02	− ·12	58
59	·0001	0	·01	...	+ ·01	− ·11	59
60	·0001		·01	...	+ ·01	− ·10	60
			14·90	15			

Table L.—Bachelors and Married Men.—*Marriage and Mortality Table.*

Age	Numbers Living		Decrements caused by			Age
	Bachelors	Married	Bachelors Dying	Bachelors Marrying	Married Dying	
x	$(bl)_x$	$(ml)_x$	$(bd)_x$	$(bm)_x$	$(md)_x$	x
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	1,000,000	...	5,886	...	..	15
16	994,114	...	5,946	...	...	16
17	988,168	..	6,010	...	...	17
18	982,158	...	5,975	...	...	18
19	976,183	...	6,033	...	...	19
20	970,150	...	5,987	2,415	7	20
21	961,748	2,408	6,015	8,115	39	21
22	947,618	10,484	5,907	14,063	108	22
23	927,648	24,439	5,854	20,124	213	23
24	901,670	44,350	5,844	26,564	362	24
25	869,262	70,552	5,862	33,138	564	25
26	830,262	103,126	5,981	37,987	816	26
27	786,294	140,297	6,175	41,177	1,107	27
28	738,942	180,367	6,142	42,860	1,427	28
29	689,940	221,800	5,390	43,260	1,794	29
30	641,290	263,266	4,755	42,609	2,040	30
31	593,926	303,835	4,170	41,128	2,264	31
32	548,628	342,699	3,691	39,013	2,456	32
33	505,924	379,256	3,304	36,683	2,656	33
34	465,937	413,283	3,002	32,492	2,955	34
35	430,443	442,820	2,820	28,731	3,324	35
36	398,892	468,227	2,733	25,321	3,785	36
37	370,838	489,763	2,723	22,319	4,338	37
38	345,796	507,744	2,744	19,671	4,896	38
39	323,381	522,519	2,788	17,355	5,338	39
40	303,238	534,536	2,824	15,325	5,769	40
41	285,089	544,092	2,880	13,536	6,136	41
42	268,673	551,492	2,900	11,963	6,483	42
43	253,810	556,972	2,915	10,573	6,820	43
44	240,322	560,725	2,904	9,343	7,135	44
45	228,075	562,933	2,893	8,254	7,377	45
46	216,928	563,810	2,881	7,286	7,553	46
47	206,761	563,543	2,870	6,426	7,657	47
48	197,465	562,312	2,859	5,661	7,748	48
49	188,945	560,225	2,849	4,977	7,878	49
50	181,119	557,324	2,823	4,366	8,001	50
51	173,930	553,689	2,798	3,822	8,162	51
52	167,310	549,349	2,792	3,334	8,364	52
53	161,184	544,319	2,818	2,897	8,612	53
54	155,469	538,604	2,872	2,521	8,940	54
55	150,076	532,185	2,949	2,198	9,354	55
56	144,929	525,029	3,048	1,923	9,846	56
57	139,958	517,106	3,162	1,692	10,457	57
58	135,104	508,341	3,291	1,513	11,225	58
59	130,300	498,629	3,453	1,381	12,133	59
60	125,466	487,877	3,617	1,279	13,253	60
61	120,570	475,903	3,778	1,181	14,500	61
62	115,611	462,584	3,936	1,086	15,797	62
63	110,589	447,873	4,084	994	17,109	63
64	105,511	431,758	4,230	906	18,403	64

TABLE L.—BACHELORS AND MARRIED MEN.—*Marriage and Mortality Table*—(continued).

Age	Numbers Living		Decrements caused by			Age
	Bachelors	Married	Bachelors Dying	Bachelors Marrying	Married Dying	
x	$(bl)_x$	$(ml)_x$	$(bd)_x$	$(bm)_x$	$(md)_x$	x
(1)	(2)	(3)	(4)	(5)	(6)	(7)
65	100,375	414,261	4,342	821	19,357	65
66	95,212	395,725	4,410	741	20,038	66
67	90,061	376,428	4,444	665	20,454	67
68	84,952	356,639	4,451	593	20,591	68
69	79,908	336,641	4,428	527	20,480	69
70	74,953	316,688	4,374	464	20,159	70
71	70,115	296,993	4,295	407	19,740	71
72	65,413	277,660	4,192	354	19,232	72
73	60,867	258,782	4,072	305	18,646	73
74	56,490	240,441	3,938	261	17,995	74
75	52,291	222,707	3,820	221	17,346	75
76	48,250	205,582	3,749	185	16,769	76
77	44,316	188,998	3,728	153	16,283	77
78	40,435	172,868	3,772	123	15,998	78
79	36,540	156,993	3,837	97	15,951	79
80	32,606	141,139	3,888	74	16,108	80
81	28,644	125,105	3,919	53	15,274	81
82	24,672	109,884	3,905	36	15,809	82
83	20,731	94,111	3,818	23	15,939	83
84	16,890	78,195	3,631	12	15,563	84
85	13,247	62,644	3,324	5	14,610	85
86	9,918	48,039	2,897	1	13,073	86
87	7,020	34,967	2,217	...	11,042	87
88	4,803	23,925	1,752	...	8,725	88
89	3,051	15,200	1,278	...	6,369	89
90	1,773	8,831	919	...	4,579	90
91	854	4,252	568	...	2,829	91
92	286	1,423	286	...	1,423	92

DISCUSSION.

The PRESIDENT (Mr. A. J. Finlaison, C.B.), in opening the discussion, said it had been very difficult to obtain authentic information of the mortality and marriage experience of members of widows' funds and their nominees, but the numerous tables now communicated would add considerably to their knowledge. The valuation of widows' and orphans' funds involved some of the most intricate problems presented to an actuary: and the valuation of funds connected with services where membership was compulsory and the nomination of several successive wives permissible were among the most intricate. A large part of the paper comprised an examination of the mortality and marriage experience of bachelors. These were very interesting subjects, but he had found it undesirable in funds with which he had been concerned to increase the annuities to be paid to nominees of married members by means of an anticipation of the

future contributions of problematical bachelors. Bachelors forsook their employment to a much larger extent than married men. Credit should of course be taken for all the contributions of bachelors actually received, but this class of revenue should not in his judgment be anticipated. Even if the future and contingent contributions of bachelors were reckoned upon to swell the annuities of widows and orphans, it would be found in most cases at a valuation—although in the Scottish banks Mr. Hewat stated that the benefits in the case of bachelors were considerable in proportion to the contributions—that the future contributions of a member who was then unmarried would be of greater worth than the contingent annuities payable to his possible widow and orphans. In other words, the value of the policies of that class of members would be negative, and the correction which should then be applied to the values would neutralize all the elaborate calculations which might be deduced from the probabilities of the marriage and re-marriage of bachelors during their service. The interesting tables of the rates of mortality among married men, their wives and widows, and the rate of re-marriage among the latter were of much greater practical importance than the tables relating to bachelors. They were told that in the Scottish Banks Widows' Funds the widows' annuities ceased in most cases on re-marriage. The extent of relief experienced by widows' funds from the cessation or suspension of annuities on re-marriage had been the subject of much discussion, and any information upon it was very welcome. The figures given tended to show that the amount of relief experienced by the Scottish Banks Fund from this cause was not so great as had been anticipated in other widows' funds. The author showed that out of 364 widows no more than 15 had hitherto re-married, or little more than 4 per-cent. Many of these ladies were no doubt still eligible for re-marriage, but thus far the percentage did not seem to indicate any considerable relief to the funds from this cause. It had been much greater in widows' funds connected with the British Indian Government Service and the Colonial Service, and also, he believed, in the funds connected with the British army. Tables were given showing unadjusted and adjusted marriage rates, but 15 marriages were too few to base any trustworthy estimates upon. He wished, however, to point out the low re-marriage rate shown in the adjusted tables from widows under 30 in comparison with the marriage rate between 30 and 35. This was contrary to the experience of a fund with whose affairs he had been acquainted for more than 30 years, in which the re-marriage rate among widows was highest between 20 and 24, and then diminished rapidly in each succeeding quinquennium of age until it ceased altogether between the ages of 50 and 54.

Mr. G. F. Hardy said the paper put them in possession of reliable statistics, having reference to special classes of lives. Their thanks were due to the authors for the labour expended in obtaining their results, and for the complete form in which they had been presented. They had been fortunate in finding a sufficient body of facts for their purpose. Professor Chrystal, in a paper read some time back at the Actuarial Society of Edinburgh, had almost ruled out of existence the problem of inverse probabilities; but in cases like the

present it often existed in a very practical sense. The actuary had frequently to decide between the use of data obviously inadequate or standard tables that might or might not be applicable. In the present paper, for example, it was found necessary to amalgamate the mortality experience of married men and widowers owing to the insufficiency of the facts relating to the latter. The paper very well showed the power of the graphic method of graduation in dealing with a limited experience, and they would all agree that in this part of their work the authors had been eminently successful. At the same time it brought out what was perhaps a weakness in the method, that in view of the smooth curves produced by the adjustment they were apt to lose sight of the slender basis of fact upon which these sometimes rested, especially as to their minor features. He might illustrate this by reference to the remark made by the authors that the mortality rates among wives were heavier over age 71 than among widows. This appeared very plainly from the graduated results, but would be found, however, to depend upon only nine deaths of wives above that age. A striking feature of the results was the very low mortality among the female lives, which, taken as a whole, were throughout better than the Government Annuity Female Experience. The widows at the younger ages were, however, subject to a somewhat higher rate. Some light was thrown upon this point by a remark made by Mr. Hewat that the heavy mortality was mainly confined to the first year or two of widowhood. This cause would operate upon a general table which took no account of duration of widowhood, in the same manner but in an opposite direction to the effect of selection upon a table such as the H^{M}. Its effect would therefore be to understate the value of annuities upon the lives of commencing widows, and this would be a point of some importance. While the series of tables produced by the authors were of great interest, the financial problems involved might perhaps have been dealt with more simply. Given certain rates of mortality, marriage, &c., operating fairly constantly, it should follow that among the numbers surviving to successive ages the proportion of married, the average age of their wives, and the average liability that would be created by the death of each member, would be practically a function of the age. A table giving this information for each age and enabling them to deal with the resultant of these various forces, in which their fluctuations would be to a large extent balanced, would probably give better results. There were two points in which the authors might perhaps supplement the valuable information they had already given. First as to the extent to which the duration of the annuities was increased by their continuance in certain cases to children after the death of the widow—and, secondly, as to the rates of withdrawal and their financial effect. In most of the problems dealt with by the actuary it was inadvisable to take this latter element into account, but in funds where membership was compulsory and withdrawals from the fund were not entirely voluntary, the rates of withdrawal were more stable and it might be legitimate to give weight to them. He would further say that the paper would be rendered more valuable and the results would be more easily grasped if the latter were set out graphically. It would be much easier to take in the main features of

the tables if this were done, than by an examination of the numerous columns of figures by which the results were at present represented.

Mr. G. H. Ryan said that the construction of a new table from original observations was such a decided gain to their common store of knowledge, that the members would feel under a special debt of gratitude to those who—like Mr. Hewat and Mr. Chatham on the present occasion—gave them the results of their work in this direction. With each new and original table the future work of the profession was placed upon an ampler and surer foundation, and any writers who might subsequently concern themselves with similar problems would have cause to be grateful to Mr. Hewat and Mr. Chatham for their paper. The first thing that struck him in examining the new probabilities of death and marriage which the authors had submitted was the very remarkable way in which they differed from all other standards, which again seemed to differ to a great extent *inter se;* and he thought in the face of this divergence they would not be justified in placing too much confidence in the new probabilities, more especially as the basis on which they rested was so slender. The tables might be the best that were available in the present condition of their knowledge, but he should think it unwise to depend too much upon them, or upon the monetary results arrived at by the use of such an instrument. The authors had stated that valuation tables based upon the new statistics were in course of preparation; and he ventured to hope that they would see their way to submit them to the Institute and to discuss the processes of valuation to be adopted in connection with widows' funds. It was a very curious thing that the important question of the valuation of widows' funds had on no occasion been treated by contributors to the proceedings of the Institute. In Dr. Sprague's 20-Volume Index there was absolutely no reference to the question of widows' funds. He (Mr. Ryan) thought it would be well if in matters where their professional interest coincided with great public interests, which was certainly the case in widows' funds, some Fellows or Associates would take such subjects into consideration. He might add that the Index to the *Journal* contained no entry relating to superannuation schemes, which was again a matter of great public importance. The members would find a most interesting discussion of many points connected with this matter by Dr. Sprague and Mr. Sutton—a kind of battle-royal between them, in fact—in the Minutes of Evidence on the School Board of London Superannuation Bill (Parliamentary Paper, No. 350, 1891). There was an appendix to the return which gave some useful original tables constructed by Mr. Sutton for the special purpose in hand. Students would find these tables of great interest, and no doubt often of service to them. He would suggest that members of the Institute desirous of contributing to their proceedings should take up such subjects as he had referred to and give reversions and graduation a long rest. By such means a greater variety would be given to their meetings, and good work done for the Institute.

Mr. R. P. Hardy thought they were indebted to the authors for bringing before them the almost-forgotten subject of pension and superannuation funds. Those funds were curious survivals of the

mediæval spirit. They were brightened at once by all the virtues, as they were disfigured with all the narrowness that attended the earlier stages of society, when association naturally grouped itself round particular rather than common interests. No doubt in that elementary way a certain amount of discipline was brought into the question of association, but it brought in its train the concomitant evil of an uncharitable exclusiveness, which was sufficiently apparent in those now happily nearly obsolete guilds. Man was learning slowly but gradually that all fruitful association must be with his brethren generally, and that all artificial limitations of interest or sympathy were opposed to the common good, and defeated the ends of their promoters. Taking such statistics, for example, as Mr. Hewat had brought them, they had a clean cut section of society, the composing units of which acquired a strange family resemblance. There was almost the microcosm within the macrocosm. They might see almost instantaneously and progressively the main elements with which the lives of the members had been moulded. They could view the operation of the many contributory causes either in a single concentrated and final result, or they could watch the slowly emerging series, each term of which grew in distinctness until it materialized in the phenomena sought. It was most interesting to trace the individual from opening manhood, with its ardent desires and its ill-regulated hopes, through all the stages of a human being's career, until that individual became fused with the heterogeneous past and became a citizen of what had been finely termed "the famous nations of the dead." Nothing lay concealed before them. The marriages, even those second marriages which a cynic styled "triumphs of hope over experience", were there, and they could trace the career of the man rising from the mere wages of 10*s*. or 15*s*. a week until a very substantial solatium had been reached. Those were the strange facts which came before them for impartial averaging or digesting into the ultimate controlling factor of measurement. These were what Zola styled "human documents", replete with pathetic interest, and fragrant with romance. It was from materials such as these that the future social historians would construct their histories. Dealing with the paper, he regretted that he might find himself at variance with the authors with reference to the method in which the partial probabilities should be dealt with. The proper way of measuring the liability under those funds was, as he believed, by a single expression that gathered up the various contributing probabilities or showed the point at which the function came to rest. He was glad to find he had the support of Mr. George Hardy. He would ask whether those problems were not of surpassing interest, and whether they did not call for the exercise of qualifications, both original and acquired, that were scarcely to be found in a single individual, and whether they did not afford scope for a cultivated ingenuity for the devisal and control of methods wholly beyond the dreams of the text-books. It would be ungracious to dwell on the necessary imperfections of tables drawn from limited data, but if he was asked to use those results as anything more than friendly guides he would have to exercise discretion. It was enough to point to the great variations in the marriage rates which Mr. Hewat and other observers had recorded, and to allude to

the scanty number of facts from which some of his deductions were drawn, or to the length of time they went back. He (Mr. Hardy) was content to have his experience of already available rates corrected or confirmed. He felt sure that the experience the authors had gained would not be lost, and would bear further fruit which would appear in the transactions of the Institute.

Mr. G. KING agreed with the Messrs. Hardy that the most convenient course in dealing with funds such as those referred to in the paper was to find out what liability was imposed on the funds on the death of the members, or when they otherwise went out of observation, by certain benefits to be received. In that way an immense amount of preliminary work was got rid of, and the necessity of dealing with rates of marriage and similar functions was avoided. He had shown, in his paper on "Family Annuities" (*J.I.A.*, xxx. 291), how the liability for an annuity to children could be obtained, and the liability for an annuity to a wife was of a similar nature. Could Mr. Hewat get some statistics from the funds he had under investigation which would help in that direction? If the authors could investigate further and ascertain the matrimonial state of the members as they went off observation, and ascertain their state as regards young children, they would be conferring a great favour on the Institute, and be giving most useful material for various investigations. In the paper, the authors said, as regards the "Existing", that the usual course was to take the dates as the anniversary in the year of membership preceding the close of observations, but that they had taken the anniversary in the succeeding years. That did not convey any clear idea to his mind. The close of the observations must either be a fixed date or the anniversary of a contract, and he did not see how the authors could mix the two. Again, how did the authors take the numbers existing as exposed to risk over half a year? He did not follow their references to the central death rates. It seemed to him that in the way it was done it might lead to an error. The authors did not explain how they got the rates that were given in later tables from the central death rates, but he assumed that they got them by the usual formulæ applicable to a mortality table where there were no secessions of any kind, and he was not sure that these formulæ were applicable when there were many secessions. In dealing with funds where it was required to get the probabilities of death and withdrawal while people were members, the matter must not be gone about in the same way as for the ordinary mortality table. When the rates of mortality for an ordinary table of mortality were obtained, these were the rates for a society where there were no withdrawals, but those rates were necessarily rather higher than the rates which prevailed during membership in a society where there were withdrawals. If they wished to value a society, allowing for withdrawals of various kinds, the matter must be gone about differently, and those who withdrew must be treated as being at risk for a whole year, instead of only for a half-year. When the work was done in that way they would get a table which accurately represented the facts—not the theoretical rate of mortality in a society where there were no disturbing elements, but the actual rates in those particular funds where valuation had to be made allowing for

withdrawals. In that way they got the same denominator for the ratio of withdrawals as for that of the deaths, and thus not only were the tables more accurate, but they were much more convenient. The paper was all the more welcome in that the authors spoke of a continuation of their labours.

Mr. W. O. Nash said that some of the speakers were of opinion that it was better to give the average cost of the benefit at each age instead of the elements on which that average cost depended. In the valuation of a fund of the character referred to in the paper, it was more convenient to proceed in that way, but they ought to give the authors full credit for having given them the underlying elements, because it was very convenient sometimes to have them separately. That was especially the case where they were dealing with sickness, mortality and secession. If they wanted to assume that the sickness altered without the mortality altering, or the rate of secession altered while the other elements remained the same, or to combine the same rate of sickness with two different rates of mortality, unless they had the elements they were unable to proceed. Therefore they ought to be grateful to the authors for having given them the elements leading up to the average cost which had been spoken of.

The President having proposed a vote of thanks to the authors, which was unanimously accorded,

Mr. Hewat, in reply, acknowledged the vote of thanks which had been given, and said no doubt, from the actuaries' point of view, it was very interesting to have statistics of some 3,700 lives, but when they came to be sub-divided into several more or less small classes the 3,700 was not quite so important after all. With regard to the graphic method, it had been adopted on this occasion after very careful consideration, but as the unadjusted results had been given in the tables, the members could form their own conclusions. The graphic method, no doubt, brought out a comparatively smooth line. No table yet published could claim to be perfect—the data on which it was based were either unreliable or too scanty. It was not likely a table would ever be framed showing what may be termed a "true" rate of mortality. As to the orphans, it was extremely difficult to get any conclusion from the statistics. If a widow died or re-married the children came on the fund in her place, but very often the children were grown up before the death or re-marriage of their mother. The idea of supplying a diagram to illustrate the results would not be lost sight of when these came to be published in some other shape. It would seem from remarks made that a good deal more was wanted from the results of the present investigation, but in Scotland they had not much time to spare for work outside of their official duties. With regard to the pension schemes referred to by Mr. Ryan, he (Mr. Hewat) had been consulted a good deal as to the framing and working of such. About 14 years ago he prepared a scheme and was instructed to show what it would cost each year during the next half century. He accordingly prepared the necessary table, and had been gratified to hear lately that during the quarter of that period which had already passed since the scheme came into operation the sums indicated in that table were practically identical with what it had actually cost

—*i.e.*, the *experience* had, thus far, confirmed the *expectation.* With regard to the blue-books referred to by Mr. Ryan, he (Mr. Hewat) had spent much profitable time in reading them. In reply to Mr. King's remarks about "preceding" and "succeeding", he had expected that point to be raised, and was prepared for it. It should be borne in mind that it was those who passed out of observation which were being dealt with, and it was assumed that these members had been on the registers on an average for six months in the year in which they withdrew. In their formula and its application they (the authors) had taken into account the existing, the withdrawals, the marriages, and the deaths of the bachelors, and they thought they had adopted a method which was suitable in the circumstances. He had been greatly interested in Mr. R. P. Hardy's remarks, because he knew what experience Mr. Hardy had in these matters. As regards widows' funds in general the bachelor members were apt to look upon the actuary with some disrespect, if not suspicion. It had been pointed out to him (Mr. Hewat) more than once that it was rather hard on the bachelors that they should have to contribute to such a fund. One member had said that if he died his sister would get nothing. The obvious answer to which was that such a provision could best be made by means of life assurance. He was in the habit of pointing out that bachelors might at any time change their condition, and then come to see the advantage of a widows' fund. A change of position generally leads to a change of view. He also congratulated the bachelors that they did not live in the palmy days of Rome, when the State thought proper to tax all bachelors to the relief of married men.

NOTE.

With regard to the remarks of several speakers, it should be borne in mind that the paper dealt with the mortality and marriage experience, and not with that of the withdrawals. The rates of withdrawal would probably be of little use to the Institute, as the rates differ so much not only in different bodies, but also at different times in the same body. A great deal had been said about the financial aspect of the question, and the tabulation of the materials as a function of the age, but that was a part of the subject outside the scope of the paper submitted. There is no doubt that a tabulation of the materials in that way would be valuable for valuation purposes, and the rates now given would be useful for that purpose; but it is questionable if such a table would be of much use to the Institute. Not only does the rate of withdrawal differ, but the age at marriage, the age of the wife, &c., are often essentially different. A table like that referred to requires to be used with caution, even in the valuation of a fund from whose experience it has been deduced, in case changes may have taken place which would render it untrustworthy.

With regard to Mr. King's remarks, as a fixed date was taken for the termination of the observations in the various funds, coinciding with the close of the financial year in each, it followed

that the "existing" would be exposed to risk for broken periods varying from 1 to 364 days in the year of membership in which they passed out of observation; that is to say, they were exposed to risk for six months on the average. When the cards came to be tabulated, the "existing" were tabulated as if they were exposed to risk during the whole of the year of membership in which they passed out of observation—that is to say, they were taken to the anniversary of the date of entry after the close of the observations, and one half of the number deducted afterwards, so as to obtain the number exposed to risk. The more usual course would have been to take them to the anniversary preceding the actual close of the observations: but this would have necessitated the leaving out of account the deaths, marriages, &c., corresponding to the period omitted. In this way a considerable body of facts would—as mentioned in the paper—have been lost, which was not desirable in the present instance, and the formula for obtaining the number at risk would not have been so simple. Had the rates been deduced in the manner mentioned by Mr. King, they would, of course, have differed from the rates usually obtained, and it would not have been possible to make comparisons.

A. H.
J. C.

LAW REPORTS.

*HAMBROUGH
versus
THE MUTUAL LIFE INSURANCE COMPANY OF NEW YORK.

IN THE SUPREME COURT OF JUDICATURE OF ENGLAND.
In the High Court of Justice.
QUEEN'S BENCH DIVISION.

DUDLEY ALBERT HAMBROUGH
(suing as Administrator of the Estate of WINDSOR DUDLEY CECIL HAMBROUGH, deceased)
versus
THE MUTUAL LIFE INSURANCE COMPANY OF NEW YORK.

PLEADINGS.

The writ was issued on the 2nd March 1894, and was endorsed with the Pleadings.

STATEMENT OF CLAIM.

The plaintiff's claim is £20,000 upon two policies of insurance upon the life of Windsor Dudley Cecil Hambrough, deceased. Statement of Claim.

* [Abridged from the transcript of the Shorthand Notes of Messrs. Cherer, Bennett & Davis, 8, New Court, Carey Street, W.C., which has kindly been placed at our disposal by the Mutual Life Insurance Company of New York, the Defendants in the suit.—Ed. *J.I.A.*]

Pleadings—*continued.*

Statement of Claim.

Particulars.

To amount due from the defendants to the plaintiff as administrator of the estate of Windsor Dudley Cecil Hambrough, deceased, under two several policies of assurance for £10,000 each on the life of the said Windsor Dudley Cecil Hambrough, granted by the defendant company on or about the 4th day of August 1893 . . . . £20,000

The plaintiff also claims interest on £20,000 at £5 per-centum per annum from the date hereof until payment or judgment.

STATEMENT OF DEFENCE.

Defence.

1. The defendants say that subject to the two policies being produced by the plaintiff and put in evidence by him at the trial they admit the execution by them of the said policies. They do not otherwise admit the same or the contents or effect thereof.

2. They do not admit that the plaintiff is the administrator of the estate of the said Windsor Dudley Cecil Hambrough, deceased.

3. They deny that the plaintiff as such administrator or at all was or is entitled to the two said policies or either of them or any moneys alleged to be payable thereunder. If he ever was so entitled, he has since ceased to be so before the commencement of this action or he holds the same as trustee for other persons, and this action is not brought for their benefit or with their authority.

4. Alternatively the said policies and the moneys alleged to be payable thereunder have been assigned or belong to some or one of the persons mentioned in certain notices in writing which have been sent to the defendants. Particulars of the said notices with the dates thereof and the names of the persons claiming thereunder appear in the schedule to an affidavit made herein by Peter Williams, the defendants' solicitor, sworn on the 14th day of April 1894, and filed on the 18th day of April 1894. The defendants refer to the said schedule as particulars.

5. Further in the alternative the policies were taken out by the deceased for the benefit of one Mrs. Monson to secure advances made or to be made by Mrs. Monson to the deceased and were assigned by the deceased to Mrs. Monson in his lifetime, and notice thereof was given to the defendants, and the said policies and the moneys alleged to be payable thereunder never formed part of the estate of the deceased and the said policies always were and still are in the possession of Mrs. Monson or persons claiming through her, who dispute the claim of the plaintiff to the said policies or money.

Pleadings—*continued.*

Defence.

6. The said policies were obtained by the fraud of the deceased and of one A. J. Monson acting as agent for the deceased.

Particulars of Fraud.

(*a*) False and fraudulent verbal representations made by the said A. J. Monson to the defendants on and between the 2nd and 8th days of August 1893, to the effect that the deceased had purchased an estate called Ardlamont, which purchase was to be completed on the said 8th day of August, and that Mrs. Monson had advanced or was about to advance large sums of money for the purpose of the said purchase, and that the policies were taken out for the purpose of securing such advances. and that he, the said A. J. Monson, was guardian and trustee for the deceased, who would succeed to a fortune of £200,000 on coming of age.

(*b*) False and fraudulent representations that no application for the insurance of the deceased's life had ever been declined or postponed. The said representations were made verbally by the deceased and the said A. J. Monson on the 2nd of August 1893, and in writing by the deceased. which writing consists of an application for insurance signed by the deceased and dated the 2nd of August 1893.

(*c*) The deceased and the said A. J. Monson fraudulently concealed from the defendants that the person whose name, E. M. Hiron, was given in the said written application as that of one of the intimate friends of the deceased was in fact the nursery governess of the said A. J. Monson. She was not a friend or an intimate friend.

(*d*) A false and fraudulent representation made verbally and in writing by the deceased to the medical examiners of the defendants on or about the 2nd and 4th of August 1893, that no unfavourable opinion upon the insurableness of his life had ever been given by a physician. The like verbally on the same occasions by the said A. J. Monson.

7. Alternatively the deceased agreed by the aforesaid application that all the statements in that application, as well as those he should make to the defendants' medical examiners in continuation of the application, were by him warranted to be true and were offered to the defendants as a consideration for the contract contained in the policies, and by the terms of the policies the applications were made part of the contract. The following statements

Pleadings—*continued.*

Defence.

and answers contained in the said application and made to the defendants' medical examiners were untrue.

(*a*) That no application for the insurance of his life had ever been declined or postponed.

(*b*) That his life had never been insured.

(*c*) That E. M. Hiron was one of his intimate friends.

(*d*) That no unfavourable opinion upon the insurableness of his life had ever been given by a physician. The application is dated the 2nd of August 1893, and the statements and answers to the medical examiners are dated the 2nd and 4th of August 1893.

By reason of the premises and by the terms of the policies the same were and are void.

R. M. BRAY.

Delivered this 30th day of April 1894, by FRESHFIELDS & WILLIAMS, of 5, Bank Buildings, Lothbury, in the City of London, Defendants' Solicitors.

PARTICULARS OF APPLICATIONS TO INSURANCE COMPANIES.

Particulars

1. The applications for insurance of the life of Windsor Dudley Cecil Hambrough are, so far as at present known, those appearing in the following schedule. The dates and results of such applications are also stated where known.

	Date of Application	Name of Office	Amount of Insurance	Result
1	Dec. 1890	Reliance Mutual	Not known	Postponed
2	March 1891	Legal and General	Ditto	Not proceeded with
3	1891	Ditto	Ditto	Ditto
4	May 1891	Scottish Amicable	£5,000	Ditto
5	28 May 1891	Caledonian	£5,000	Declined
6	Nov. 1892	British Empire	Not known	Not proceeded with
7	Nov. 1892	Scottish Amicable	£2,000	Declined
8	May 1892	Liverpool, London & Globe	£15,000	Not proceeded with
9	28 July 1893	Ditto	£50,000	Withdrawn
10	31 July 1893	Ditto	£26,000	Declined

2. The defendants are unable at present to give the names of the offices in which Windsor Dudley Cecil Hambrough was insured. Pleadings—*continued.* Particulars.

3. The names of the offices, the physicians of which gave an unfavourable opinion, with date, are as follows:

The Reliance Mutual Life Assurance Society, in the months of December 1890, and January and February 1891.

Delivered this 19th day of July 1894, by FRESHFIELDS & WILLIAMS, 5, Bank Buildings, E.C., Defendants' Solicitors.

REPLY.

The plaintiff takes and joins issue on the defence except so far as the same consists of admissions, and in further answer to paragraph 5 of the defence says that the deceased Windsor Dudley Cecil Hambrough had not attained the age of 21 years at the times therein referred to, and died before he had attained the age of 21 years. Reply.

W. P. G. BOXALL.

Delivered the 16th day of June 1894, by PRINCE & PLUMBRIDGE, of 9, Fleet Street, E.C., Solicitors for the Plaintiff.

The case came on for trial on the 3rd December 1894, before the LORD CHIEF JUSTICE OF ENGLAND (Lord Russell of Killowen) and a Special Jury.

Mr. W. WILLIS, Q.C., Mr. BOXALL and Mr. ROSKILL for the Plaintiff.

Mr. J. P. MURPHY, Q.C., Mr. R. B. FINLAY, Q.C., Mr. R. M. BRAY and Mr. J. HARVEY MURPHY for the Defendants.

Mr. HUGH STURGES watched the case on behalf of Mr. A. J. Monson.

Mr. KING FARLOW, Mr. LE BRETON, Mr. LACEY SMITH and Mr. GOLDNESS watched the case on behalf of other persons interested.

* * * *

The LORD CHIEF JUSTICE: Gentlemen of the jury, this is an extraordinary case, and presented to you under extraordinary circumstances. The plaintiff is suing as administrator of the estate of his deceased son in respect Lord Russell, L.C.J.

Lord Russell, L.C.J.—*continued.* of two policies of insurance said to have been properly effected with the defendant company, each for the sum of £10,000, dated the 4th of August, but in fact executed on the 5th of August 1893. The plaintiff is not, and apparently never has been, in possession of those policies at all. Each of them was produced here by witnesses called upon a process known as *subpœna duces*. I am not going to ask you any question as to the plaintiff's right to sue if these policies are valid policies; but I think that the position of things, and also the previous history (and a very strange one it is) of attempts sometimes successful, sometimes not successful, to insure the life of Cecil Hambrough has some bearing on the question, what is the true view to take of Mr. Monson's position in relation to the effecting of the policies in question?

This young man's history, which we have only had in a very sketchy fashion, it is not necessary to say much about. What we do know we know mainly from the evidence of Miss Hiron, a nursery governess in the employment of the Monsons. He was born in 1873 in the month of May and had no estate or means in possession. There were three estates in respect of which he had a considerable interest. The estates were in the Isle of Wight and Middlesex. Of those, his father, the present plaintiff, was tenant-for-life, and he was tenant-in-tail in remainder, and if he had lived to come of age, that is to say to be twenty-one, he would have been in a position, by concurring with his father, to break the entail. There is one other estate, the Pipewell Estate, as to which he had an interest of a different kind. That interest was not expectant on the death of his father, but, as I gathered, on the death of his uncle. It is not very material to dwell upon that. His intimate acquaintance with the Monsons began about the year 1890. What his exact position was, whether he was boarded at the house or taught at the house, we are not clearly told, but from 1890 he lived continuously under the roof and as part of the family of the Monsons, and from that period till the event of his death on the 10th of August 1893, he was in close and intimate relationship with these persons, and he was engaged in conjunction with one or other of these persons in a number of attempts of insurance upon his own life. He seems from 1890 to have been entirely impecunious, and to have been in part financed through the Monsons by the aid of a person who has been described as a money-lender, Mr. Loftus Tottenham. At the period with which we are chiefly concerned here, there is no doubt his means had fallen to a very low ebb, that Tottenham was very slow in making advances, and some letters which have been put in show that demands were made for very small sums indeed from Mr. Tottenham. From 1890 till 1893 there seem to

have been insurances, or attempts at insurances, or enquiries with a view to insurance at as many as eight or nine offices.

The first proposal was made to the Reliance in December 1890, and was signed by Cecil Hambrough, but the matter was put forward by Tottenham. Again, in March 1891, the proposal is signed by this young man and put forward also by Tottenham to the Legal and General. In May 1891 there is a proposal to the Scottish Amicable, signed by Cecil, Monson is made the referee, and the correspondence shows that Monson was the person who was negotiating with the insurance company on the subject of that intended insurance. That was the first proposal made to the Scottish Amicable; there was another in October 1892. The proposal again was signed by the deceased Cecil Hambrough, but again the correspondence is carried on by Monson with the addition of the fact that a portion of the correspondence is in the handwriting of and signed by Mrs. Monson. In May 1891, an application was made to the Caledonian; there the proposal is signed by Cecil Hambrough; Monson is again one of the referees. In November 1892 an application is made to the British Empire; that was a proposal to insure for the benefit of Mrs. Monson. The proposal is signed by the deceased Cecil Hambrough. In July 1893 there is an application to the Scottish Provident—a very large proposal—£50,000. That again is a proposal for the benefit of Mrs. Monson; but again the correspondence is carried on by Mr. Monson. Then comes the Liverpool, London and Globe proposal, as regards which the correspondence begins as early as May 1891, and that proposal was unquestionably declined by the decision of the Liverpool, London and Globe directors in Liverpool, on the 1st August, and the communication of that decision was conveyed by telegram on the morning of Tuesday, the 2nd August. As regards the policies which are the subject matter of this enquiry, we know that those policies were executed by Mr. Haldeman, provisionally to cover the life for a period of sixty days. They were executed on the 5th August and reached Glasgow upon the 7th August, and on the 7th or 8th were, acting upon the authority in writing of Mr. Hambrough, delivered to Monson for his wife, Hambrough stating in his communication to the office that he had assigned for proper consideration his interest in those policies to Mrs. Monson. We know the subsequent events which gave rise to this question. On the 10th August, this young man meets with his death. The learned counsel for the plaintiff correctly stated there is no ground apparently for suggesting suicide; therefore he met his death either by the accidental discharge of his own or somebody

Lord Russell, L.C.J.—*continued.* else's gun, or by the intentional discharge of a gun. We are not in any way concerned in this case to attempt to penetrate that mystery. So far as this is concerned, you cannot—when you come to the question of fraudulent misrepresentations—shut your eyes to the considerations which may have prompted the making of those fraudulent misrepresentations, if they were made, which you will have to determine; but so far as the most important question in this case is concerned, namely, the question of what is the legal effect of the answers which the deceased himself made, and which he signed with his own hand—the question, as it has been called, of warranty or condition on the one hand or collateral agreement on the other hand, so far as that is concerned, I quite join in the suggestion made to you by the learned counsel for the plaintiff that you should consider it as if it were the only question in the case. And in considering that, you ought not to be affected directly or indirectly by any suggestion which may be occurring to your mind of fraudulent misrepresentations inducing this contract—those must be considered by themselves and upon their own merits.

Now, the defendants are called upon to pay this insurance, or to give a legal answer why they will not pay. They give two answers. They say, first of all: He, the deceased, warranted that certain statements to which he put his name were true, and thereby made those statements the basis upon which the contract or policy was to rest, and we contend that if those statements were untrue, then there is an end of this policy. Now, I shall, at the proper time, as I conceive it, express my view on the point of law; but so far as you are concerned I propose to ask you only the question of fact, and that is, is there or is there not a mis-statement of fact in the proposal signed by Hambrough or in the answers which he made to the medical examiner? Now, gentlemen, that brings us to the first, and, as I conceive, the more important of these questions. You recollect what happened. On the 2nd of August Mr. Monson came again to the defendants' office in Glasgow in the morning. He had made an enquiry at the Globe office as to whether the proposal then under consideration by the Globe office was accepted or declined. He had received an answer to a telegram which he himself had sent, and sent in the name of the Globe from Glasgow, that it had been declined. That answer he undoubtedly received before he paid his second visit to the defendants' office. On that occasion he discussed the matter with Mr. McLean, in the first instance young Cecil Hambrough not being there, but later on Cecil Hambrough comes. Before Cecil Hambrough is come,

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Mr. Monson has undertaken to answer questions which would more properly be questions to be answered by the person whose life was to be insured; but what is stated to have taken place, and what you must accept as being the fact is, that McLean filled up the answers to those questions before Cecil Hambrough appeared; that when Cecil Hambrough appeared he read out the questions and read out the answers which had been suggested by Mr. Monson; he then handed the paper to Cecil Hambrough, who read them, and himself assented to those questions and answers, and signed his name to them; and you must assume, as a fixed datum, that those are statements which Hambrough made just as distinctly as if Monson had never appeared upon the scene at all. Those questions are: Full name, residence, present occupation, address, when he was born, for whose benefit the insurance is to be effected, &c. Then comes the important question: The following are the companies or associations to which I have ever applied for any life insurance which has been refused on the plan asked for or postponed. Answer: I have never applied (I am enlarging it and explaining—you will say whether correctly or not) for any life insurance as to which my application has either been declined or postponed. The defendants say it is untrue in both respects; that as regards the Reliance, that was an application which was postponed; as regards the Globe, that was an insurance or an application for insurance which was refused.

Now the facts in relation to the Reliance may be stated to you very briefly. The story of the Reliance is told you principally by Mr. Wilkinson of the Reliance Office, and by Dr. John Ogle. Mr. Wilkinson tells us that application was made in May 1890 to insure the plaintiff's life through Tottenham, and declined. On the 23rd December 1890 comes the proposal signed by Cecil Hambrough, who was examined on four or five occasions by Dr. Ogle, by Dr. Pye Smith, by Dr. Ramskill, those examinations being on the 23rd December, 15th January, 28th January, and the 12th February. Therefore the examinations were extending from the day on which the proposal was made until the middle, or towards the middle, of the month of February 1891. Now, what caused this (to use a neutral word for a moment) delay was this: One of the doctors, who was yesterday in the witness-box, came to the conclusion that there was albumen in the urine of this young man, which would be in some views of it a very serious matter to consider in connection with life insurance and in any event would be a matter that would require caution and further consideration and examination. Accordingly, this gentleman reports that he discovered

Lord Russell, L.C.J.—*continued.* this presence of albumen. Apparently Dr. Pye Smith does not differ from him. Dr. Ramskill a few days later takes a different view, but it is not unimportant to bear in mind that although he came to the conclusion that there was not albumen on the day on which he examined, which was January the 25th or 28th, he does this: He is asked to classify the case, and instead of simply saying "first class", he says "on the papers and examination, first class", and appends this note: "If on re-examination there should be found intermittent albumen he should be placed in the third class, and he ought to be examined again and more than once." You will, therefore, observe that there is not a marked conflict of opinion between Dr. Ogle on the one hand and Dr. Ramskill on the other, because it may be an intermittent presence of albumen. Now, gentlemen, what did the company do? The company have this before them. If there is, so to speak, no hitch in the business, and if it is a perfectly good life, as to which no question has arisen, insurance companies, as indeed this case illustrates in several of its circumstances, are only too anxious apparently to accept risks. If there is no hitch, therefore, the policy would be carried through in the course of a few days, and you must ask yourselves this question: What is the meaning or object in putting these questions whether there has been any postponement in dealing with the application for an insurance? Is it too much to say that that would be what a prudent company would do where any question of doubt arises, where they are willing enough to accept the risk if they can see their way safely to do it, but think that they must wait a little longer? They must have the result of the further examination and have the benefit of further reports at a later period before they arrive at a definite conclusion. In other words, ask yourselves, is not this question addressed in order to elicit the fact of whether there is or has not been by and in relation to the postponement of an application for an insurance something out of the usual ordinary course which has called for the attention and further examination of the insurance company? Now what did the company do? They got the various reports of the medical men from time to time. The first of these examinations I have told you was on the 23rd of December. On that same day the report came before the Board, and the minute of the Board, which is the authentic evidence of the decision that the Board arrived at, is headed: "Cecil Hambrough; to come up again." On the 29th of January the minute is "Deferred for another examination in a fortnight." 12th February, "Further deferred." Gentlemen, you must ask yourselves, if that is not a postponement of acting

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upon an application for insurance, what is it? Is there in this case any difference between deferring acting upon an application or postponing acting upon an application—you must say. Counsel for the plaintiff has suggested—and I should be glad if you had appreciated it more fully than I did—that there was some difference between deferring acting upon an application and postponing acting on an application; and that postponing involved some decision on the part of the Board, whereas deferring did not involve any decision on the part of the Board. I was not myself able to appreciate the distinction, but I think it right to draw your attention to it as the learned counsel dwelt upon it. You must address yourselves to the object with which this question was addressed—tell us if in relation to your application to some other insurance company there has not been a hitch or delay in granting which throws a little suspicion upon the eligibility of the risk you are offering. In relation to that you will be good enough to tell me whether or not it is the fact that the application of this gentleman for insurance with the Reliance was or was not postponed. That is the first question I will ask you.

The other question relates to the Globe, and as to the Globe the defendants say that was a case in which the Globe declined. Now, in relation to the Globe, counsel for the plaintiff makes two contentions. He says in the first instance, this question only applies to cases in which the person whose life is to be insured has himself applied for the life insurance; and he says in the case of the Globe he did not himself apply for the life insurance. Next, he says that the true meaning of the question, and the true meaning of the answer to the question is: I do not know that I have been refused or declined. Those are the two contentions which the learned counsel, as I understand him, submitted as regards the Globe. Now the state of the case is this. You will recollect in the case of the Globe there are two proposals; but it is only with the second of those we have to do in this matter. There was a proposal in May 1892, which was a proposal by A. J. Monson for an insurance in the sum of £15,000 on the life of Cecil Hambrough. Nothing was done in relation to that. As far as I recollect, that insurance has no bearing on the question now discussed. The circumstances of the other proposal in July 1893 are as follow. A long time before, application had been made by Monson to Wardle, who was the agent of the Globe at Leeds, for the purpose of insuring the life of Cecil Hambrough. In the earlier stages of these communications the insurance was proposed to be for the benefit of Monson, and in August 1892 Monson became bankrupt. Accordingly, when the question of re-insuring the life of Cecil Hambrough

Lord Russell, L.C.J.—*continued.* crops up again in July 1893, the person put forward as then interested in the life is not Monson, but Monson's wife, and a correspondence ensued between Wardle and Monson, to which reference has been made. It is therefore quite true that the application in July, 1893, was an application to insure for the benefit of Mrs. Monson. Here is the document of the 24th of July, 1893, signed "Agnes Maud Monson", and then she describes what her desire is for insuring his life: "And I declare", she says, "that I have an interest in the life of the said Windsor Dudley Cecil Hambrough to the extent of £50,000." It is not unimportant, however, to observe that the witness to this proposal—the form of the Globe company, differing in that respect from other companies whose papers one has seen, apparently requiring a witness to the proposal, and accordingly the proposal is witnessed by Cecil Hambrough himself. In addition to that he visits the office of the Globe at Glasgow, there sees Dr. Turner and Dr. Finlayson, and gives them a detailed statement with a view to the carrying out of what was undoubtedly in form, and probably intended to be in substance, the application and proposal of Mrs. Monson to insure his life. You will be good enough to tell me what you say of this. The point that is made, as you understand, is that this question only relates to where "I" have applied. Do you or do you not think that, if the application is made by Mrs. Monson to insure in her interest, that where that application is put forward in the proposal which he recognized by witnessing it, and which he aids the effecting of by submitting himself to the medical officers for examinations—do you or do you not think that that is a case in which he may be said to have applied? I put this question to you, because the learned counsel has more than once pointed out he considers these are matters for you to answer. About the substance of the thing there can be no manner of doubt; but this is a matter which ought to be strictly construed. I shall express my own opinion about it when I conceive the proper time has come, but I wish your opinion about it meanwhile. Is it an application by Cecil Hambrough, if the proposal is put forward and purports to be for the benefit of Mrs. Monson, his part in connection with it being recognizing that proposal by witnessing it and helping to effect the object of the insurance by submitting himself to be examined by the medical men?

The next point which is made by the plaintiff's counsel on this point is that this answer must be judged of (and therein he is quite right, as it seems to me) by reference to the character of the question, and he contends, looking to the character of the question, that all that is meant to be answered is what the man knows at the time he is called upon to give the answer. I myself cannot

concur. He is asked a question: Has any application of your own for insurance ever been declined? He may say: So far as I know, no. He may say: To the best of my knowledge I do not think so. But if he undertakes to say positively No, then, if the fact were otherwise, he has stated what is not true, and it does not seem to me that there is any room whatever for argument on that part of the case. Therefore you will, as regards this question of the Globe, tell me, first, whether it is in your judgment, taking a fair view of the question and answer, a case at all in which he could be said to have applied for insurance to the Globe; and, next, whether the insurance was declined by the Globe in fact. Well, the latter fact is beyond dispute. The state of the case is this. This proposal for £50,000 coming forward, demand was made for particulars justifying the interest which Mrs. Monson professed to have had in the life of young Cecil Hambrough, a demand all the more justifiable because, as their agent, Mr. Wardle, at Leeds, knew, a much smaller extent of interest had been stated in the life of Cecil Hambrough in the previous year, and that interest had been stated to be not the interest of Mrs. Monson, but of Monson the husband. Then come vague answers, entirely unsatisfactory answers, as to this interest, and thereupon the company decisively, on the 1st of August, declined the policy, and information, as I have already told you, of that declining of the policy reached Mr. Monson at Glasgow upon the 2nd of August.

Now, gentlemen, in my judgment, these questions which I have up to this put before you are the most important questions upon the so-called warranty upon which the defendants rely, namely, the warranty of the statements in the proposal. There is not now put forward the mis-statement that E. M. Hiron was one of his intimate friends. It would undoubtedly have been more candid when Mr. and Mrs. Monson's attention was called to the fact that the insurance company was under the impression that E. M. Hiron was a gentleman who was addressed as esquire to see that the company was set right on that; but it is a matter to which I should not advise you to attach very much importance.

In connection with the question as to the Globe and how far young Cecil Hambrough could have been said to be a party applying to the Globe Insurance, there was one other matter which I do not think I mentioned. In nearly all these insurances, whether in the name of Cecil Hambrough or of Mrs. Monson or of Mr. Monson, Mr. Monson had correspondence with the insurance company, but in relation to the Globe there is a letter from Mr. Hambrough. I have already drawn your attention to his signature in the proposal and also to the report of the

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medical examiners: but on the 31st of July, 1893, he writes this letter from Ardlamont, addressed to the manager of the Liverpool, London and Globe: "I am requested by Mrs. Agnes Maud Monson to write and inform you that she has an interest on my life to the extent of £26,000. I have given her an undertaking under which I have agreed to pay her this sum after my attaining twenty-one if I should live until then—Yours truly, W. D. C. Hambrough." I think that exhausts the story of his connection with that policy.

The remaining point is upon the question of warranty. I was telling you that the answers given by the person about to be insured to the medical examiners are also made a part of the proposal and part of the contract, and it is said that an unfavourable opinion upon the insurableness of the life of Cecil Hambrough was given, and therefore the statement which he made, and is proved to have made, to Dr. Barr and Dr. Broadfoot, namely, that no such unfavourable opinion ever was given, was untrue. The question is: Has any unfavourable opinion upon the insurableness of your life ever been given by a physician; and the answer is: No.—The same to Dr. Broadfoot.

Now the defendants say that it is not true, because Dr. Ogle had expressed such an opinion, and they go on further to say that when this answer was given by Cecil Hambrough, and before it was given, he knew that that opinion had been expressed. They suggest that, because he referred to what took place as to the Reliance in the answer he gave to the Globe Insurance question. It is a little involved. I do not know if you follow it, but it involves three different insurances. First the insurance here in question—the answer to Dr. Barr and Dr. Broadfoot, the answer or report by Dr. Ogle in the case of the Reliance, and that answer or report of Dr. Ogle referred to by the assured himself.

I will read the words in the medical reports of the Globe, which I think is better, in order to be quite clear. There are four questions, and they are these: "1.—Have you been previously proposed for insurance? —Yes. 2.—If so, in what office or offices and when? Reliance about three years ago and British Empire last year. 3.—Were you accepted at the ordinary or at an increased rate of premium? Proposal deferred by Reliance on account of albumen being seen in the urine. Accepted by British Empire at ordinary premium. 4.—Have you ever been declined by any office, if so by what office or offices, and when?—No." This needs a little particularity of examination. In the first instance I believe the fact is that there is no evidence that young Hambrough ever saw Dr. Ogle's report, but the fact that there was a delay in dealing with the Reliance, and that ultimately nothing

Lord Russell, L.C.J.—*continued.*

was done, and that it was deferred was obviously brought to his attention, and obviously he did learn somehow or other, because he here states that the proposal was deferred by the Reliance on account of the albumen being seen, and this statement has a very important bearing on the first of the points which I have already disposed of. The question here put is not whether the proposal was deferred by the Reliance, but whether any unfavourable opinion upon the insurableness of his life had ever been given by a physician, if so, state full particulars. Then you have to ask yourselves this question. He has not seen Dr. Ogle's report; there is no evidence that he had, but he does know that the matter of the Reliance has not gone through. He knows it has been deferred, and he knows it has been deferred on account of albumen being seen in his urine. Then you must say whether or not in your judgment that is or is not an unfavourable opinion upon the insurableness of his life given by a physician. That is the next question. Now, gentlemen, I will leave the question of so-called warranty altogether.

But the defendants further say, apart from this question of warranty, they have been induced to enter into this contract by false and fraudulent misrepresentations by someone for whose misrepresentations the assured is responsible. Now the considerations applying to this part of the case are entirely different from the considerations as to mis-statements in the proposal, and so far as mis-statements in the proposal are concerned, in one view it is a matter of law at all events. It is quite immaterial whether those mis-statements are made with a fraudulent intent or not—it is quite immaterial whether they are made innocently or ignorantly. If they were made then they have a certain definite legal effect; but as regards fraudulent misrepresentations as to matters which are not embodied in the contract, it is then the defendants, relying upon them to escape liability upon a contract, have to establish, first, that the representations were in fact made; that the representations were false and fraudulent; that they were material to the subject-matter; and that they influenced the party to whom they were made in the matter to which they related. Therefore, the question is a matter of fact.

Now, upon this part of the case there does not seem to be any difference between the statements of the learned counsel for the plaintiff and for the defendants. The main contests on the part of the learned counsel as regards this part of the case were two. First, that these false representations were not made by any person for whom Cecil Hambrough was responsible; next, that they were not material, and did not influence the action of the defendants. That they were made,

Lord Russell, L.C.J.—*continued.* therefore, seems to be conceded by counsel for the plaintiff, and, indeed, it is not a concession of very much importance. This part of the case renders it necessary that I should say a word or two more than I have yet said as to the circumstances which led up to the insurance, especially in order to determine what is the true view to take of the position of Mr. Monson, and the true view to take of his real relation, as regards the effect of this insurance, to young Hambrough. I have pointed out to you the number of attempts that were made in relation to the insurance of this young man, and I have pointed out to you that, except in the case of the earlier two of those insurances, with everyone of them Mr. Monson had direct concern either as being the person corresponding with the insurance company, or as the person who was to be the beneficiary under the policy, or as the person, referee, or friend, for the purpose of answering questions relating to the intending assured, and in some cases his connection was based upon more than one of those positions. Now, in July, an attempt was made to get the Liverpool, London and Globe to insure this young man's life for £50,000, Mrs. Monson to be the beneficiary of that insurance. The amount was reduced subsequently to £26,000, but the statement as to interest being entirely unsatisfactory the Liverpool, London and Globe would have nothing whatever to do with the matter, and you may think probably that the action of the Liverpool and London and Globe is not without its value in judging of the materiality of the statements made as to the object with which the insurance was intended to be effected. That being so, Mr. Monson goes to Glasgow, he visits the Globe office, he visits the office also of the defendant company, the Mutual. At the Globe he is told that the decision of the directors will not be sent there, but to the office from which the proposal had emanated, namely, the Leeds office. The suggestion was made to him: here is a telegraph form, if you choose to telegraph, you may learn in that way, and thereupon he sends the telegram, which has been produced, in which he, Monson, telegraphs in the name of the Globe to the Globe office in Liverpool, and the answer comes back to the Globe Office in Glasgow: "Declined." We know now the declination was the act and resolution of the Board on the evening before, namely, 1st August. In that state of things he goes back to the defendant's office, the Mutual. He there makes a series of statements, all of which appear to be false, namely, that Hambrough is about to purchase the estate of Ardlamont; that Mrs. Monson is going to advance a considerable sum towards the purchase of that estate; that young Hambrough is a man who at twenty-one will succeed to £200,000; and various statements of

that kind, every one of those statements being false. Were they fraudulent? You cannot doubt that they were fraudulent. Were they material? Counsel for the plaintiff says it is moonshine to suppose that those statements as to object or as to the circumstances of the person effecting the insurance are material; that the only thing an insurance company acts upon or ought to regard or will regard is the value of the life that is offered; that it is an insurance of a life, and that object, position, and circumstances—all these things—are not material, and do not and ought not to affect the judgment of an intending insurer. But you must consider whether that will for a moment bear the test of examination. If a rustic as healthy as you please walks into an insurance office, especially if he is accompanied by a friend who says that he wants an insurance to be made for his benefit, consider whether it is not most material. There are risks which people will be slow to put into formal language, but which are nevertheless such as will occur to men's minds under such circumstances. Is it not likely that the friend of the rustic would be asked, why do you want to insure his life in £20,000, what is he to you, what interest have you in his life? That was exactly the course, in somewhat different circumstances, which the Liverpool, London and Globe took, because, as the learned counsel for the plaintiff pointed out yesterday, except for the suggestion of Dr. Ogle, there seems to be an unanimity of opinion, judging by his apparent health and his constitution upon examination, that this young man had in the natural order of events a long life before him; and yet, though those were the reports which the Liverpool, London and Globe had before them they declined this insurance, and declined it because there was no satisfactory account given of the interest which it was pretended was to be covered by that insurance. The fact that the Liverpool, London and Globe acted in that way is not to determine your judgment in this matter. It certainly is a proper matter to call to your attention when it is suggested that it is out of all reason to suppose that directors of insurance companies would not regard anything pertaining to the object or circumstances of the person to be insured, or would have only regard to the circumstances of the value of his life judged as a man.

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Now the next thing to consider is, if you believe the statements were falsely and fraudulently made, and were material statements, then did they influence the action of the assured? Gentlemen, I strongly advise you, if you arrive at the conclusion that the statements were false and fraudulent, and that those statements were material, that you should have very

Lord Russell, L.C.J.—*continued.*

little difficulty indeed in arriving at the third conclusion, that they did influence those to whom they were addressed. People cannot analyze after the event the particular things which have and which have not a direct sensible operation on their minds, so that they are able to explain. The question is, would you, if you were in the position of an insurer carrying on such business as is ordinarily and reasonably carried on, not be influenced by considerations as to the object with which the insurance was effected, and as to the interest that the person for whose benefit it was to be effected had in that life, because this is one of the peculiarities of this case. So far as the evidence goes, nothing can be clearer than this, that the intention was that the insurance, if it could have been done, would have been carried out and effected in the name of Mrs. Monson, and not of Cecil Hambrough at all. That was the suggestion originally made by Mr. Monson to Mr. McLean or Mr. Herbert, I forget which. I therefore repeat what I have said, that if you come to the conclusion that these statements were false and fraudulent, and were, looking to the circumstances of this case, material, and that they would operate on the mind of an ordinary reasonable man either in declining the risk or in moderating or modifying the terms upon which he would accept it; then, if you arrive at those two conclusions, you should have very little difficulty in arriving at the concluding one, that these transactions were influenced by these false and fraudulent statements made to influence them, and addressed to a material matter.

Now, gentlemen, the remaining question is this: so far as the statements to the insurance agents, McLean and Herbert, were the statements of Monson, is this policy to be affected by those statements? It is said for the plaintiff, and I think rightly said, that it has to be shown that Monson was acting in effecting this policy for Cecil Hambrough, or in conjunction with Cecil Hambrough, and with his acquiescence. Now, was he? Here, again, it is important that I should recur to some of the very extraordinary circumstances of this case. I have told you the story of his previous attempts at insurance. You know the position in which young Hambrough stood at this time to the Monsons; you know the impecunious condition to which from that correspondence it is apparent they were reduced, and you must ask yourselves whether or not, whatever the reason was, the position of things was not such that young Hambrough was willing to lend himself in any way in which he was required—to lend himself for the purpose of effecting insurances at the instance of or for the purposes or interest of the Monsons.

Lord Russell, L.C.J.—*continued.*

It is not possible to blink this consideration at all. I find it impossible to treat this as an insurance effected for Cecil Hambrough's benefit; I find it quite impossible to ask you to consider it disconnected with the earlier history of the connection with these people, because if you arrive at the conclusion that the position of things was such that this young man was a mere creature in the hands of Monson and his wife, ready to sign a proposal in his own name, or to back up a proposal in his own name, or in her name, then you must say whether he was not an acquiescing party putting forward, or recognizing Monson when he put himself forward in effecting this insurance. If so, statements made by Monson ought in good sense, and I believe in point of law, to affect this insurance, if they were false and fraudulent, and if they were material and calculated to influence and did influence the insurers. Therefore, the question I will ask you as to this remaining point is this: as regards the statements of Monson, first of all was Monson the party who substantially effected this insurance, and in the next place did he act in effecting this insurance with the knowledge and acquiescence of Hambrough? I say nothing about agency, which may be actually misleading. Did he, Monson, in effecting this insurance, act with the acquiescence and knowledge of Hambrough? If he did, it seems to me impossible that a policy, if you should arrive at the conclusion that it was obtained, induced by false and fraudulent statements, should be allowed to stand on the mere suggestion (for which there is no warranty on the facts of this case) that this was an insurance for the benefit of Cecil Hambrough. It is as clear as anything can be in this case that this young man was lending himself to any scheme that these persons suggested, not necessarily with a guilty or fraudulent knowledge on his part, for he was as so much putty in their hands. Whether the insurance was in his or her or Monson's name, he was equally ready to do as his friends or supposed friends asked him. These are all the questions I shall submit to you at present, gentlemen. You will be good enough to give me your answers to them.

A Juror: Your Lordship mentioned something with reference to agency. Would the fact of Monson being paid or having promised to be paid 15 per-cent commission constitute him an agent of the company?

The Lord Chief Justice: No.

The Verdict.

The jury retired and returned with the following verdict.

(1) Was there any untrue statement in the proposal upon the point of the policy being declined or postponed? *We find that the application was postponed.*

The Verdict—*continued.*

(2) Did he apply to the Globe? *We find that he did apply to the Globe.*

(3) Was he declined? *We find he was.*

(4) Was he aware of any unfavourable opinion having been expressed by any physician as to the insurability of his life? *We find he was as to the matter of albumen.*

(5) We divide this question into three heads.

(*a*) Were false or fraudulent statements made? *We say "Yes."*

(*b*) Were they material?—*Yes.*

(*c*) Did they influence the insurers?—*Yes.*

(6) Was Monson the party substantially affecting the insurance on Cecil Hambrough's life, and did he act with the knowledge and acquiescence of Cecil Hambrough?—*Yes.*

Judgment of Lord Russell, L.C.J.

The Lord Chief Justice then delivered Judgment in the following terms:

The Lord Chief Justice: Then it may be desirable for the jury to know that I agree in the finding of the jury, and it now becomes my duty to say what I conceive to be the legal effect of those findings so far as the proposal or warranty or condition is concerned. In my judgment the proposal was made the basis of the contract. The legal effect of the express warranty of the truth of the statement of the proposer is this, that if any one of those statements is untrue the policy cannot be enforced by anyone. In my judgment it is immaterial in this case to consider whether the truth of those statements is a condition precedent to the existence of any contract at all, or whether the untruth of such statements prevents the contract or policy being enforceable. I say that that is immaterial, because the only practical effect of deciding that no contract at all existed, would be to make the premium paid on the occasion of the insurance recoverable unless there were a stipulation in the contract to the contrary, namely, as is not an unusual stipulation, that it should be forfeited. But if it be necessary to decide the point, I should say that there was no contract, that the risk never attached, that it was a void contract *ab initio*. I am of opinion, and I decide, that the policy is not enforceable by reason of the untruth of the statements in the proposal. I therefore give judgment for the defendants.

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